NSRL Time of Flight Study using Carbon Ions at 120 MeV/n

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A Carbon ion beam was prepared at an energy of about 120 MeV/n. The Large Binary Filter was used to measure a Bragg peak for the beam. The results in Figure 1 show a range of 332.5 mm of High Density Polyethylene obtained with QC3 located at the 92 inch marker on the NSRL rails. Correcting for half of QC3 (-0.4 mm H_2O equivalent) yields a kinetic energy of 117.6 MeV/amu at 92" according to the SRIM dE/dx calculator.

The Bragg peak beam energy was compared to a Time-of-Flight beam energy measurement.
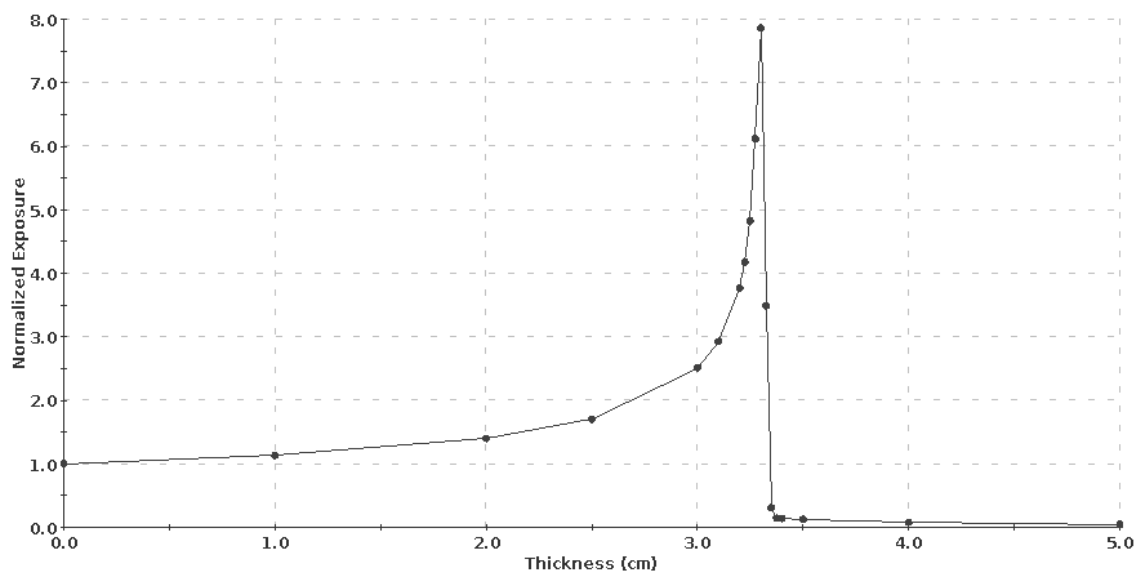

Figure 1: Bragg peak measurement of the Carbon ion beam with the Large Binary Filter.

Req. (cm)	Deliv. (cm)	H2O (mm)	Ring 1	Ring 2	Ring 3	Ring 4	Ring 5	Ring 6	Ring 7	Ring 8	IC
0.0	0.0	0.000	11.519	9.325	5.541	2.336	0.630	0.098	0.014	0.005	1.000
1.0	1.0	10.245	13.120	10.630	6.261	2.614	0.708	0.114	0.019	0.007	1.132
2.0	2.0	20.488	16.098	13.068	7.708	3.232	0.892	0.153	0.028	0.011	1.400
2.5	2.5	25.630	19.419	15.787	9.338	3.934	1.096	0.193	0.036	0.015	1.700
3.0	3.0	30.733	28.578	23.284	13.789	5.822	1.629	0.285	0.051	0.019	2.511
3.1	3.1	31.635	33.292	27.149	16.082	6.792	1.896	0.326	0.054	0.020	2.925
3.2	3.2	32.869	41.939	34.477	20.627	8.799	2.515	0.453	0.077	0.029	3.765
3.225	3.225	33.057	46.667	38.376	22.923	9.744	2.768	0.488	0.078	0.028	4.174
3.25	3.25	33.273	53.815	44.384	26.545	11.276	3.194	0.550	0.081	0.028	4.823
3.275	3.275	33.461	68.110	56.535	33.817	14.301	4.027	0.670	0.085	0.027	6.120
3.3	3.3	33.772	89.050	73.097	43.490	18.404	5.107	0.820	0.090	0.026	7.868
3.3	3.3	33.772	88.872	72.955	43.436	18.395	5.109	0.818	0.090	0.026	7.859
3.325	3.325	33.959	42.479	32.103	19.000	8.235	2.227	0.375	0.062	0.025	3.486
3.35	3.35	34.176	3.497	2.399	1.446	0.699	0.259	0.098	0.046	0.023	0.309

Table 1: Data for Figure 1.

Two scintillators were placed on the beamline. The first was a 1 cm^2 trigger scintillator that provided the TDC-START, placed upstream, at approximately 80" on the rails. The second was 10 cm^2 ToF scintillator that provided the TDC-STOP. It was placed at 2 locations: upstream at 115" and downstream at 242" for a total flight distance of 127" = 322.6 +/- 0.1 cm.

The TDC-START triggered the DAQ and started the TDC clock. The TDC-STOP recorded the stop time, with the change in stop times representing the time of flight over the 3.2258 meters. The TDC had a factory calibration of 35 ps/channel. We checked the calibration with the precision pulser over the same TDC range. The pulser moved from 35 ns (channel 583) to 155 ns (channel 3880) for a difference of 120 ns (3297 channel) for a calibration of (120ns/3297ch) 36.4 ps/channel.

The TDC-STOP pulse moved from channel 658 to channel 1377.8 for 719.8 channels, representing 26.13 ns for the time of flight.

This corresponds to a velocity of 3.2258 m/26.13 ns = 1.234×10^8 m/s or $\beta = 0.4118$. The kinetic energy for a carbon ion at this velocity is $K = (\gamma-1)m$ where is $\gamma = (1-\beta^2)^{-1/2}$ and m is the atomic mass unit of 931.49 MeV. This comes out to be 90.68 MeV/amu.

This represents the average energy of the C ion as it travels from the upstream to downstream scintillator, i.e. 115" to 242". Energy loss in air is quite small for C ions at this energy, with the expected energy loss for a 100 MeV/amu C ion through 3.2258 m is 7.57 MeV/amu. The energy loss will be treated linearly, with 90.68 MeV/amu representing the energy at the midpoint between the two scintillators. Correcting for the energy loss in 1.6129 m of air, plus the transit from the location of the QC3 chamber where the Bragg peak was measured (115" – 92" = 23", 0.5842 m) for a total transit in air of 2.1971 m yields a C ion energy of 95.90 MeV/amu.

We need to account for energy loss in the upstream scintillator as well. Its dimensions are $1 \times 1 \times 0.5$ cm^3, plus the light tight tube and black tape.   The entire material stack-up is difficult to estimate accurately, so we tried measuring it using a Bragg curve technique instead.

The beam tuned to a pencil beam (point-to-parallel) and collimated down to a spot smaller than the scintillator size, $< 1 \times 1$ cm^2 using the Tungsten collimator. We measured the polyethylene thickness required to get to the Bragg peak using the small binary filter with the added 25.6 mm panel in place.
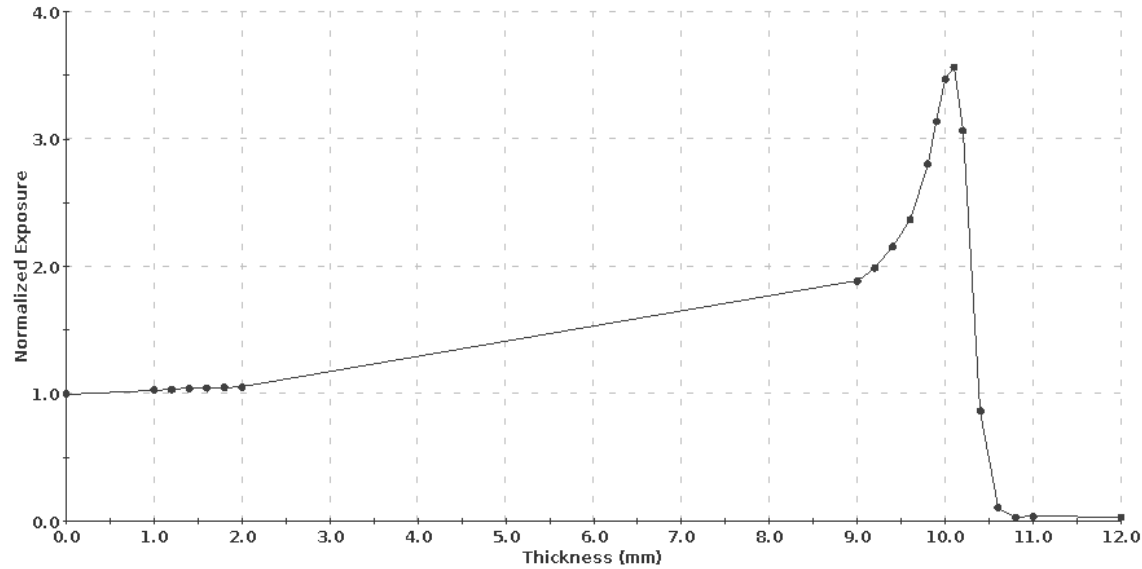
Figure 2: Bragg peak measurement for a 1 cm2 beam spot through the Tungsten collimator.

Peak location is (10.1 + 25.6 =) 35.7 mm of non-high density poly. This measurement was taken with QC3 at 55" rather than the 92" of the previous Bragg curve. Calculating the C ion energy using this Bragg peak yields 119.49 MeV/amu @ 55".

The Bragg peak measurement was repeated after placing the TDC-START scintillator upstream of the collimator with the scintillator completely covering the hole in the collimator.
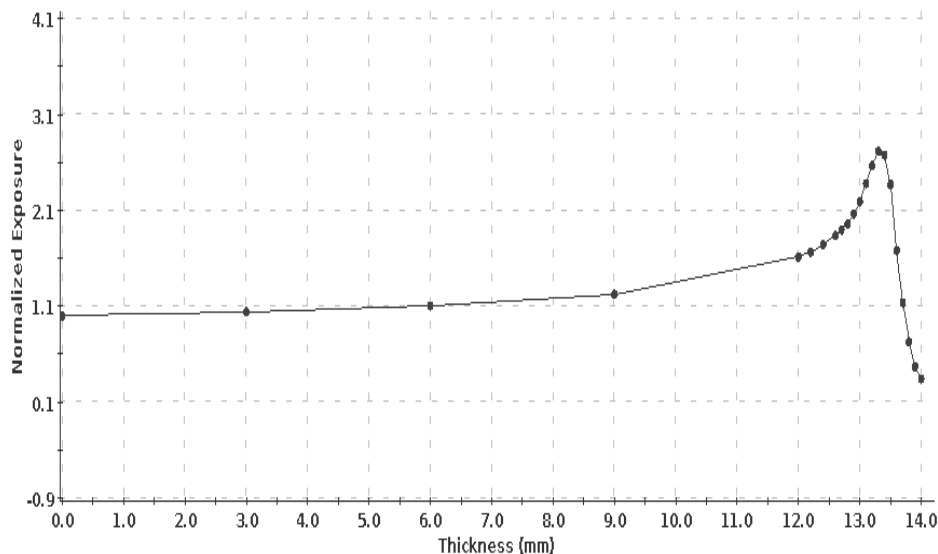

Figure 3: Bragg peak measurement through the TDC_Start scintillator.

The Bragg peak is located at (13.3 + 12.8 =) 26.1 mm. This shows that the scintillator and all wrapping material represents 9.6 mm of polyethylene equivalent. Using this to correct for energy loss in the scintillator yields 115.5 MeV/amu at 92". This is to be compared to 117.6

MeV/amu measured using the Large Binary Filter. The discrepancy between the two energy measurements amounts to 2.1 MeV/amu, or the equivalent of 1.1 mm of poly.

Req. (mm)	Deliv. (cm)	H2O (mm)	Ring 1	Ring 2	Ring 3	Ring 4	Ring 5	Ring 6	Ring 7	Ring 8	IC
0.0	0.0	0.000	117.433	14.643	0.763	0.214	0.037	0.023	0.023	0.014	1.000
3.0	3.0	30.733	111.682	16.580	0.744	0.208	0.042	0.016	0.032	0.017	1.040
6.0	6.0	61.619	107.211	18.989	0.888	0.194	0.050	0.013	0.018	0.016	1.103
9.0	9.0	92.715	105.779	22.736	1.077	0.234	0.009	0.021	0.006	0.018	1.222
12.0	12.0	123.601	120.602	32.717	1.365	0.115	0.021	0.004	0.000	0.012	1.596
12.0	12.0	123.601	120.725	32.785	1.408	0.152	0.018	0.029	0.005	0.016	1.610
12.2	12.2	125.585	123.984	34.184	1.423	0.131	0.024	0.003	0.022	0.016	1.664
12.4	12.4	127.624	127.781	35.848	1.471	0.102	0.040	0.020	0.023	0.022	1.741
12.6	12.6	129.607	133.211	38.167	1.548	0.101	0.030	0.013	0.030	0.014	1.835
12.7	12.7	130.509	136.548	39.680	1.564	0.109	0.023	0.030	0.019	0.012	1.896
12.8	12.8	130.706	139.673	41.239	1.619	0.120	0.035	0.004	0.019	0.016	1.957
12.9	12.9	131.608	146.073	43.456	1.759	0.105	0.033	0.015	0.013	0.013	2.061
13.0	13.0	132.689	154.131	46.524	1.883	0.106	0.013	0.000	0.010	0.019	2.192
13.1	13.1	133.592	164.322	50.622	2.058	0.124	0.033	0.008	0.013	0.023	2.379
13.2	13.2	134.728	175.871	55.009	2.258	0.127	0.015	0.005	0.008	0.010	2.565
13.3	13.3	135.631	183.904	58.282	2.440	0.126	0.013	0.017	0.013	0.014	2.714
13.4	13.4	136.712	180.852	57.378	2.563	0.090	0.010	0.008	0.019	0.016	2.677
13.5	13.5	137.614	158.224	50.357	2.409	0.086	0.023	0.038	0.018	0.021	2.370
13.6	13.6	138.968	112.563	35.327	1.953	0.083	0.025	0.008	0.011	0.010	1.682
13.7	13.7	139.870	75.274	23.436	1.450	0.068	0.017	0.001	0.026	0.013	1.134
13.8	13.8	140.951	47.798	14.549	0.999	0.063	0.031	0.025	0.023	0.016	0.727
13.9	13.9	141.853	30.409	9.152	0.712	0.073	0.032	0.009	0.010	0.015	0.468
14.0	14.0	142.990	22.540	6.338	0.561	0.081	0.005	0.016	0.022	0.012	0.339

Table 2: Data from Bragg peak measurement in Figure 3.

Error Analysis

The kinetic energy K is defined as $K = (\gamma-1)m$ where is $\gamma = (1-\beta^2)^{-1/2}$. Note that the error on K is the same as the error on the total energy, $E = \gamma m$, because there is effectively no error on m.

$$E = \gamma m = m(1 - \beta^2)^{-1/2} = m \left(1 - \frac{L^2}{(cT\alpha)^2}\right)^{-\frac{1}{2}}$$

where L is the distance between the two TDC-STOP scintillator locations, T is the number of TDC channels between the two TDC-STOP values, α is the calibration constant for the TDC, and c is the speed of light.

$E(L,T,\alpha)$ is a function of the three variables, L, T, and α. The uncertainty in E can be determined from the uncertainty on each of the three measured quantities by looking at the partial derivatives.

In general, the uncertainty in E is given in terms of the independent variables, L, T, and α by:

$$\delta E^2 = \left(\frac{\partial E}{\partial L}\right)^2 \delta L^2 + \left(\frac{\partial E}{\partial T}\right)^2 \delta T^2 + \left(\frac{\partial E}{\partial \alpha}\right)^2 \delta \alpha^2$$

In spite of being taught never to differentiate in public…

$$\frac{\partial E}{\partial L} = m \left(-\frac{1}{2}\right)(1 - \beta^2)^{-\frac{3}{2}}(-2\beta)(1/cT\alpha) = m\gamma^3 L/(cT\alpha)^2 = \beta^2\gamma^2 E/L$$

or this can be written as:

$$\frac{\partial E}{E} = \beta^2 \gamma^2 \left(\frac{\partial L}{L}\right)$$

showing that the fractional uncertainty in the distance measurement is scaled by $\beta^2\gamma^2$ to give the fractional uncertainty in the energy.

The same calculation shows that the contributions to the uncertainty from T and α are also scaled by $\beta^2\gamma^2$ to contribute to the total energy uncertainty.

$$\frac{\partial E}{E} = \beta^2 \gamma^2 \left(\frac{\partial T}{T}\right)$$

$$\frac{\partial E}{E} = \beta^2 \gamma^2 \left(\frac{\partial \alpha}{\alpha}\right)$$

The estimate of uncertainty in the distance measurement, δL = 1 mm, so $\delta L/L = 3 \times 10^{-4}$.

The TDC calibration was obtained by driving the TDC with a precision pulser. The pulser was delayed in 20 ns steps from 0 to 120 ns, and the TDC response was fit to a Gaussian profile. An example of the pulser fit is shown below for a pulser delay of 120 ns. The fit gave a centroid to the peak at 3880 ± 1.2 channels. Similar fits were obtained for all 7 pulser settings. Figure 2 summarizes the data.
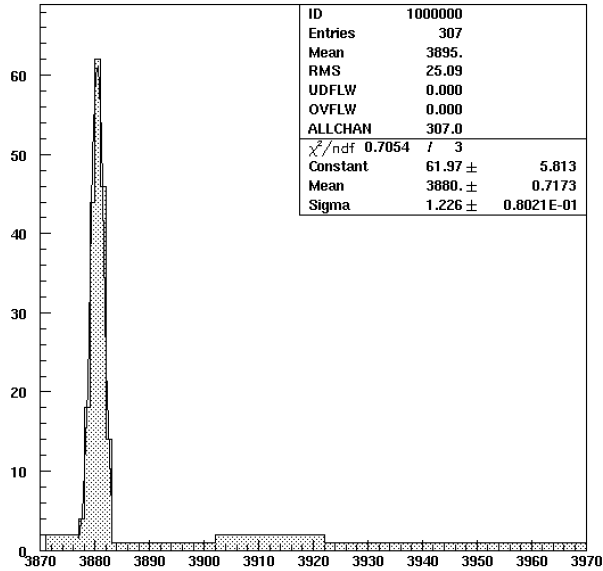

ID	1000000	
Entries	307	
Mean	3895.	
RMS	25.09	
UDFLW	0.000	
OVFLW	0.000	
ALLCHAN	307.0	
χ^2/ndf 0.7054	3	
Constant	61.97 ±	5.813
Mean	3880. ±	0.7173
Sigma	1.226 ±	0.8021E-01

Figure 4: TDC response to the precision pulser with a pulser delay of 120 ns.

Time	TDC Counts	σ	Linear fit	parameters	Δ	Δ²/σ²
0	583.70	0.96	586.10	586.10	-2.40	6.27
20	1134.00	1.50	1137.03	36.30	-3.03	4.07
40	1689.00	1.20	1687.95		1.05	0.76
60	2243.40	1.50	2238.87		4.53	9.10
80	2797.00	1.30	2789.80		7.20	30.69
100	3349.00	1.30	3340.72		8.28	40.54
120	3880.20	1.20	3891.65		-11.45	90.99
dof	5.00				χ²/dof	36.48

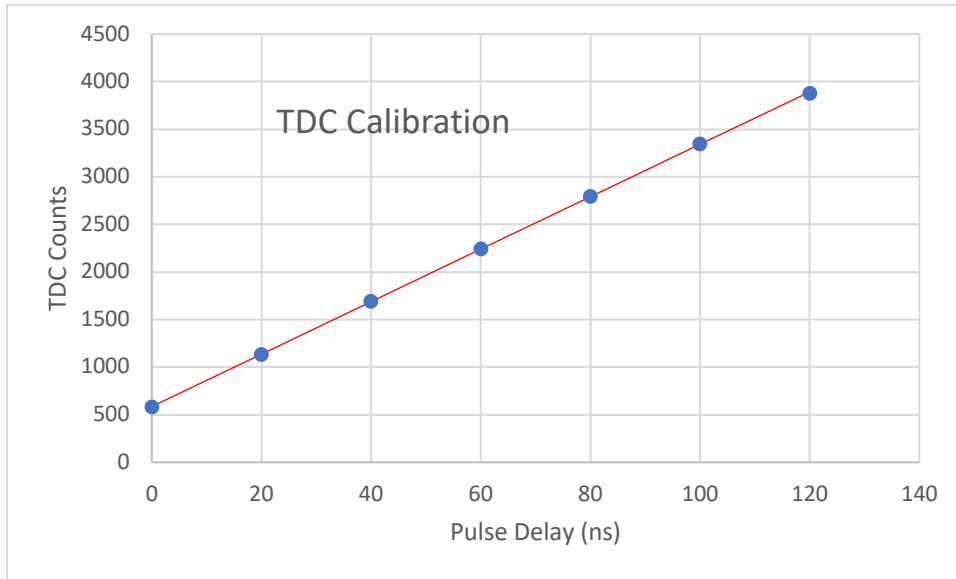Table 3: Precision Pulser calibration of TDC.

Figure 5: Fit results of TDC calibration.

Results of the linear fit to the calibration gives a TDC channel calibration of 36.30 ± 0.02 ps/channel, to compare to the manufacturer's 35 ps/ch. The uncertainty in the calibration constant came from a variation in the slope required to increase the χ^2 per degree of freedom by 1.0. This uncertainty, $\delta\alpha/\alpha = 5.5 \times 10^{-4}$, is comparable to the uncertainty on each of the fits to the pulser peaks.

Lastly, the uncertainty in the location of the Upstream and Downstream TDC distributions were determined by Gaussian fits to those two peaks. The fit results are $T_{up} = 658.0$ channels with $\sigma_{up} = 4.7$ channels, and $T_{dn} = 1377.8$ channels with $\sigma_{dn} = 5.0$ channels. The uncertainty on the TDC fit results is given by $\sigma/\sqrt{N}$ where N is the number of events used in the fit; 15000 for T_{up} and 6000 for T_{dn}. This gives $T_{up} = 658.0 \pm 0.04$ channels and $T_{dn} = 1377.8 \pm 0.07$ channels. The

uncertainty on the difference is $T_\Delta = T_{dn} - T_{up} = 719.8 \pm .08$. The fractional uncertainty is then given by $\delta T/T = 1 \times 10^{-4}$.

Summing the three contributions to the uncertainty in quadrature, and scaling with $\beta^2\gamma^2$ gives

$$\frac{\partial E}{E} = \beta^2\gamma^2 \left\{ \left(\frac{\partial\alpha}{\alpha}\right)^2 + \left(\frac{\partial T}{T}\right)^2 + \left(\frac{\partial L}{L}\right)^2 \right\}^{\frac{1}{2}} = 1.3 \times 10^{-4}$$

Or $\delta E = 1.3 \times 10^{-4} E = 0.12$ MeV. Since there is effectively no uncertainty on the Carbon ion rest mass, the energy uncertainty on the total energy is the same as the uncertainty on the kinetic energy, 0.12 MeV/amu.

This energy uncertainty is small compared to the discrepancy between the Bragg peak measurement and the Time-of-Flight measurement of 2.1 MeV/amu.

I suspect that the most significant discrepancy comes from the SRIM evaluation of the Carbon ion energy, combined with the Bragg peak measurement. Uncertainties in polyethylene density could be on the order of 1%. SRIM results should be reliable at the ~5% level[1].

With these considerations in mind, the agreement between the Bragg peak measurement and the Time-of-Flight measurement is rather good. The Time-of-Flight result may well be the better of the two.

It would be worthwhile repeating the Time-of-Flight measurement to see how reproducible they are. This would give confidence in the error analysis.

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Data for the TDC calibration and Time of Flight measurements are located in the files:
C_108_31_5_2022_004.ascii for the pulser run.
C_108_31_5_2002_007.ascii for the ToF run.
The directory is /home/cfsb/nsrl/data/nsrl.
Ntuples are in /home/cfsb/nsrl/daq/nt
With kumac files c4.kumac and c7.kumac in the same directory on the acn linux farm.

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[1] Private Conversation, J. F. Ziegler