BNL-222987-2022-TECH

EIC-ADD-TN-029

Proton-electron focusing in EIC ring cooler

S. Seletskiy

May 2022

Electron-Ion Collider

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC), Nuclear Physics (NP) (SC-26)

DISCLAIMER

Proton-electron focusing in EIC Ring Cooler

S. Seletskiy*, A. Fedotov, D. Kayran, J. Kewisch

May 6, 2022

1 Introduction

In this note we consider an effect of the proton-electron focusing on an emittance of e-bunches in the ring cooler.

It will be shown that for the optimized parameters of the ring cooler, when the proton and the electron beams are well-matched in the cooling section (CS), there is no significant emittance growth from a proton-electron space charge (SC) kick.

2 EIC Ring Cooler parameters

Table 1 shows parameters of the EIC Ring Cooler, which are important for the following considerations. The design parameters are taken from the preconceptual design [1], the optimized parameters are the result of a recent optimization [2, 3].

3 Space charge focusing

The space charge, either the self-SC or the space charge of a co-traveling beam in the cooling section, introduces an additional focusing. Since such a focusing is nonlinear, it can not be fully compensated by the linear optics. Hence, the SC causes filamentation of the transverse phase space of the e-bunches, which can be described as an emittance growth.

Let us consider a Gaussian, circularly symmetric transverse density distributions for both the e-bunch and the p-bunch. The equation of motion of an individual electron in this case is [4]:

*seletskiy@bnl.gov

Table 1: EIC cooler parameters

relativistic γ	293.1	
Number of protons per bunch (N_i)	$6.88 \cdot 10^{10}$	
rms p-bunch length (σ_{zp}) [cm]	6	
p-bunch peak current ($I_p = \frac{ecN_p}{\sqrt{2\pi}\sigma_{zp}}$) [A]	22	
p-bunch rms momentum spread ($\sigma_{\delta p}$)	$6.6 \cdot 10^{-4}$	
p-bunch emittance ($\varepsilon_{px,y}$) [nm]	9.6, 1.5	
Ring circumference [m]	449.1	
e-ring tunes $Q_{x,y}$	59.92, 59.85	
Cooling section length [m]	170	
rms e-bunch length (σ_{ze}) [cm]	12	
e-bunch rms momentum spread ($\sigma_{\delta e}$)	$8.9 \cdot 10^{-4}$	
	design	optimized
Number of electrons per bunch (N_e)	$3 \cdot 10^{11}$	$2 \cdot 10^{11}$
e-bunch peak current ($I_e = \frac{ecN_e}{\sqrt{2\pi}\sigma_{ep}}$) [A]	48.3	32.2
p-bunch CS beta function ($\beta_{px,py}$) [m]	200, 200	200, 1200
e-bunch CS beta function ($\beta_{ex,ey}$) [m]	170, 280	100, 100
e-bunch emittance ($\varepsilon_{x,y}$) [nm]	21, 18	15, 15
cooling times [1] ($\tau_{x,y,z}$) [min]	123, 61, 25	118, 118, 19 [2]

$$r'' = K_e \frac{1}{r}\left[\left(1 - e^{-\frac{r^2}{2\sigma_e^2}}\right) - \frac{I_p}{I_e}\left(1 - e^{-\frac{r^2}{2\sigma_p^2}}\right)\right] - \kappa r \tag{1}$$

Here generalized perviance $K_e = \frac{2I_e}{I_A(\gamma\beta)^3} \approx \frac{2I_e}{I_A\gamma^3}$ (in this formula γ and β are the relativistic factors, and we will assume $\beta = 1$ for further considerations), $I_A = \frac{4\pi\varepsilon_0 m_e c^3}{e} \approx 17 \cdot 10^3$ A is Alfven current, $\sigma \equiv \sigma_x = \sigma_y$, I_e and I_p are the respective peak currents, and κ represents the gradient of the linear focusing elements.

We are interested in a proton-electron focusing in the cooling section, so for this exercise we are dropping the self space charge term:

$$r'' = -K_e \frac{I_p}{I_e}\frac{1}{r}\left(1 - e^{-\frac{r^2}{2\sigma_p^2}}\right) - \kappa r \tag{2}$$

Notice, that a defocusing caused by the self SC of the e-bunches alleviates the p-e focusing in the cooling section to some extent. Therefore, Eq. (2)

[1]No redistribution of cooling decrements is assumed

[2]Loss of cooling due to a partial bunches' overlap is taken into account in calculations of $\tau_{x,y,z}$

overestimates the space charge effect on beam dynamics.

Rewriting Eq. (2) for x and y coordinates and we get:

$$\begin{cases} x'' = -K_p \frac{1}{x^2+y^2}\left(1-e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right)x - \kappa x \\ y'' = -K_p \frac{1}{x^2+y^2}\left(1-e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right)y - \kappa y \end{cases} \tag{3}$$

where $K_p = K_e I_p / I_e$.

Expanding Eq. (3) with respect to a small parameter $(x^2+y^2)/\sigma_p^2$ we get linearized equations of motion:

$$\begin{cases} x'' + \left(\kappa + \frac{K_p}{2\sigma_p^2}\right)x = 0 \\ y'' + \left(\kappa + \frac{K_p}{2\sigma_p^2}\right)y = 0 \end{cases} \tag{4}$$

4 Motion in Courant-Snyder coordinates

It is convenient to consider the effects on the bunch emittance using the Courant-Snyder coordinates (ξ, ζ):

$$\begin{aligned} \xi &= x/\sqrt{\beta} \\ \zeta &= x\alpha/\sqrt{\beta} + x'\sqrt{\beta} \end{aligned} \tag{5}$$

An independent variable in the (ξ, ζ)-coordinates is a betatron phase φ. A betatron motion in such a phase space is represented by a circle of the radius $\sqrt{2J}$, where an action $J = (\xi^2 + \zeta^2)/2$.

For the Gaussian bunch, the distribution function in the (J, φ) variables is given by:

$$f_{J,\varphi} = \frac{1}{2\pi\varepsilon} e^{-J/\varepsilon} \tag{6}$$

and the average value of the action is:

$$\langle J \rangle = \int_0^\infty \int_0^{2\pi} J f_{J,\varphi} \, d\varphi \, dJ = \varepsilon \tag{7}$$

One turn in the storage ring in the Courant-Snyder coordinates is represented by a simple rotation matrix:

$$\begin{pmatrix} \xi_{h,v1} \\ \zeta_{h,v1} \end{pmatrix} = \begin{pmatrix} \cos(2\pi\nu_{h,v}) & \sin(2\pi\nu_{h,v}) \\ -\sin(2\pi\nu_{h,v}) & \cos(2\pi\nu_{h,v}) \end{pmatrix} \cdot \begin{pmatrix} \xi_{h,v0} \\ \zeta_{h,v0} \end{pmatrix} \tag{8}$$

where ν is a fractional tune of the ring, and h, v indexes correspond to the horizontal and the vertical motion respectively.

We will assume that the p-e focusing in the CS is weak enough to be represented simply by an instantaneous change in the electron angles ($\delta x'$, $\delta y'$) after each turn in the storage ring. This assumption will be justified later. From Eq. (3) we get:

$$\begin{aligned} \delta x' &= -\tfrac{K_p L_{CS} x}{x^2+y^2}\left(1-e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right) \\ \delta y' &= -\tfrac{K_p L_{CS} y}{x^2+y^2}\left(1-e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right) \end{aligned} \tag{9}$$

where L_{CS} is the length of the cooling section.

The linear part of the SC focusing kick can be represented by a transfer matrix:

$$M = \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}, \quad k \equiv -\frac{K_p L_{CS}}{2\sigma_p^2} \tag{10}$$

Denoting the variables before the kick with an index 1 and after the kick with an index 2 we can write for the Twiss parameters:

$$\begin{aligned} \beta_2 &= \beta_1 \equiv \beta_{CS} \\ \alpha_2 &= -k\beta_1 + \alpha_1 \end{aligned} \tag{11}$$

where β_{CS} is an average e-beam beta-function in the cooling section

Then, from Eqs. (5) and (11) we get:

$$\begin{aligned} \xi_{h2} &= \xi_{h1} \\ \zeta_{h2} &= \zeta_{h1} + \sqrt{\beta_{x1}}(\delta x' - kx_1) \end{aligned} \tag{12}$$

An equation for the vertical motion is identical to Eq. (12).

If the SC kick was linear, then we would have $\delta x' = kx_1$, which would mean that $J_2 = J_1$ as it should be for the linear focusing.

Instead, we have a nonlinear kick, which gives us the following expression for $\delta\zeta = \zeta_2 - \zeta_1$:

$$\delta\zeta_h = \sqrt{\beta_{x1}}(\delta x' - kx_1) = -K_p L_{CS} x \sqrt{\beta_{x1}} \left(\frac{1-e^{-\frac{x_1^2+y_1^2}{2\sigma_p^2}}}{x_1^2+y_1^2} x_1 - \frac{x_1}{2\sigma_p^2} \right) \tag{13}$$

Again, an equation for $\delta\zeta_v$ is identical to Eq. (13).

Finally, substituting Eq. (5) into Eq. (13) , we get:

$$\delta\zeta_{h,v} = -K_p L_{CS}\left(\frac{1-e^{-\frac{\beta_{CS}}{2\sigma_p^2}(\xi_{h1}^2+\xi_{v1}^2)}}{\xi_{h1}^2+\xi_{v1}^2} - \frac{\beta_{CS}}{2\sigma_p^2}\right)\xi_{h,v1} \tag{14}$$

Under our assumptions, the motion of each electron in (ξ,ζ)-phase space is equivalent to the motion of a harmonic oscillator experiencing a weak nonlinear driving force:

$$\begin{cases} \xi'_{h,v} &= \zeta_{h,v} \\ \zeta'_{h,v} &= -\xi_{h,v} + \delta\zeta_{h,v}\sum_n \delta_D(\varphi_{h,v} - 2\pi Q_{h,v} n) \end{cases} \tag{15}$$

where δ_D is the Dirac delta function, and Q_h and Q_v are the full horizontal and vertical tunes of the storage ring.

The easiest and the fastest way to simulate the turn-by-turn motion of an electron is to apply the transformation (8) to coordinates of a particle from the previous turn and to add the kick (14) to the obtained result. Solving Eq. (15) numerically gives the same result, of course, but slows down the simulations.

5 Single pass effect

Before considering dynamics of an ensemble of nonlinear oscillators we will check that the SC effect is indeed small for a single pass through the cooling section.

The change in action due to a single nonlinear kick ($\delta J = J_2 - J_1$) is:

$$\delta J = \frac{\xi_1^2 + \zeta_1^2 + 2\zeta_1\delta\zeta + \delta\zeta^2}{2} - J_1 = J_1\sin(\varphi)\delta\zeta + \frac{\delta\zeta^2}{2} \tag{16}$$

Assuming that the beam distribution doesn't change over one pass through the CS, we get the change in the emittance ($\delta\varepsilon$) by substituting Eq. (16) into Eq. (7):

$$\delta\varepsilon = \int_0^\infty\int_0^{2\pi} f_{J,\varphi}(J\sin(\varphi)\delta\zeta + \delta\zeta^2/2)d\varphi dJ \tag{17}$$

To simplify further estimates we will drop the x-y coupling terms in Eq. (14):

$$\delta\zeta \approx -K_p L_{CS}\left(\frac{1-e^{-\frac{\beta_{CS}}{2\sigma_p^2}\xi^2}}{\xi} - \frac{\beta_{CS}\xi}{2\sigma_p^2}\right) \tag{18}$$

Finally, substituting Eq. (18) into Eq. (17) and noticing that $\xi = \sqrt{2J}\cos(\varphi)$ and that $\forall J$ the integral $\int\limits_0^{2\pi} f_{J,\varphi} J \sin(\varphi)\delta\zeta d\varphi = 0$, we get the following expression for the fractional emittance change over one pass through the CS:

$$\delta\varepsilon/\varepsilon = \frac{I_p^2 L_{CS}^2}{2\pi\varepsilon^2 I_A^2 \gamma^6} \int\limits_0^{\infty}\int\limits_0^{2\pi} \frac{e^{-J/\varepsilon}}{J} \frac{\left(1 - e^{-\frac{J\beta}{\sigma_p^2}\cos^2\varphi} - \frac{J\beta}{\sigma_p^2}\cos^2\varphi\right)^2}{\cos^2\varphi} d\varphi dJ \tag{19}$$

Introducing a dimensionless variable $j \equiv J/\varepsilon$ we can rewrite Eq. (19) in a little bit more symmetric form:

$$\begin{aligned} \delta\varepsilon/\varepsilon &= \frac{L_{CS}^2 I_p^2}{2\pi\varepsilon^2 I_A^2 \gamma^6} \Xi(\sigma_e/\sigma_p) \\ \Xi(d) &= \int\limits_0^{\infty}\int\limits_0^{2\pi} \frac{e^{-j}}{j\cos^2\varphi} \left(1 - e^{-d^2 j \cos^2\varphi} - d^2 j \cos^2\varphi\right)^2 d\varphi dj \end{aligned} \tag{20}$$

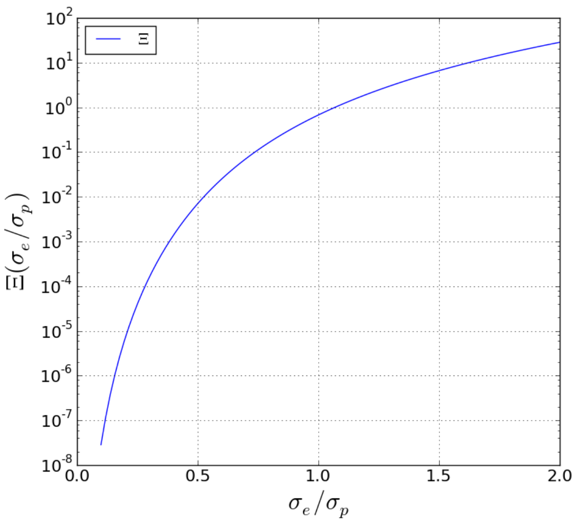

Figure 1: An overlap function Ξ.

Notice that the integral Ξ is simply a function of the ratio of the two beam sizes (σ_e/σ_p) in the cooling section. The plot in Fig. 1 demonstrates that this dependence is rather strong.

For the optimized parameters $\delta\varepsilon/\varepsilon \approx 6 \cdot 10^{-3}$. The effect of the proton-electron space charge kick on a single pass through the CS is small. This justifies our approach of small nonlinear excitations.

For the original design parameters we would have $\delta\varepsilon/\varepsilon \approx 1.6$, which would be a very strong effect on the emittance in a single pass through the cooling section.

6 Beam dynamics in presence of p-e focusing

Let us now consider the motion of a single electron in the storage ring. We will follow the simulations' recipe given at the end of the Section 4. Figure 2 shows a phase space trajectory for a particle with $J = \varepsilon$ tracked in the storage ring for 67k turns (or $\approx$ 0.1 s). For clarity, we also show a phase space trajectory of a particle with the same amplitude for the case of an undisturbed motion.

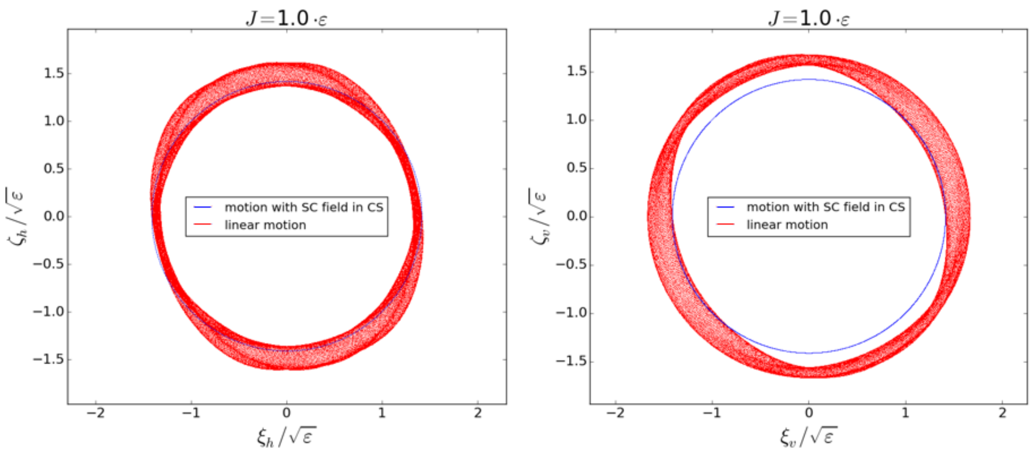

Figure 2: Phase space trajectory of an electron in the presence of the proton-electron focusing in the CS (red trace) and for the undisturbed motion (blue trace).

We can already see from Fig. 2 that for the optimized parameters the p-e focusing in the cooling section must not introduce any noticeable emittance growth. Tracking an ensemble of 10k particles for 20k turns, which is equivalent to $\approx$ 30 ms, confirms this conclusion (see Fig. 3).

7 Conclusion

We studied the effect of the proton-electron space charge focusing in the cooling section on the e-beam emittance in the EIC ring cooler.

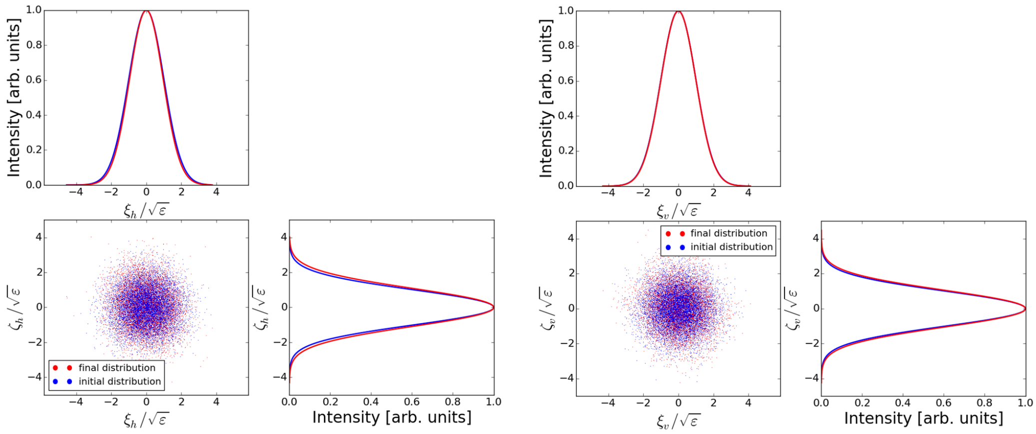

Figure 3: An ensemble of 10^4 particles tracked for 20k turns in the electron ring in the presence of the p-e focusing in the CS.

We conclude that for the optimized parameters the emittance growth associated with this effect is negligible.

8 Acknowledgments

We are grateful to Valeri Lebedev for inspiring these studies.

We would like to thank Michael Blaskiewicz for pointing out an importance of coupling between horizontal and vertical degrees of freedom for our simulations.

References

[1] H. Zhao, J. Kewisch, M. Blaskiewicz, and A. Fedotov, Ring-based electron cooler for high energy beam cooling, Phys. Rev. Accel. Beams 24, 043501 (2021).

[2] S. Seletskiy, Optimizing space charge and cooling in EIC Ring Cooler, presentation at EIC Ring Cooler Meeting series, BNL, February 17, 2022.

[3] K. Kewisch, Ring cooler cooling rates, presentation at EIC Ring Cooler Meeting series, BNL, March 17, 2022.

[4] S. Seletskiy, A. Fedotov, D. Kayran, Beam dynamics in non-magnetized electron cooler with strong hadron-electron focusing, BNL-216225-2020-TECH (2020).