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Proton-electron focusing in EIC ring cooler

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1 Introduction

In this note we consider an effect of the proton-electron focusing on an emittance of e-bunches in the ring cooler.

It will be shown that for the optimized parameters of the ring cooler, when the proton and the electron beams are well-matched in the cooling section (CS), there is no significant emittance growth from a proton-electron space charge (SC) kick.

2 EIC Ring Cooler parameters

Table 1 shows parameters of the EIC Ring Cooler, which are important for the following considerations. The design parameters are taken from the pre-conceptual design [1], the optimized parameters are the result of a recent optimization [2, 3].

3 Space charge focusing

The space charge, either the self-SC or the space charge of a co-traveling beam in the cooling section, introduces an additional focusing. Since such a focusing is nonlinear, it can not be fully compensated by the linear optics. Hence, the SC causes filamentation of the transverse phase space of the e-bunches, which can be described as an emittance growth.

Let us consider a Gaussian, circularly symmetric transverse density distributions for both the e-bunch and the p-bunch. The equation of motion of an individual electron in this case is [4]:

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Table 1: EIC cooler parameters

relativistic γ	293.1	
Number of protons per bunch (N_i)	$6.88 \cdot 10^{10}$	
rms p-bunch length (σ_{zp}) [cm]	6	
p-bunch peak current ($I_p = \frac{ecN_p}{\sqrt{2\pi}\sigma_{zp}}$) [A]	22	
p-bunch rms momentum spread ($\sigma_{\delta p}$)	$6.6 \cdot 10^{-4}$	
p-bunch emittance ($\varepsilon_{px,y}$) [nm]	9.6, 1.5	
Ring circumference [m]	449.1	
e-ring tunes $Q_{x,y}$	59.92, 59.85	
Cooling section length [m]	170	
rms e-bunch length (σ_{ze}) [cm]	12	
e-bunch rms momentum spread ($\sigma_{\delta e}$)	$8.9 \cdot 10^{-4}$	
	design	optimized
Number of electrons per bunch (N_e)	$3 \cdot 10^{11}$	$2 \cdot 10^{11}$
e-bunch peak current ($I_e = \frac{ecN_e}{\sqrt{2\pi}\sigma_{ep}}$) [A]	48.3	32.2
p-bunch CS beta function ($\beta_{px,py}$) [m]	200, 200	200, 1200
e-bunch CS beta function ($\beta_{ex,ey}$) [m]	170, 280	100, 100
e-bunch emittance ($\varepsilon_{x,y}$) [nm]	21, 18	15, 15
cooling times [1] ($\tau_{x,y,z}$) [min]	123, 61, 25	118, 118, 19 [2]

$$r'' = K_e \frac{1}{r}\left[\left(1 - e^{-\frac{r^2}{2\sigma_e^2}}\right) - \frac{I_p}{I_e}\left(1 - e^{-\frac{r^2}{2\sigma_p^2}}\right)\right] - \kappa r \tag{1}$$

Here generalized perviance $K_e = \frac{2I_e}{I_A(\gamma\beta)^3} \approx \frac{2I_e}{I_A\gamma^3}$ (in this formula γ and β are the relativistic factors, and we will assume $\beta = 1$ for further considerations), $I_A = \frac{4\pi\varepsilon_0 m_e c^3}{e} \approx 17 \cdot 10^3$ A is Alfven current, $\sigma \equiv \sigma_x = \sigma_y$, I_e and I_p are the respective peak currents, and κ represents the gradient of the linear focusing elements.

We are interested in a proton-electron focusing in the cooling section, so for this exercise we are dropping the self space charge term:

$$r'' = -K_e \frac{I_p}{I_e}\frac{1}{r}\left(1 - e^{-\frac{r^2}{2\sigma_p^2}}\right) - \kappa r \tag{2}$$

Notice, that a defocusing caused by the self SC of the e-bunches alleviates the p-e focusing in the cooling section to some extent. Therefore, Eq. (2)

[1] No redistribution of cooling decrements is assumed

[2] Loss of cooling due to a partial bunches' overlap is taken into account in calculations of $\tau_{x,y,z}$

overestimates the space charge effect on beam dynamics.

Rewriting Eq. (2) for x and y coordinates and we get:

$$\left\{ \begin{array}{lcl} x'' & = & -K_p \frac{1}{x^2+y^2} \left(1 - e^{-\frac{x^2+y^2}{2\sigma_p^2}} \right) x - \kappa x \\ y'' & = & -K_p \frac{1}{x^2+y^2} \left(1 - e^{-\frac{x^2+y^2}{2\sigma_p^2}} \right) y - \kappa y \end{array} \right. \tag{3}$$

where $K_p = K_e I_p / I_e$.

Expanding Eq. (3) with respect to a small parameter $(x^2 + y^2)/\sigma_p^2$ we get linearized equations of motion:

$$\left\{ \begin{array}{l} x'' + \left(\kappa + \frac{K_p}{2\sigma_p^2} \right) x = 0 \\ y'' + \left(\kappa + \frac{K_p}{2\sigma_p^2} \right) y = 0 \end{array} \right. \tag{4}$$

4 Motion in Courant-Snyder coordinates

It is convenient to consider the effects on the bunch emittance using the Courant-Snyder coordinates (ξ, ζ):

$$\begin{array}{lcl} \xi & = & x/\sqrt{\beta} \\ \zeta & = & x\alpha/\sqrt{\beta} + x'\sqrt{\beta} \end{array} \tag{5}$$

An independent variable in the (ξ, ζ)-coordinates is a betatron phase φ. A betatron motion in such a phase space is represented by a circle of the radius $\sqrt{2J}$, where an action $J = (\xi^2 + \zeta^2)/2$.

For the Gaussian bunch, the distribution function in the (J, φ) variables is given by:

$$f_{J,\varphi} = \frac{1}{2\pi\varepsilon} e^{-J/\varepsilon} \tag{6}$$

and the average value of the action is:

$$\langle J \rangle = \int_0^\infty \int_0^{2\pi} J f_{J,\varphi} d\varphi dJ = \varepsilon \tag{7}$$

One turn in the storage ring in the Courant-Snyder coordinates is represented by a simple rotation matrix:

$$\begin{pmatrix} \xi_{h,v1} \\ \zeta_{h,v1} \end{pmatrix} = \begin{pmatrix} \cos(2\pi\nu_{h,v}) & \sin(2\pi\nu_{h,v}) \\ -\sin(2\pi\nu_{h,v}) & \cos(2\pi\nu_{h,v}) \end{pmatrix} \cdot \begin{pmatrix} \xi_{h,v0} \\ \zeta_{h,v0} \end{pmatrix} \tag{8}$$

where ν is a fractional tune of the ring, and h, v indexes correspond to the horizontal and the vertical motion respectively.

We will assume that the p-e focusing in the CS is weak enough to be represented simply by an instantaneous change in the electron angles ($\delta x'$, $\delta y'$) after each turn in the storage ring. This assumption will be justified later. From Eq. (3) we get:

$$\begin{aligned} \delta x' &= -\tfrac{K_p L_{CS} x}{x^2+y^2}\left(1 - e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right) \\ \delta y' &= -\tfrac{K_p L_{CS} y}{x^2+y^2}\left(1 - e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right) \end{aligned} \tag{9}$$

where L_{CS} is the length of the cooling section.

The linear part of the SC focusing kick can be represented by a transfer matrix:

$$M = \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}, \quad k \equiv -\frac{K_p L_{CS}}{2\sigma_p^2} \tag{10}$$

Denoting the variables before the kick with an index 1 and after the kick with an index 2 we can write for the Twiss parameters:

$$\begin{aligned} \beta_2 &= \beta_1 \equiv \beta_{CS} \\ \alpha_2 &= -k\beta_1 + \alpha_1 \end{aligned} \tag{11}$$

where β_{CS} is an average e-beam beta-function in the cooling section

Then, from Eqs. (5) and (11) we get:

$$\begin{aligned} \xi_{h2} &= \xi_{h1} \\ \zeta_{h2} &= \zeta_{h1} + \sqrt{\beta_{x1}}(\delta x' - k x_1) \end{aligned} \tag{12}$$

An equation for the vertical motion is identical to Eq. (12).

If the SC kick was linear, then we would have $\delta x' = k x_1$, which would mean that $J_2 = J_1$ as it should be for the linear focusing.

Instead, we have a nonlinear kick, which gives us the following expression for $\delta\zeta = \zeta_2 - \zeta_1$:

$$\delta\zeta_h = \sqrt{\beta_{x1}}(\delta x' - k x_1) = -K_p L_{CS} x \sqrt{\beta_{x1}} \left(\frac{1 - e^{-\frac{x_1^2+y_1^2}{2\sigma_p^2}}}{x_1^2 + y_1^2} x_1 - \frac{x_1}{2\sigma_p^2} \right) \tag{13}$$

Again, an equation for $\delta\zeta_v$ is identical to Eq. (13).

Finally, substituting Eq. (5) into Eq. (13) , we get:

$$\delta\zeta_{h,v} = -K_p L_{CS} \left(\frac{1 - e^{-\frac{\beta_{CS}}{2\sigma_p^2}(\xi_{h1}^2+\xi_{v1}^2)}}{\xi_{h1}^2 + \xi_{v1}^2} - \frac{\beta_{CS}}{2\sigma_p^2} \right) \xi_{h,v1} \tag{14}$$

Under our assumptions, the motion of each electron in (ξ, ζ)-phase space is equivalent to the motion of a harmonic oscillator experiencing a weak nonlinear driving force:

$$\begin{cases} \xi'_{h,v} &= \zeta_{h,v} \\ \zeta'_{h,v} &= -\xi_{h,v} + \delta\zeta_{h,v} \sum\limits_n \delta_D(\varphi_{h,v} - 2\pi Q_{h,v} n) \end{cases} \tag{15}$$

where δ_D is the Dirac delta function, and Q_h and Q_v are the full horizontal and vertical tunes of the storage ring.

The easiest and the fastest way to simulate the turn-by-turn motion of an electron is to apply the transformation (8) to coordinates of a particle from the previous turn and to add the kick (14) to the obtained result. Solving Eq. (15) numerically gives the same result, of course, but slows down the simulations.

5 Single pass effect

Before considering dynamics of an ensemble of nonlinear oscillators we will check that the SC effect is indeed small for a single pass through the cooling section.

The change in action due to a single nonlinear kick ($\delta J = J_2 - J_1$) is:

$$\delta J = \frac{\xi_1^2 + \zeta_1^2 + 2\zeta_1\delta\zeta + \delta\zeta^2}{2} - J_1 = J_1 \sin(\varphi)\delta\zeta + \frac{\delta\zeta^2}{2} \tag{16}$$

Assuming that the beam distribution doesn't change over one pass through the CS, we get the change in the emittance ($\delta\varepsilon$) by substituting Eq. (16) into Eq. (7):

$$\delta\varepsilon = \int_0^\infty \int_0^{2\pi} f_{J,\varphi}(J\sin(\varphi)\delta\zeta + \delta\zeta^2/2)d\varphi dJ \tag{17}$$

To simplify further estimates we will drop the x-y coupling terms in Eq. (14):

$$\delta\zeta \approx -K_p L_{CS} \left(\frac{1 - e^{-\frac{\beta_{CS}}{2\sigma_p^2}\xi^2}}{\xi} - \frac{\beta_{CS}\xi}{2\sigma_p^2} \right) \tag{18}$$

Finally, substituting Eq. (18) into Eq. (17) and noticing that $\xi = \sqrt{2J}\cos(\varphi)$ and that $\forall J$ the integral $\int\limits_0^{2\pi} f_{J,\varphi} J \sin(\varphi)\delta\zeta d\varphi = 0$, we get the following expression for the fractional emittance change over one pass through the CS:

$$\delta\varepsilon/\varepsilon = \frac{I_p^2 L_{CS}^2}{2\pi\varepsilon^2 I_A^2\gamma^6}\int\limits_0^{\infty}\int\limits_0^{2\pi}\frac{e^{-J/\varepsilon}}{J}\frac{\left(1-e^{-\frac{J\beta}{\sigma_p^2}\cos^2\varphi}-\frac{J\beta}{\sigma_p^2}\cos^2\varphi\right)^2}{\cos^2\varphi}d\varphi dJ \qquad (19)$$

Introducing a dimensionless variable $j \equiv J/\varepsilon$ we can rewrite Eq. (19) in a little bit more symmetric form:

$$\begin{aligned} \delta\varepsilon/\varepsilon &= \frac{L_{CS}^2 I_p^2}{2\pi\varepsilon^2 I_A^2\gamma^6}\Xi(\sigma_e/\sigma_p) \\ \Xi(d) &= \int\limits_0^{\infty}\int\limits_0^{2\pi}\frac{e^{-j}}{j\cos^2\varphi}\left(1-e^{-d^2 j\cos^2\varphi}-d^2 j\cos^2\varphi\right)^2 d\varphi dj \end{aligned} \qquad (20)$$

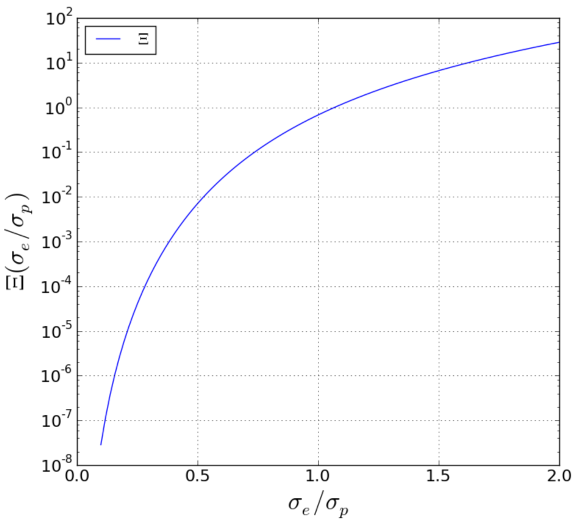

Figure 1: An overlap function Ξ.

Notice that the integral Ξ is simply a function of the ratio of the two beam sizes (σ_e/σ_p) in the cooling section. The plot in Fig. 1 demonstrates that this dependence is rather strong.

For the optimized parameters $\delta\varepsilon/\varepsilon \approx 6\cdot 10^{-3}$. The effect of the proton-electron space charge kick on a single pass through the CS is small. This justifies our approach of small nonlinear excitations.

For the original design parameters we would have $\delta\varepsilon/\varepsilon \approx 1.6$, which would be a very strong effect on the emittance in a single pass through the cooling section.

6 Beam dynamics in presence of p-e focusing

Let us now consider the motion of a single electron in the storage ring. We will follow the simulations' recipe given at the end of the Section 4. Figure 2 shows a phase space trajectory for a particle with $J = \varepsilon$ tracked in the storage ring for 67k turns (or $\approx$ 0.1 s). For clarity, we also show a phase space trajectory of a particle with the same amplitude for the case of an undisturbed motion.

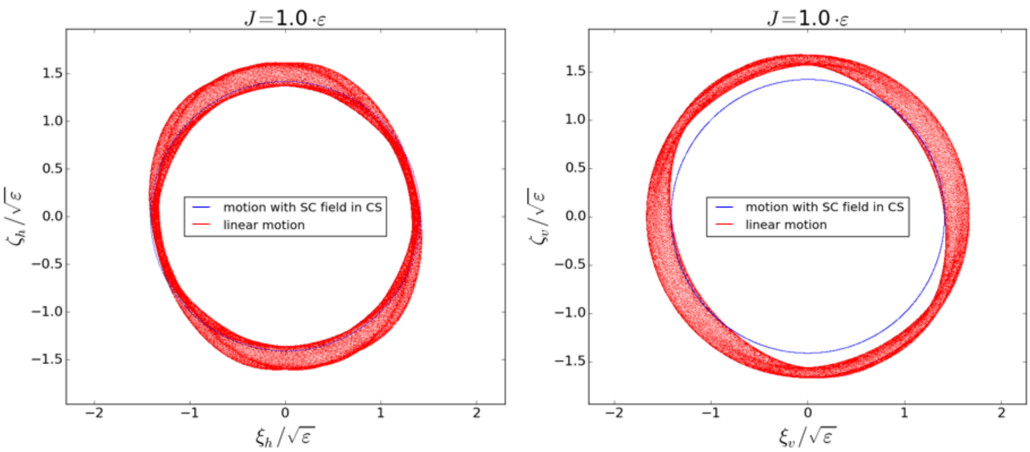

Figure 2: Phase space trajectory of an electron in the presence of the proton-electron focusing in the CS (red trace) and for the undisturbed motion (blue trace).

We can already see from Fig. 2 that for the optimized parameters the p-e focusing in the cooling section must not introduce any noticeable emittance growth. Tracking an ensemble of 10k particles for 20k turns, which is equivalent to $\approx$ 30 ms, confirms this conclusion (see Fig. 3).

7 Conclusion

We studied the effect of the proton-electron space charge focusing in the cooling section on the e-beam emittance in the EIC ring cooler.

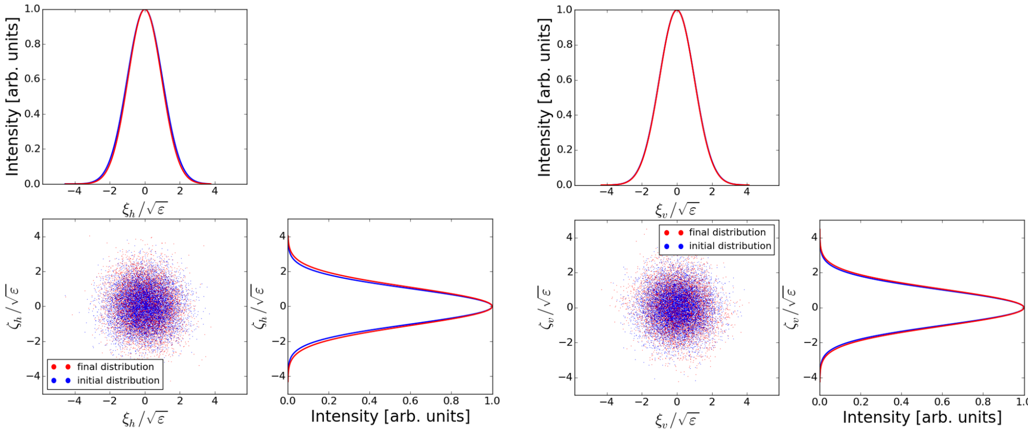

Figure 3: An ensemble of 10^4 particles tracked for 20k turns in the electron ring in the presence of the p-e focusing in the CS.

We conclude that for the optimized parameters the emittance growth associated with this effect is negligible.

8 Acknowledgments

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References

[1] H. Zhao, J. Kewisch, M. Blaskiewicz, and A. Fedotov, Ring-based electron cooler for high energy beam cooling, Phys. Rev. Accel. Beams 24, 043501 (2021).

[2] S. Seletskiy, Optimizing space charge and cooling in EIC Ring Cooler, presentation at EIC Ring Cooler Meeting series, BNL, February 17, 2022.

[3] K. Kewisch, Ring cooler cooling rates, presentation at EIC Ring Cooler Meeting series, BNL, March 17, 2022.

[4] S. Seletskiy, A. Fedotov, D. Kayran, Beam dynamics in non-magnetized electron cooler with strong hadron-electron focusing, BNL-216225-2020-TECH (2020).