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Absorptive corrections to the forward elastic proton-proton analyzing power A_N

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In the RHIC Polarized Atomic Hydrogen Gas Jet Target (HJET) measurements of the proton-proton analyzing power $A_N(t)$, absorptive corrections to the electromagnetic form factor were not considered in the data analysis. Recently, these corrections were evaluated theoretically. However, the results were presented in form of integral expressions which were calculated numerically. In this note, the expressions are simplified to make them more suitable for the HJET data analysis. Possible effect of the absorptive corrections in the $A_N(t)$ measurement is discussed.

I. INTRODUCTION

For RHIC spin program, theoretical analysis of the high energy forward elastic proton-proton analyzing power $A_N(t)$, developed in [1, 2], was systematized in Ref. [3]. The expression for

$$A_N(t) = \frac{\sqrt{-t}}{m_p} \times \frac{[\kappa_p(1-\delta_C\rho) - 2(I_5 - \delta_C R_5)] \frac{t_c}{t} - 2(R_5 + \rho I_5)}{\left(\frac{t_c}{t}\right)^2 - 2(\rho + \delta_C) \frac{t_c}{t} + 1 + \rho^2}, \quad (1)$$

became a commonly used standard which was referred to in the development of the RHIC proton beam polarimetry [4] and in the experimental studies of $A_N(t)$ [5, 6]. In Eq. (1), $\kappa_p = \mu_p - 1 = 1.793$ is anomalous magnetic of a proton, $\rho = -0.079$ is forward real/imaginary ratio, $\delta_C \sim 0.02$ is Coulomb phase, and $t_c = -8\pi\alpha/\sigma_{\text{tot}} = -1.86 \times 10^{-3} \text{ GeV}^2$ is expressed via total pp cross section. The numerical estimates are given for an $E_{\text{Lab}} = 100 \text{ GeV}$ proton beam.

Experimental precision achieved at the HJET [4, 6] required a consideration of some small corrections [7] to Eq. (1) which were neglected in Ref. [3].

Since absorptive corrections for non-flip and spin-flip electromagnetic form factors of a proton were unknown, the HJET measurements of $r_5 = R_5 + iI_5$ may need to be adjusted and, consequently, results of the Regge fit should be revisited.

Recently, theoretical evaluation of the absorptive correction was done [8]. However, the results were presented as integrals, which were calculated numerically. Thus, it is not so easy to adapt these calculations for analysis of an experimental data.

Here, expressions for absorptive correction given in Ref. [8] will be reviewed with the goal to interpret them to a form convenient for the HJET data analysis. Similar study has already been done for Coulomb phase in the spin-flip amplitude [9]. Several typos was found in expressions given in Ref. [8].

II. ABSORPTIVE CORRECTION FOR THE NON-FLIP pp AMPLITUDE

Omitting terms of about $\mathcal{O}(\alpha^2)$, Eqs. (17,19,24,25) in Ref. [8] can be presented in the following ‘‘standard’’, i.e. normalized by factor $1/2\pi$, Fourier integrals for electromagnetic (C)

$$\begin{aligned} \phi_+^{em}(q_T) &= \frac{i}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \left[1 - e^{i\chi_C^{nf}(b)} \right] \\ &\approx \frac{1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \left[\chi_C^{nf} + i\chi_C^{nf} \chi_C^{nf}/2 \right] \end{aligned} \quad (2)$$

$$\begin{aligned} \chi_C^{nf}(b) &= \frac{-\alpha}{2\pi} \int d^2q_T \frac{F_1^2(q_T^2)}{q_T^2 + \lambda^2} e^{-i\vec{q}_T \vec{b}} \\ &\rightarrow \frac{1}{2\pi} \int d^2q_T e^{-i\vec{q}_T \vec{b}} f_C(q_T) \end{aligned} \quad (3)$$

and for hadronic, (N)+(NC),

$$\begin{aligned} \phi_+^h(q_T) &= \frac{i}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \gamma_N^{nf}(b) e^{i\chi_C^{nf}(b)} \\ &\approx \frac{1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \left[(i\gamma_N^{nf}) + i(i\gamma_N^{nf})\chi_C^{nf} \right] \end{aligned} \quad (4)$$

$$i\gamma_N^{nf}(b) = i \times \frac{i}{2\pi} \int d^2q_T F_h^{nf}(q_T^2) e^{-i\vec{q}_T \vec{b}} \quad (5)$$

$$= \frac{1}{2\pi} \int d^2q_T e^{-i\vec{q}_T \vec{b}} f_N(q_T) \quad (6)$$

amplitudes. Blue color was used for expressions given exactly as in [8].

Here, amplitude $\phi_+ = \phi_1 + \phi_3$ is defined as sum of two non-flip helicity amplitudes [3]. Therefore [10], the electromagnetic amplitude in Born approximation is

$$f_C(q_T) = \frac{-2\alpha}{q_T^2 + \lambda^2} e^{-\tilde{B}q_T^2/2}, \quad \tilde{B} = \frac{2}{3}r_E^2 = 12.1 \text{ GeV}^2, \quad (7)$$

where $r_E = 0.841 \text{ fm}$ [11] is rms charge radius of a proton. Using the same normalization,

$$f_N(q_T) = \frac{2\alpha}{q_c^2} (i + \rho) e^{-Bq_T^2/2} \quad (8)$$

$$= -(i + \rho) f_C(q_T) \frac{q_T^2}{q_c^2} e^{(\tilde{B}-B)q_T^2/2} \quad (9)$$

where [3]

$$q_c^2 = -t_c = 8\pi\alpha/\sigma_{\text{tot}}. \quad (10)$$

For 100 GeV proton beam, $B = 11.2 \text{ GeV}^{-2}$ [12]. Eq. (8) leads to

$$i\gamma_N^{nf}(b) = (i + \rho) \frac{\sigma_{\text{tot}}}{4\pi B} e^{-b^2/2B}. \quad (11)$$

Born amplitude $f_C(q_T)$, implicitly defined in Eq. (19) of [8], is factor 2 smaller than given in Ref. [10]. It should be considered as typo.

For the pp scattering, the absorptive corrections can be approximated [13, 14] by factor

$$S(b) = 1 - \frac{\sigma_{\text{tot}}}{4\pi B} e^{-b^2/2B} = 1 - \gamma_N(b, \rho=0) \quad (12)$$

in the impact space. For used here definition of $\gamma_N(b)$ [Eq. (11)], Eq. (30) in [8] can be written as

$$\tilde{\phi}_+^{em}(q_T) = I_C(q_T) + I_{CC}(q_T) + I_{CN}(q_T) + I_{CCN}(q_T), \quad (13)$$

where, using results of Ref. [10] and assuming $\rho = 0$

$$I_C(q_T) = \frac{1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{nf} = f_C(q_T), \quad (14)$$

$$\begin{aligned} I_{CC}(q_T) &= \frac{i}{4\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{nf} \chi_C^{nf} \\ &= i f_C(q_T) \Phi_C(q_T), \end{aligned} \quad (15)$$

$$\Phi_C(q_T) = \alpha \left[\ln \frac{\lambda^2}{q_T^2} + \mathcal{O}(\tilde{B} q_T^2) \right], \quad (16)$$

$$I_{CN}(q_T) = \frac{i}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{nf} i\gamma_N^{nf} \quad (17)$$

$$\begin{aligned} &= i f_N(q_T) \frac{-\alpha}{\pi} \int \frac{d^2q_1}{q_1^2 + \lambda^2} \\ &\quad \times \exp \left[-(B + \tilde{B}) q_1^2/2 + B \vec{q}_1 \vec{q}_T \right] \end{aligned} \quad (18)$$

$$= i f_N(q_T) \Phi_{NC}(q_T), \quad (19)$$

$$\Phi_{NC}(q_T) = \alpha \left[\ln \frac{\lambda^2}{q_T^2} + \ln \frac{(B + \tilde{B}) q_T^2}{2} + \gamma + \mathcal{O}(B q_T^2) \right], \quad (20)$$

$$\gamma = 0.5772\dots \quad (\text{Euler's constant}), \quad (21)$$

$$I_{CCN}(q_T) = \frac{-1}{4\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{nf} \chi_C^{nf} i\gamma_N^{nf} \quad (22)$$

Using only the first three terms in sum (13) and substituting $f_N(t) \approx -i f_C(t) t/t_c$ (9), one readily finds

$$\tilde{\phi}_+^{em}(t) = f_C(t) \times [1 + i\Phi_C(t) + \Phi_{NC}(t) t/t_c] \quad (23)$$

In this approach, an absorptive correction a to the electromagnetic form factor,

$$\exp(\tilde{B}t/2) \rightarrow \exp(\tilde{B}t/2 + a t/t_c), \quad (24)$$

can be evaluated as

$$a_{\text{nf}} = \Phi_{NC} = \alpha \left[\ln \frac{\lambda^2}{|t_c|} + \ln \frac{(B + \tilde{B})|t_c|}{2} + \gamma + \mathcal{O}(Bt) \right]. \quad (25)$$

The result logarithmically depends on photon mass λ , but there is no dependence on t (if one neglects terms $\mathcal{O}(\alpha Bt)$ in Φ_{NC}). In such an approximation, using Eqs. (17,19), one finds $\chi_C^{nf}(b) \gamma_N^{nf} = a_{\text{nf}}^2 \gamma_N^{nf}(b)$ and, consequently,

$$I_{CCN}(q_T) = i f_C(q_T) a_{\text{nf}}^2 t/2t_c. \quad (26)$$

$I_{CCN}(q_T)$, which depends on the photon mass as $(\ln \lambda)^2$, contributes only to phase of $f_C(q_T)$.

To summarize, we cannot conclude that photon mass λ disappears from the expression used to calculate the absorptive correction a_{nf} to the non-flip amplitude.

Following QED prescriptions, one can expect that consideration of soft photon emission will eliminate the dependence on λ in (25). If so, the dominant term in the Coulomb-nuclear interference part of the cross-section can be approximated as

$$d\sigma_{el}^{\text{CN}}(q_T^2) \rightarrow d\sigma_{el}^{\text{CN}}(q_T^2) \times \left[1 + A \frac{q_T^2}{q_c^2} \ln \frac{q_{\text{max}}^2}{q_{\text{min}}^2} \right] \quad (27)$$

Since $q_{\text{min}}^2 \sim \lambda^2$ and $q_{\text{max}}^2 \sim 2/(B + \tilde{B})$, such an elimination of the photon mass term will result in absorptive correction of

$$a_{\text{nf}} = \alpha\gamma \approx 0.004 \quad (28)$$

Obviously, Eq. (27) is oversimplified. Therefore, at minimum, we should use less strict estimate for a_{nf} , e.g.

$$|a_{\text{nf}}| \lesssim \mathcal{O}(\alpha). \quad (29)$$

In terms of the correction to the electromagnetic slope, $\tilde{B} \rightarrow \tilde{B} + \Delta\tilde{B}$, this estimate can be written as

$$|\Delta\tilde{B}_{\text{nf}}| = \left| \frac{2a_{\text{nf}}}{t_c} \right| \lesssim \mathcal{O} \left(\frac{\sigma_{\text{tot}}}{4\pi} \right) \approx \mathcal{O}(7 \text{ GeV}^{-2}) \quad (30)$$

III. ABSORPTIVE CORRECTION FOR THE SPIN-FLIP pp AMPLITUDE

Using the same approach as for the non-flip amplitudes, the spin-flip ones can be presented as

$$\begin{aligned} \phi_5^{em}(q_T) &= \frac{1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{sf}(b) e^{i\chi_C^{nf}(b)} \\ &\approx \frac{1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \left[\chi_C^{sf} + \chi_C^{sf} i\chi_C^{nf} \right], \end{aligned} \quad (31)$$

$$\begin{aligned} \chi_C^{sf}(b) &= \frac{-\alpha\kappa_p}{4\pi m_p} \int d^2q_T \frac{F_1(q_T^2) F_2(q_T^2)}{q_T^2 + \lambda^2} \frac{\vec{q}_T \vec{b}}{b} e^{-i\vec{q}_T \vec{b}} \\ &\rightarrow \frac{1}{2\pi} \int d^2q_T e^{-i\vec{q}_T \vec{b}} f_C^{sf}(\vec{q}_T), \end{aligned} \quad (32)$$

$$f_C^{sf}(\vec{q}_T) = \frac{\kappa_p}{2} \frac{(\vec{q}_T \vec{n})}{m_p} f_C(q_T, \tilde{B}^{sf}) \quad (33)$$

$$\begin{aligned}\phi_5^h(q_T) &= \frac{i}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \gamma_N^{sf}(b) e^{i\chi_C^{nf}(b)} \\ &\approx \frac{1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \left[(i\gamma_N^{sf}) + i(i\gamma_N^{sf})\chi_C^{nf} \right],\end{aligned}\quad (34)$$

$$\begin{aligned}i\gamma_N^{sf}(b) &= i \frac{i}{2\pi} \int d^2q_T F_h^{sf}(q_T^2) e^{-i\vec{q}_T \vec{b}} \\ &= \frac{1}{2\pi} \int d^2q_T e^{-i\vec{q}_T \vec{b}} f_N^{sf}(\vec{q}_T),\end{aligned}\quad (35)$$

$$f_N^{sf}(\vec{q}_T) = \frac{r_5}{i + \rho} \frac{(\vec{q}_T \vec{n})}{m_p} f_N(q_T, B^{sf})\quad (36)$$

Unit vector \vec{n} used is perpendicular to the beam proton momentum and spin [9]. Generally, the spin-flip slopes \tilde{B}^{sf} and B^{sf} , substituted to amplitudes (7,8), are not the same as the non-flip ones, e.g. $\tilde{B}^{sf} = (r_E^2 + r_M^2)/3$, where $r_M = 0.851 \pm 0.026$ fm [15] is rms magnetic radius of a proton.

Spin-flip electromagnetic amplitude $\tilde{\phi}_5^{em}(q_T)$ introduced in Eq. (31) of Ref. [8] to evaluate the absorptive correction can be re-written as

$$\tilde{\phi}_5^{em}(q_T) = I_C^{sf}(q_T) + I_{CC}^{sf}(q_T) + I_{CN}^{sf}(q_T) + I_{CCN}^{sf}(q_T),\quad (37)$$

where, using calculations above and results of Ref. [9],

$$I_C^{sf}(q_T) = \frac{1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{sf} = f_C^{sf}(q_T),\quad (38)$$

$$\begin{aligned}I_{CC}^{sf}(q_T) &= \frac{i}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{nf} \chi_C^{sf} \\ &= i f_C^{sf}(q_T) \Phi_C(q_T),\end{aligned}\quad (39)$$

$$I_{CN}^{sf}(q_T) = \frac{i}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{sf} i\gamma_N^{nf}\quad (40)$$

$$\approx i f_N^{sf}(q_T) \times \alpha B \left(B + \tilde{B}_{sf} \right),\quad (41)$$

$$I_{CCN}^{sf}(q_T) = \frac{-1}{2\pi} \int d^2b e^{i\vec{q}_T \vec{b}} \chi_C^{nf} \chi_C^{sf} i\gamma_N^{nf}\quad (42)$$

Comparing Eqs. (35) and (41), one finds $\chi_C^{sf}(b)\gamma_N^{nf}(b) \times \alpha B / (B + \tilde{B}^{sf})$. Thus,

$$\begin{aligned}\tilde{\phi}_5^{em}(q_T) &= f_C(q_T) [1 + i\Phi_C(q_T)] \\ &\times \left[1 + \frac{\alpha B}{B + \tilde{B}^{sf} t} \right].\end{aligned}\quad (43)$$

Consequently, the spin-flip absorptive correction is

$$a_{sf} = \frac{\alpha B}{B + \tilde{B}^{sf}} \approx 0.003\quad (44)$$

and

$$\Delta \tilde{B}^{sf} = 2a_{sf}/t_c \approx 3 \text{ GeV}^{-2}.\quad (45)$$

IV. DISCUSSION

Absorptive corrections to the electromagnetic spin-flip amplitude effectively change [7] real part of r_5 in Eq. (1)

$$R_5 \rightarrow R_5 - a_{sf} \kappa_p / 2\quad (46)$$

in Eq. (1). This correction of about 3×10^{-3} is significant compared to the experimental accuracy ($\pm 0.5_{\text{stat}} \pm 0.8_{\text{sf}} \times 10^{-3}$, achieved at HJET [6]). Thus, the absorptive corrections should be applied to the already published values of r_5 as well as the Regge fit of $r_5(s)$ should be revisited.

Absorptive correction to the non-flip amplitude does not alter r_5 in (1) [7]. However, it may bias a measured value of $\rho^{\text{meas}} = \rho + a_{nf}$. Estimate of a_{nf} , given in Eq. (29), does not exclude that the combined fit of $\rho(s)$ and $\sigma_{\text{tot}}(s)$ should be revisited (after precise determination of a_{nf}).

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- [1] B. Kopeliovich and L. Lapidus, *Yad. Fiz.* **19**, 218 (1974), [*Sov. J. Nucl. Phys.* **19**, 114 (1974)].
- [2] N. H. Buttimore, E. Gotsman, and E. Leader, *Phys. Rev. D* **18**, 694 (1978), [Erratum: **35**, 407 (1987)].
- [3] N. H. Buttimore, B. Kopeliovich, E. Leader, J. Soffer, and T. Trueman, *Phys. Rev. D* **59**, 114010 (1999), [arXiv:hep-ph/9901339](https://arxiv.org/abs/hep-ph/9901339).
- [4] A. Zelenski *et al.*, *Nucl. Instrum. Meth. A* **536**, 248 (2005); A. A. Poblaguev, A. Zelenski, G. Atoian, Y. Makedisi, and J. Ritter, *Nucl. Instrum. Meth. A* **976**, 164261 (2020), [arXiv:2006.08393](https://arxiv.org/abs/2006.08393) [*physics.ins-det*].
- [5] H. Okada *et al.*, *Phys. Lett. B* **638**, 450 (2006), [arXiv:nucl-ex/0502022](https://arxiv.org/abs/nucl-ex/0502022); I. G. Alekseev *et al.*, *Phys. Rev. D* **79**, 094014 (2009); L. Adamczyk *et al.* (STAR), *Phys. Lett. B* **719**, 62 (2013), [arXiv:1206.1928](https://arxiv.org/abs/1206.1928) [*nucl-ex*].
- [6] A. A. Poblaguev *et al.*, *Phys. Rev. Lett.* **123**, 162001 (2019), [arXiv:1909.11135](https://arxiv.org/abs/1909.11135) [*hep-ex*].
- [7] A. A. Poblaguev, *Phys. Rev. D* **100**, 116017 (2019), [arXiv:1910.02563](https://arxiv.org/abs/1910.02563) [*hep-ph*].
- [8] B. Z. Kopeliovich, M. Krelina, and I. K. Potashnikova, *Phys. Lett. B* **816**, 136262 (2021), [arXiv:2102.01595](https://arxiv.org/abs/2102.01595) [*hep-ph*].
- [9] A. Poblaguev, *Coulomb phase corrections to the transverse analyzing power $A_N(t)$ in high energy forward proton-proton scattering*, Tech. Rep. BNL-222205-2021-TECH, C-A/AP/652 (Brookhaven National Laboratory, 2021).
- [10] B. Z. Kopeliovich and A. V. Tarasov, *Phys. Lett. B* **497**, 44 (2001), [arXiv:hep-ph/0010062](https://arxiv.org/abs/hep-ph/0010062).
- [11] P. Zyla *et al.* (Particle Data Group), *PTEP* **2020**, 083C01 (2020).
- [12] V. Bartenev *et al.*, *Phys. Rev. Lett.* **31**, 1088 (1973); *Phys. Rev. Lett.* **31**, 1367 (1973).
- [13] C. Bourrely, J. Soffer, and E. Leader, *Phys. Rept.* **59**, 95 (1980).
- [14] M. Krelina and B. Kopeliovich, *PoS SPIN2018*, 033

(2019).

[15] G. Lee, J. R. Arrington, and R. J. Hill, *Phys. Rev. D* **92**, 013013 (2015), [arXiv:1505.01489](https://arxiv.org/abs/1505.01489) [hep-ph].