BNL-222221-2021-TECH

EIC-ADD-TN-023

Instabilities driven by the fundamental crabbing mode

M. Blaskiewicz

October 2021

Electron-Ion Collider Project

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC), Nuclear Physics (NP) (SC-26)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No.DE-SC0012704 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Instabilities driven by the fundamental crabbing mode

M. Blaskiewicz
BNL 911B, Upton, NY 11973, USA

Instabilities driven by the fundamental mode of a crabbing cavity are discussed. For nominal parameters in the EIC it is necessary to provide strong RF feedback to reduce the apparent impedance of this mode.

INTRODUCTION

The Electron Ion Collider (EIC) requires crab cavities to correct for crossing angle effects [1]. These cavities produce a time varying horizontal kick that makes the collision head on in the center of mass frame of the bunches. This results in a luminosity that is very close to the the luminosity obtained without a crossing angle. Producing the kick requires a transverse RF field. This field requires a superconducting resonator and reasonable power levels require loaded quality factors of order a million. This results is a very large transverse impedance for the crabbing mode. The horizontal beta function is large and this, coupled with the large transverse impedance of the resonator can lead to strong instabilities.

In the first section of the paper a simple model is used to describe the situation without feedback. There are large ranges of tune leading to strong instabilities in both the electron and hadron storage rings of the EIC. These estimates are confirmed with simulations.

In the second section, RF feedback is used to reduce the apparent impedance of the crabbing mode. Simulating the actual feedback would require both a significant rewrite of the tracking code and a significant reduction in computational speed. Instead, the apparent impedance is fitted using one pole filters, which allows use of the existing simulation code.

In the third section a transverse damping system is outlined.

A SIMPLE MODEL

A simple model that captures the physics for strong instabilites can be obtained analytically. We take the transverse wake potential of all the crab cavities to be

$$W_x(t) = W_0 \sin(\omega_r t)\exp(-\alpha t), \tag{1}$$

where $W_0 = (R_x/Q)\omega_r > 0$, $\alpha = \omega_r/2Q$ and we do not distinguish between ω_r and $\tilde{\omega}$ [2]. The variable t is the delay between the driving and kicked particle. It is positive by causality. We use the coordinate θ to denote azimuth and time as the evolution variable. The dipole moment of the beam is $D(\theta,t) = I(\theta,t)x(\theta,t)$ with I the instantaneous current and x the instantaneous offset of the beam. We use the smooth approximation for the wake so that the transverse force per unit charge is

$$(E + v\times B)_x(\theta,t) \equiv \tilde{E}(\theta,t) = \int_0^\infty \frac{d\tau}{2\pi R} W_x(\tau) D(\theta, t-\tau), \tag{2}$$

where $2\pi R$ is the circumference of the accelerator. The bunches are modeled as two macroparticles separated by a fixed delay τ_b. There are M bunches that fill the ring symmetrically. The situation is shown in Figure 1.

The dipole moment is

$$D(\theta,t) = q\omega_0(N_b/2)\sum_{k=0}^{M-1} x_{1,k}(t)\delta_p(\theta - 2\pi k/M - \omega_0 t) + x_{2,k}(t)\delta_p(\theta - 2\pi k/M - \omega_0(t-\tau_b)), \tag{3}$$

where N_b is the number of particles per bunch, q is the ion charge, $\omega_0 = 2\pi f_0$ is the angular revolution frequency, and δ_p is the periodic delta function,

$$\delta_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - 2\pi k) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \exp(ikx).$$

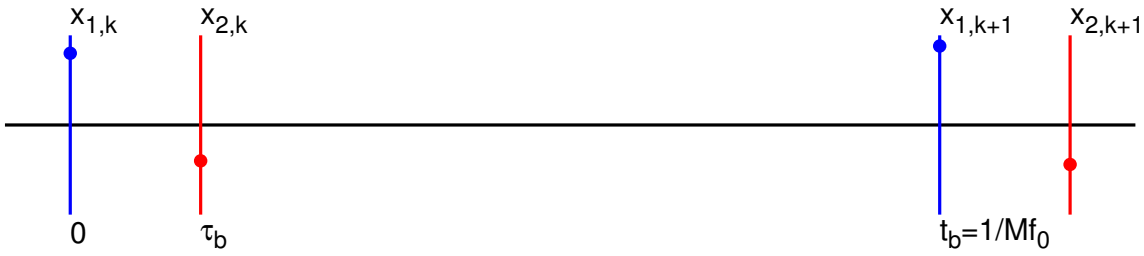


FIG. 1: For the simplest model the bunches are given by two macroparticles spaced in time by τ_b. The ring is filled symmetrically with M bunches and the revolution frequency is f_0. The bunch spacing is t_b.Each macroparticle executes betatron oscillations, drives the wake field, and is kicked by the resulting $\tilde{E}$.

Assume a single coupled bunch mode with $x_{1,k}(t) = \hat{x}_0 \exp(-2\pi iks/M - i\Omega t)$ with coherent frequency Ω and coupled bunch mode s. Make the ansantz $x_{2,k}(t) = rx_{1,k}(t)$ with r constant. Then

$$D(\theta,t) = qf_0(N_b/2)\hat{x}_0 \sum_{p=-\infty}^{\infty}\sum_{k=0}^{M-1}\left(1 + re^{-ip\omega_0\tau_b}\right)e^{-ip(\theta - 2\pi k/M - \omega_0 t) - i2\pi ks/M - i\Omega t} \tag{4}$$

The coupled bunch mode number limits the values of p to $p = nM + s$ with integer n.

$$D(\theta,t) = Mqf_0\hat{x}_0(N_b/2)\sum_{n=-\infty}^{\infty}\left(1 + re^{-i(nM+s)\omega_0\tau_b}\right)e^{-i(nM+s)(\theta-\omega_0 t) - i\Omega t}. \tag{5}$$

The dipole moment creates a transverse wake

$$\tilde{E}(\theta,t) = \frac{\bar{I}\hat{x}_0}{4\pi R}\sum_{n=-\infty}^{\infty}\left(1 + re^{-i(nM+s)\omega_0\tau_b}\right)e^{-i(nM+s)(\theta-\omega_0 t) - i\Omega t}\tilde{W}(\Omega - (nM+s)\omega_0), \tag{6}$$

where $\bar{I} = qf_0MN_b$ is the DC beam current and

$$\tilde{W}(\omega) = \int_0^{\infty} e^{i\omega t}W_x(t)dt = \frac{R_x}{Q}\frac{\omega_r^2}{\omega_r^2 - \omega^2 - i\omega\omega_r/Q} = iZ_x(\omega), \tag{7}$$

where Z_x is the transverse impedance. At the macroparticles $x_{1,k}$ we have $\tilde{E}_k^1 = \tilde{E}(2\pi k/M + \omega_0 t, t)$. Similarly $\tilde{E}_k^2 = \tilde{E}(2\pi k/M - \omega_0\tau_b + \omega_0 t, t)$. The equations of motion are

$$\gamma m\left(\ddot{x}_{j,k}(t) + \omega_\beta^2 x_{j,k}(t)\right) = q\tilde{E}_k^j(t) \tag{8}$$

where ω_β is the unperturbed betatron frequency, γ is the Lorentz factor, and m is the mass of the ion. Equality holds for $j = 1, 2$. We have $\ddot{x}_{j,k}(t) = -\Omega^2 x_{j,k}(t)$. The forces at the particles are manipulated to make them proportional to $x_{j,k}$, resulting in two dispersion relations,

$$\omega_\beta^2 - \Omega^2 = \frac{q\bar{I}}{4\pi\gamma mR}\sum_n\left(1 + re^{-i(nM+s)\omega_0\tau_b}\right)\tilde{W}(\Omega - (nM+s)\omega_0), \tag{9}$$

$$\omega_\beta^2 - \Omega^2 = \frac{q\bar{I}}{4\pi\gamma mR}\sum_n\left(1 + \frac{1}{r}e^{i(nM+s)\omega_0\tau_b}\right)\tilde{W}(\Omega - (nM+s)\omega_0), \tag{10}$$

where (9) pertains to $x_{1,k}$. The two sums on the right must be equal, giving an equation for r.

From equation (1)

$$\tilde{W}(\omega) = \frac{W_0}{2i}\left(\frac{1}{\alpha - i\omega - i\omega_r} - \frac{1}{\alpha - i\omega + i\omega_r}\right) \tag{11}$$

The line width of the resonator is small compared to the bunching frequency so the sums in equations (9) and (10) contain only two terms. We limit discussion to the mode that has betatron sidebands close to ω_r. set $\Omega = \epsilon\omega_\beta + \Delta\omega$ where $\epsilon = \pm 1$. Set $\omega_\beta = m\omega_0 + \omega_1$ with integer m and $|\omega_1| < \omega_0/2$. There is no steady state beam loading on the cavity so the resonant frequency satisfies $\omega_r = NM\omega_0$ with integer N. Then

$$\begin{aligned}
&\sum_n \left(1 + re^{-i(nM+s)\omega_0\tau_b}\right)\tilde{W}(\Omega - (nM+s)\omega_0 \\
&\approx \left(1 + re^{i(NM-s)\omega_0\tau_b}\right)\frac{iW_0/2}{\alpha - i\epsilon\omega_1 - i\Delta\omega} \\
&- \left(1 + re^{-i(NM+s)\omega_0\tau_b}\right)\frac{iW_0/2}{\alpha - i\epsilon\omega_1 - i\Delta\omega} \\
&\approx r\sin(\omega_r\tau_b)\frac{W_0/2}{\alpha - i\epsilon\omega_1 + \Delta\omega}
\end{aligned} \tag{12}$$

Likewise

$$\sum_n \left(1 + \frac{1}{r}e^{i(nM+s)\omega_0\tau_b}\right)\tilde{W}(\Omega - (nM+s)\omega_0) \approx \frac{-1}{r}\sin(\omega_r\tau_b)\frac{W_0/2}{\alpha - i\epsilon\omega_1 - i\Delta\omega}$$

Hence $r = \epsilon_2 i$ with $\epsilon_2 = \pm 1$.

Finally we assume the single sideband approximation with $\omega_\beta^2 - \Omega^2 \approx -2\epsilon\omega_\beta\Delta\omega$ yielding the final dispersion relation

$$2\epsilon\omega_\beta\Delta\omega = -\frac{q\bar{I}}{4\pi\gamma mR}\frac{i\epsilon_2 W_0\sin(\omega_r\tau_b)}{\alpha - i\epsilon\omega_1 - i\Delta\omega}. \tag{13}$$

This is a quadratic equation in $\Delta\omega$. Define the coherent tune shift $\Delta Q_c = \Delta\omega/\omega_0$, the fractional tune $\Delta Q_x = \epsilon\omega_1/\omega_0$, $\tilde{\alpha} = \alpha/\omega_0$ and

$$K = \frac{\bar{I}\beta_c(R_x/Q)\sin(\omega_r\tau_b)}{8\pi(E_T/q)}\frac{\omega_r}{\omega_0} \tag{14}$$

where the beta function at the crab cavity is β_c and $E_T = \gamma mc^2$ is the total energy per particle. With these definitions the coherent tune shift is given by

$$\Delta Q_c = -\frac{\Delta Q_x + i\tilde{\alpha}}{2} \pm \frac{\sqrt{(\Delta Q_x + i\tilde{\alpha})^2 - 4K}}{2} \tag{15}$$

In equation (14) the original analysis has been modified to make the product of the beta function and the transverse impedance correct in the final expression. Positive values of $Im(\Delta Q_c)$ lead to instability.

Figure 2 shows $Im(\Delta Q_c)$ for some nominal parameters. Simulations using a modified version of TRANFT [3] are compared with the formula in Figure 3. This figure also shows the result of a simulation using two, longitudinally fixed particles per bunch with all the impedance localized in a single thin lens. The nominal horizontal tunes are 0.2 and 0.08 for protons and electrons, respectively. The large space charge tune shifts for the protons might require tunes below 0.125 at injection. In this case no damping system can contend with such growth. The apparent impedance of the cavity must be reduced. This is considered in the next section.

REDUCING APPARENT IMPEDANCE USING FEEDBACK

The reduction of RF impedance using feedback is a well known and powerful technique [4, 5]. This is typically done for accelerating cavities which have only a longitudinal voltage. Crab cavities have both longitudinal and transverse voltage and some care will be needed to keep things straight. For a pure dipole impedance the cavity has a transverse voltage which gives a horizontal momentum kick $\Delta p_x = qV_x(t)/c$. Using the Panofsky-Wenzel theorem the longitudinal voltage and kick are $c\Delta p_z = qV_z(t) = -qx\dot{V}_x/c$. For structures that are short compared to c/ω, V_x is in phase with the magnetic field of the cavity and V_z is in phase with the electric field. Therefore any coupling of V_x will typically involve V_z too. For our resonator we have

$$\ddot{V}_x + \omega_r^2 V_x + 2\alpha\dot{V}_x = \frac{R_x}{Q}\omega_r^2 D_T(t), \tag{16}$$

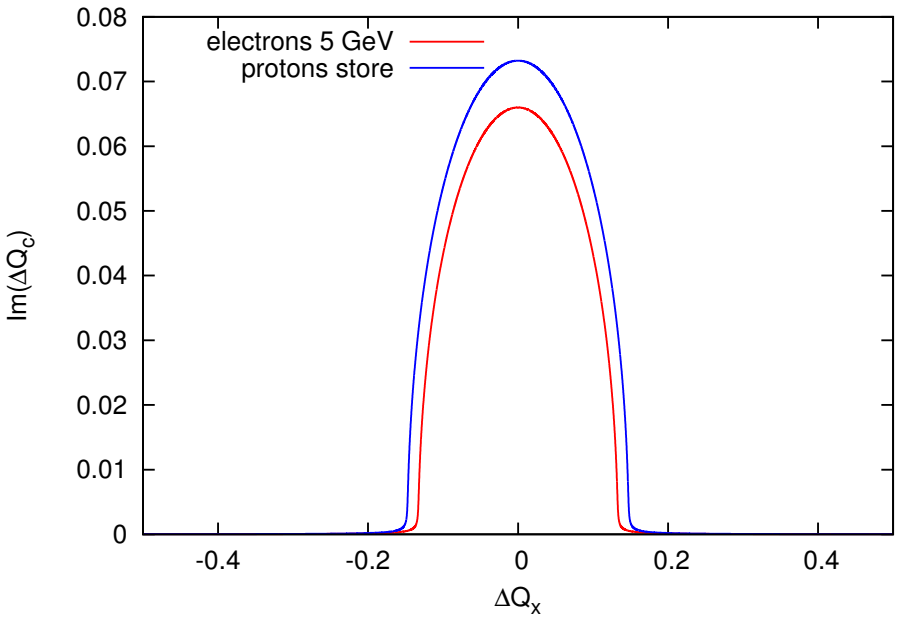


FIG. 2: Imaginary part of the tune versus fractional tune for electrons at 5 GeV and protons at 275 GeV with nominal parameters.

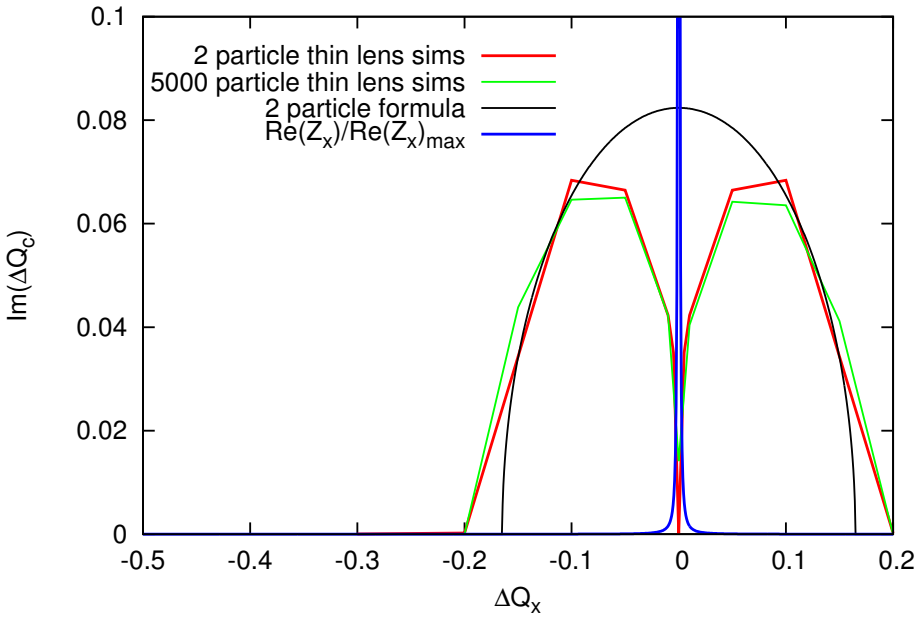


FIG. 3: Imaginary part of the tune versus fractional tune for 275 GeV protons. The red line is a simulation using two, longitudinally fixed particles per bunch with all the transverse impedance in a single thin lens. The green line uses a modified version of TRANFT that allows for multibunch wakes. The long range impedance is in a single thin lens. The black line is the estimate from equation (15). The blue line is the ratio of the transverse resistance to the peak value of the transverse resistiance.

where D_T is the total dipole moment driving the cavity. To include low level drive and feedback take

$$D_T(t) = D_B(t) + x_0 I_{LL}(t) - x_0 Y_{op} V_x(t - T_d), \tag{17}$$

where D_B is the dipole moment from the beam, x_0 is the effective offset of the coupling loop, I_{LL} is the current generated by the low level drive that includes a variety of multiturn feedback loops, Y_{op} is the operator generating the input signal from the cavity onto the feedback transmission line, and T_d is the loop delay of the feedback. Assume things vary in time as $F(t) = \hat{F}\exp(-i\omega t)$. Then

$$\hat{V}_x\left[\frac{-i\omega}{\omega_r R_x} + \frac{Q}{R_x}\left(1 - \frac{\omega^2}{\omega_r^2}\right) + x_0\tilde{Y}_{op}(\omega)e^{i\omega T_d}\right] = \hat{D}_b + x_0\hat{I}_{LL}. \tag{18}$$

Take $x_0\tilde{Y}_{op}(\omega) = -i\omega/(\omega_r R_{eff})$. On resonance this puts R_{eff} in parallel with R_x and the differentiation is automatic with a coupling loop. This leads to a final expression for the transverse impedance

$$Z_x(\omega) = \left\{\frac{\omega}{\omega_r}\left(\frac{1}{R_x} + \frac{e^{i\omega T_d}}{R_{eff}}\right) + i\frac{Q}{R_x}\left(1 - \frac{\omega^2}{\omega_r^2}\right)\right\}^{-1}. \tag{19}$$

For simulation purposes the transverse wake is modeled as a sum of narrow band resonators,

$$W_x(t) = \sum_{k=1}^{K} W_k \sin(\omega_k t)\exp(-\alpha t) \qquad \text{for } t > 0,$$

where the ω_k are on a uniform grid. The W_k are real numbers found using least squares. One considers the sum

$$\chi^2(W_1, W_2, \ldots W_K) = \sum_m \left| Z_x(\omega_m) - \sum_{k=1}^{K} W_k \frac{\omega_k}{i(\omega_k^2 - \omega_m^2) + 2\alpha\omega_m}\right|^2 \tag{20}$$

where the ω_m are on a grid in the vicinity of the resonance and we have made the same approximations as in equation (1). The absolute value squared is just the sum of the real part squared and the imaginary part squared so equation (20) is a multivariable quadratic form with real coefficients. There will be a unique minimum. It is found by setting $\partial\chi^2/\partial W_k = 0$ for $k = 1, 2, \ldots K$. This results in a set of K equations that are solved numerically for the W_ks. The result of fitting for a 197 MHz cavity with $T_d = 75/f_{res}$ and $R_{eff} = 300R_x/Q$ is shown in Figure 4.

To use this model in multiparticle tracking simulations one obtains the dipole moment $D_B(t)$ for a bunch as it passes a fiducial location. For the wake a complex representation in used so that each term will have a simple behavior in the gap between the bunches.Updating the wakes within the bunch is done using a difference equation with the appropriate driving term. Between the bunches there are large gaps of length T_g without mesh. Each term in the sum is multplied by $\exp(-\alpha T_g \pm i\omega_k T_g)$ to do the update exactly. The ratio of meshed to unmeshed length can be 1000, with a comeasurate reduction in required memory and computational time.

For multi bunch modes the simulation code has two options. One can use a few, or even one, bunch and assume a symmetric fill with a single coupled bunch mode. Another option is to use a full turn of beam, including the abort gap. Longitudinal modes can be included so that transient beam loading on the main RF can be included in the simulation. For this case there is no need to simulate the 1000+ bunches. Instead one can reduce the number of bunches by a fixed factor and increase the impedance by that same factor.

TABLE I: Nominal proton parameters for one IP

E_T/q	275 GV	100 GV	41 GV	25 GV
$\bar{I}$	1 A	1 A	1 A	1 A
β_c	1300 m	250 m	250 m	130 m
$\sigma_z = c\tau_b/2$	6 cm	7 cm	7.5 cm	80 cm
$\sigma(p)/p$	6.8×10^{-4}	9.7×10^{-4}	10.3×10^{-4}	
Q_s	0.01	0.01	0.01	0.001
N_{sym}	1260	1260	1260	315
R_x/Q	197 MHz	$8\times2400\Omega$/m		
R_x/Q	394 MHz	$4\times1270\Omega$/m		

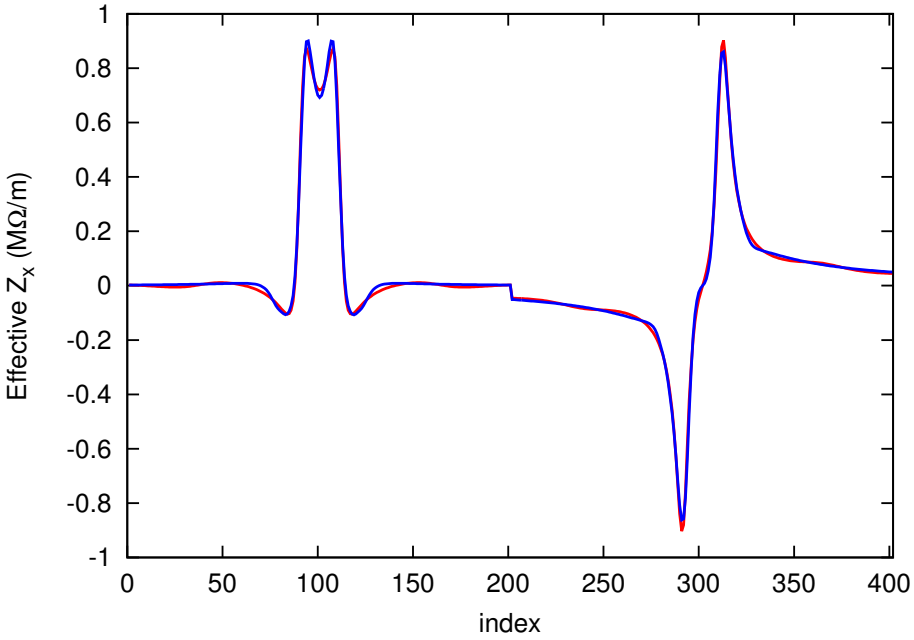


FIG. 4: Transverse impedance of one 197 MHz crab cavity with strong feedback. The red curve corresponds to equation (18). The first 201 points are $Re(Z_x)$ and the second 201 point are $Im(Z_x)$. The blue curve is a least squares fit using 10 resonators of the form shown in equation (1).

TRANSVERSE DAMPERS

Table I shows some relevant parameters for the hadron storage ring. During store the synchrotron tune is large and the beam-beam force from the electrons helps stabilize the protons. Simulations using a simple resonator model for the cavities show that an effective quality factor of 300 leads to stable beams during store. However, during injection and ramping the beam is unstable. Simulations using a single crab cavity frequency have been done with a transverse kicker voltage of the form

$$V_x(t) = a(t)\sin(\omega_{crab}t) + b(t)\cos(\omega_{crab}t). \tag{21}$$

In the simulations, $a(t)$ and $b(t)$ were constant within the bunch and allowed to vary freely from one bunch to the next. The values of of the coefficients were calculated using knowledge of the macroparticle angles at the kicker. These simulations resulted in stable beams but it is clearly not possible to implement this scheme in a real machine. For the real machine one could envision stripline pickups and kickers. For the 197 MHz system an effective quality factor of 300 is shown in Figure 4. The full range of the fit is 10 MHz, which is certainly broad enough for the feedback bandwidth. Suppose we use the difference signal from stripline pickup for the damper. The output signal is $V(t) = Z_x[D(t) - D(t - 2L/c)]$ where L is the length of the stripline and Z_x is the transverse transfer impedance of the stripline. In the frequency domain $\hat{V} = Z_x\hat{D}[1 - \exp(i\omega\tau)]$, with $\tau = 2L/c$. Setting $f_0\tau = 1/2$ makes the term in square brackets 2 and maximizes the signal at $f_0 = 197$ MHz. This gives $L = 38$ cm. At the edge frequencies of 197 ± 5 MHz the term in square brackets is $2.00 \pm 0.080i$. This is only a 0.04 radian phase error, far less than the typical $\pi/8$ threshold. Hence, optimized stripline pickups should work very well. For the kicker consider a pair of matched striplines run in difference mode. For a drive current $I_d(t)$ the transverse voltage is

$$V_x(t) = \int_{t-2L/c}^{t} \frac{I_d(t_1)}{C_k} dt_1, \tag{22}$$

where C_k is an effective capacitance. In the frequency domain $\hat{V}_x = [1 - \exp(i\omega\tau)]/(-i\omega C_k)$. The factor of i leads to a phase shift of $\pi/2$ that is easlily corrected for by a quarter wavelength delay. Assuming a simple, wide band system the phase errors of the pickup and kicker will add. Setting the sum to $\pi/8$ implies $\pi/16$ for the pickup and kicker

independently. The maximum half bandwidth satisfies

$$\frac{\sin(\Delta\omega\tau)}{1+\cos(\Delta\omega\tau)} = \tan(\pi/16).$$

This gives $\Delta\omega\tau \approx 0.4$ which corresponds to a full bandwidth of nearly 50 MHz for the 197 MHz system. Figure 5 shows an implementation of this scheme employing a traversal filter to extend the output pulse from the stripline. The output of the traversal filter is given by

$$S_{out}(t) = \sum_{k=0}^{4} S_{in}(t - 2k\tau),$$

and there is a time shift of -4τ to center things. The pulse train on the left is at the center frequence and the one on the right is 50 MHz higher in frequency. Even for this large frequency difference the phase shift looks OK although the amplitude out of the traversal filter is reduced.

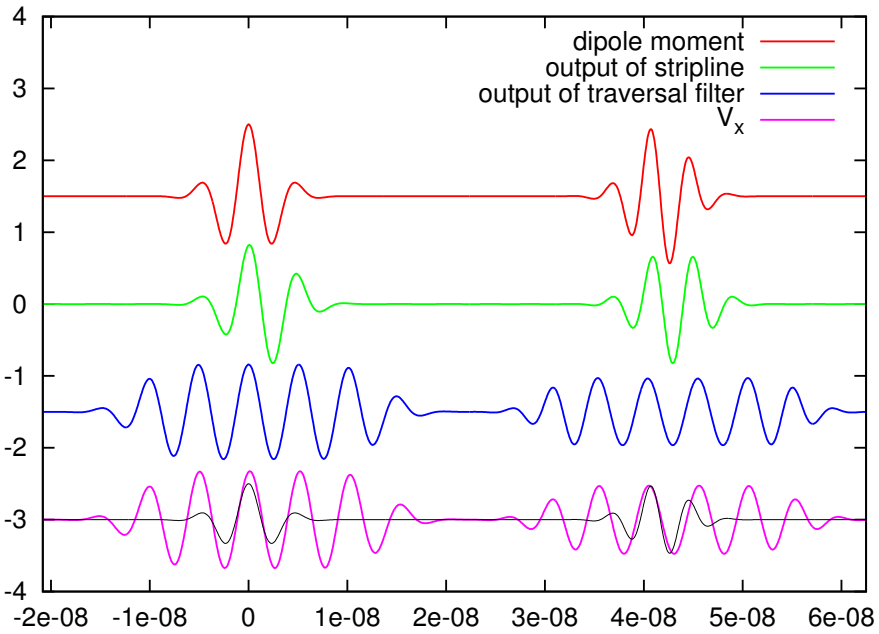


FIG. 5: Outline of a damping system. The red traces show the beam dipole moments at the pickup. The trace on the left is at the 197 MHz center frequency while the on on the right is 50 MHz higher. The spacing corresponds to the 24 MHz RF frequency. The green traces are the signals out of a pair of striplines in difference mode. The blue traces are the output of a 5 stage traversal filter with a delay of 2τ. The magenta traces are the voltage at the kicker with a time shift of $\tau/2$ and the light black traces are copies of the dipole moment.

A 394 MHz damping system may also be needed in the HSR. Possible interference between the damping systems will need to be carefully monitored. This is left for future work.

CONCLUSIONS

The crab cavities in the ESR and HSR can lead to very strong transverse instabilites. RF feedback can reduce the apparent impedance. When combined with large synchrotron tunes and beam-beam tune spread the beams appear to be stable during store. During injection and ramping in the HSR, transverse damping is required. Stripline pickups and kickers offer adequate bandwidth.

ACKNOWLEDGEMENTS

Thanks to Binping Xiao and Subashini De Silva for supplying the crab cavity parameters. Thanks to Qiong Wu for careful reading of the revised manuscript and for finding errors in the equations (4),(5) and (6).

[1] S.U. De Silva *et. al.* MOPAB393 IPAC2021 (2021).
[2] A. Chao 'Physics of Collective Beam Instabilites in High Energy Accelerators' Wiley (1993).
[3] M. Blaskiewicz 'The TRANFT User's Manual' BNL-77074-2006-IR 2006, see also M. Blaskiewicz PAC07, THPAS090, p3690, (2007).
[4] D. Boussard, G. Lambert, REDUCTION OF APPARENT IMPEDANCE OF WIDE BAND ACCELERATING CAVITIES BY RF FEEDBACK, PAC83, p2239 (1983).
[5] O. Aberle *et al* 'High-Luminosity Large Hadron Collider (HL-LHC) Technical Design Report' CERN-2020-010 (2020).