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Coulomb phase corrections to the transverse analyzing power AN(t) in high energy forward proton-proton scattering

A. A. Poblaguev

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Coulomb phase corrections to the transverse analyzing power $A_{\mathrm{N}}(t)$ in high energy forward proton-proton scattering

A.A. Poblaguev*
Brookhaven National Laboratory, Upton, New York 11973, USA
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Study of the polarized proton-proton elastic scattering in the Coulomb Nuclear interference region allows one to measure forward hadronic spin-flip amplitude including the phase. However, in a precision experimental data analysis, a phase shift correction δ_C due to the long distance Coulomb interaction should be taken into account. For the unpolarized scattering, δ_C is commonly considered as well established. Here, we evaluate the Coulomb phase shifts for the spin-flip electromagnetic and hadronic amplitudes. The difference between spin-flip and non-flip Coulomb phase shifts was found to be negligible for current experimental accuracy in the high energy transverse spin elastic pp measurements. However, effective alteration of the hadronic spin-flip amplitude by the long distance electromagnetic corrections can be noticeable.

I. INTRODUCTION

It is well known that an experimentally determined hadronic scattering amplitude is effectively altered by the accompanying electromagnetic interaction. In the Coulomb-nuclear interference (CNI) region, the proton-proton amplitude can be approximated by the following sum of hadronic $\phi(s,t)$ and electromagnetic, t_c/t, amplitudes [1]

$$\phi_{pp}^{\mathrm{CNI}}(s,t) = \mathrm{Im}\,\phi(s,0)\left[(i+\rho)e^{Bt/2}+\frac{t_c}{t}e^{i\delta_C+\widetilde{B}t/2}\right]. \quad (1)$$

Here, $\delta_C(s,t)$ stands for difference between the electromagnetic and hadronic phases induced by the long-distance Coulomb interaction. Following QED calculations and the optical theorem, $t_c = -8\pi\alpha/\sigma_{\mathrm{tot}}(s)$ [2], where α is the fine structure constant and σ_{tot} is the total pp cross section. Generally, the amplitudes are functions of total energy squared s and momentum transfer squared t.

In this paper, numerical estimates will be done for a 100 GeV proton beam (typical for the Relativistic Heavy Ion Collider or the future Electron Ion Collider) scattering on a fixed proton target. Therefore, $\rho = -0.079$ [3], $\sigma_{\mathrm{tot}} = 39.2\,\mathrm{mb}$ [3], $t_c = -1.86\times10^{-3}\,\mathrm{GeV}^2$, and $B = 11.2\,\mathrm{GeV}^{-2}$ [4]. For the sake of simplicity, we do not distinguish between the hadronic B and electromagnetic $\widetilde{B}$ slopes in expressions for the elastic differential cross section

$$\frac{d\sigma^{el}}{dt} \propto \left[\left(\frac{t_c}{t}\right)^2 - 2\left(\rho+\delta_C\right)\frac{t_c}{t} + 1 + \rho^2\right]e^{Bt}, \quad (2)$$

and the analyzing power $A_{\mathrm{N}}(t)$ [Eq. (8)]. We also do not consider small corrections [5] due to Dirac and Pauli form factors and due to the absorption [6].

* poblaguev@bnl.gov

For the unpolarized scattering, theoretical understanding of $\delta_C(t)$ was developed in many works, in particularly, [7–10].

In Ref. [9], the Coulomb phase was evaluated as

$$\delta_C = -\alpha\left[\ln\frac{-(B+\widetilde{B})t}{2} + \gamma - \frac{\widetilde{B}t}{2}\ln\frac{-\widetilde{B}t}{2} + \frac{\widetilde{B}t}{4}\right], \quad (3)$$

where $\gamma = 0.5772$ is Euler's constant and $\widetilde{B} = 8/\Lambda^2 = 11.2\,\mathrm{GeV}^{-2}$ was derived from the ep electromagnetic form factor in dipole form, $\mathcal{F}_{ep} = (1-t/\Lambda^2)^{-2}$, $\Lambda^2 = 0.71\,\mathrm{GeV}^2$ [11]. Up to now, this result is commonly used in experimental data analysis.

Neglecting the slopes B and $\widetilde{B}$ dependence on t and omitting the $\mathcal{O}(\alpha^2)$ terms, analytical solution for δ_C was found in Ref. [10]

$$\delta_C = \alpha\times\left\{\ln\frac{\widetilde{B}^2}{B^2} + Ei\left(\frac{2B^2/\widetilde{B}}{B+\widetilde{B}}w\right) - Ei(w) + e^{2w}\left[2E_1(2w) - E_1(w)\right]\right\}, \quad (4)$$

where $w = \left|\widetilde{B}t\right|/4$ and

$$\widetilde{B} = \frac{2}{3}r_E^2 = 12.1\,\mathrm{GeV}^{-2} \quad (5)$$

was expressed via rms charge radius of a proton, $r_E = 0.841\,\mathrm{fm}$ [12]. In terms of B and $\widetilde{B}$, Eqs. (3) and (4) are in agreement for low t.

It was shown in Ref. [13] that Coulomb phase should be independent of the helicity structure of the scattering amplitudes. However, the conclusion made was actually relevant only for the leading term $\sim \ln|t|$ in $\delta_C(t)$.

In this paper we adapt formulas derived in Ref. [10] to evaluate Coulomb phases in the spin-flip scattering. Consequent alteration of the expression [14] for forward elastic pp analyzing power $A_{\mathrm{N}}(t)$ will be discussed.

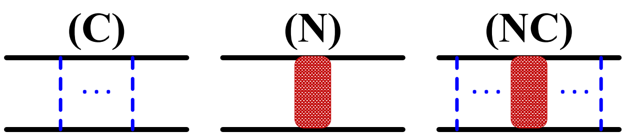

FIG. 1. Three types of the elastic pp scattering: (C) electromagnetic including multiple photon exchange, (N) bare hadronic, and (NC) combined hadronic and electromagnetic.

II. HIGH ENERGY FORWARD ELASTIC pp ANALYZING POWER

For a CNI elastic scattering of a vertically polarized proton beam $p^\uparrow p$, the analyzing power A_N is caused by interference of the non-flip (nf) and spin-flip (sf) amplitudes [2, 13, 14]

$$A_N = \frac{2\,\mathrm{Im}\left[\widetilde{\phi}_{sf}\phi^*_{nf} + \phi_{sf}\widetilde{\phi}^*_{nf} + \phi_{sf}\phi^*_{nf}\right]}{\left|\phi_{nf} + \widetilde{\phi}_{nf}\right|^2}. \tag{6}$$

Here, hadronic ϕ and electromagnetic $\widetilde{\phi}$ parts of an amplitude are discriminated by tilde symbol.

The sf amplitudes can be simply related to the nf ones:

$$\widetilde{\phi}_{sf}/\widetilde{\phi}_{nf} = \frac{\kappa_p}{2}\sqrt{-t}/m_p, \quad \phi_{sf}/\phi_{nf} = \frac{r_5}{i+\rho}\sqrt{-t}/m_p, \tag{7}$$

where $\kappa_p = \mu_p - 1 = 1.973$ is anomalous magnetic moment of a proton, $|r_5| = |R_5 + iI_5| \sim 0.02$ [15] is hadronic spin-flip parameter [14], and m_p is a proton mass. Consequently,

$$A_\mathrm{N}(t) = \frac{\sqrt{-t}}{m_p}\times \frac{\left[\kappa_p(1-\delta_C^{em}\rho) - 2(I_5 - \delta_C^h R_5)\right]\frac{t_c}{t} - 2(R_5 + \rho I_5)}{\left(\frac{t_c}{t}\right)^2 - 2\left(\rho + \delta_C\right)\frac{t_c}{t} + 1 + \rho^2}, \tag{8}$$

where non-flip phase δ_C is given in Eq. (4) while δ_C^{em} and δ_C^h are spin-flip phase shifts in $\widetilde{\phi}_{sf}\phi^*_{nf}$ and $\phi_{sf}\widetilde{\phi}^*_{nf}$ interference terms, respectively.

All recent experimental studies [15, 16] of the forward elastic proton-proton $A_\mathrm{N}(t)$ were based on Eq. (8) with $\delta_C^{em} = \delta_C^h = \delta_C$.

III. SPIN-FLIP COULOMB PHASES IN ELASTIC $p^\uparrow p$ ANALYZING POWER

To evaluate spin-flip δ_C^{em} and δ_C^h, we adapt the expressions which were derived in Ref. [10] to calculate δ_C.

A. The theoretical approach [10] used to calculate Coulomb corrections to the non-flip amplitudes

Considering multiple photon exchange in the elastic pp scattering and neglecting the higher order corrections $\mathcal{O}(\alpha^3)$, the net long range Coulomb (C) amplitude (see Fig.1) can be presented as [10]

$$f_C(q_T) = \frac{i}{2\pi}\int d^2b\, e^{i\vec{q}_T\vec{b}}\left[1 - e^{i\chi_C^{nf}(b)}\right], \tag{9}$$

$$= \hat{f}_C(q_T) + \frac{i}{2\pi}\int d^2b\, e^{i\vec{q}_T\vec{b}}\left[\chi_C(b)\right]^2/2 \tag{10}$$

where $q_T \approx \sqrt{-t}$ is transverse momentum and the eikonal phase

$$\chi_C^{nf}(b) = \frac{1}{2\pi}\int d^2q_T\,\hat{f}_C(q_T)e^{-i\vec{q}_T\vec{b}} \tag{11}$$

is a Fourier transform of the Coulomb part of the amplitude calculated in Born approximation [10]

$$\hat{f}_C(q_T) = \frac{-2\alpha}{q_T^2 + \lambda^2}\,e^{-\widetilde{B}q_T^2/2} \tag{12}$$

Here, the amplitude is defined as sum of two non-flip helicity amplitudes $\langle ++\,|\,++\rangle$ and $\langle +-\,|\,+-\rangle$ [14]. A small photon mass λ was included to Eq. (12) to keep the integrals finite.

The multi-photon exchange results in an acquired Coulomb phase $\Phi_C(q_T)$

$$f_C(q_T) = \hat{f}_C(q_T)\,e^{i\Phi_C(q_T)} \tag{13}$$

Assuming $\Phi_C(q_T) \ll 1$, one finds

$$\Phi_C(q_T) = -i\left[f_C(q_T)/\hat{f}_C(q_T) - 1\right] \tag{14}$$

$$= \frac{1}{4\pi}\int d^2q_1 d^2q_2\,\delta(\vec{q}_T - \vec{q}_1 - \vec{q}_2)\frac{\hat{f}_C(q_1)\hat{f}_C(q_2)}{\hat{f}_C(q_T)}, \tag{15}$$

Similarly, to calculate the Coulomb corrections to the hadronic amplitude

$$\hat{f}_N(q_T) = \frac{(i+\rho)\sigma_\mathrm{tot}}{4\pi}\,e^{-Bq_T^2/2} \tag{16}$$

one can use the following relations:

$$f_\mathrm{NC}(q_T) = \frac{i}{2\pi}\int d^2b\, e^{i\vec{q}_T\vec{b}}\gamma_N^{nf}(b)e^{i\chi_C^{nf}(b)} \tag{17}$$

$$= \hat{f}_N(q_T) + \frac{i}{2\pi}\int d^2b\, e^{i\vec{q}_T\vec{b}}\gamma_N^{nf}\chi_C^{nf}, \tag{18}$$

$$\gamma_N^{nf}(b) = \frac{-i}{2\pi}\int d^2q_T e^{-i\vec{q}_T\vec{b}}\,\hat{f}_N(q_T), \tag{19}$$

$$\Phi_\mathrm{NC}(q_T) = -\frac{\alpha}{\pi}\int\frac{d^2q_1}{q_1^2 + \lambda^2} \times \exp\left[-(B + \widetilde{B})q_1^2/2 + B\vec{q}_1\vec{q}_T.\right] \tag{20}$$

Eqs. (15) and (20) were analytically integrated in Ref. [10]. Both, $\Phi_C(q_T)$ and $\Phi_\mathrm{NC}(q_T)$, contains the divergent term $\ln q^2/\lambda^2$ which, however, is canceled in the final expression for the Coulomb phase difference

$$\delta_C(t) = \Phi_C(t) - \Phi_\mathrm{NC}(t) \tag{21}$$

displayed in Eq. (4).

B. Coulomb corrections to the spin-flip amplitudes

To find the Coulomb corrected spin-flip amplitudes $f_C^{sf}(q_T)$ and $f_{\rm NC}^{sf}(q_T)$, one can use the following eikonal phases [17]

$$\chi_C^{sf}(b) = \frac{1}{2\pi}\int d^2q e^{-i\vec{q}\vec{b}} \times \frac{\kappa_p}{2m_p}\,(\vec{n}\vec{q})\,\hat{f}_C(q)/2 \tag{22}$$

and

$$\gamma_N^{sf}(b) = \frac{-i}{2\pi}\int d^2q e^{-i\vec{q}\vec{b}} \times \frac{r_5}{(i+\rho)m_p}\,(\vec{n}\vec{q})\,\hat{f}_N(q)/2 \tag{23}$$

respectively. Here, $\vec{n} \propto \vec{p}_{\rm beam}\times\vec{s}$ is a unit vector orthogonal to the beam momentum $\vec{p}_{\rm beam}$ and the proton spin $\vec{s}$.

Considering the spin flip amplitudes for $\vec{q}_T = \vec{n}q_T$, one can readily determine the spin-flip phase $\Phi_C^{sf}(q_T)$ by adding factor $2(\vec{q}_T\vec{q}_1)/q_T^2$ or $2(\vec{q}_T\vec{q}_2)/q_T^2$ to integral (15). Since $\vec{q}_1\vec{q}_T+\vec{q}_2\vec{q}_T=q_T^2$, we immediately find

$$\Phi_C^{sf}(q_T)=\Phi_C(q_T), \tag{24}$$

which leads to

$$\delta_C^{em}(t) = \delta_C(t). \tag{25}$$

To calculate $\Phi_{\rm NC}^{sf}$, factor $(\vec{q}_2\vec{q}_T)/q_T^2 = 1-(\vec{q}_1\vec{q}_T)/q_T^2$ should be added to Eq. (20), which gives

$$\Phi_{\rm NC}^{sf}(q_T) = \Phi_{\rm NC}(q_T) - \frac{\alpha B}{B+\widetilde{B}}\times\Delta_{\rm NC}^{sf}(\eta), \tag{26}$$

$$\Delta_{\rm NC}^{sf}(\eta) = \int_0^\infty \frac{du}{\eta}e^{-u^2/4\eta}\int_{-\pi}^{\pi}\frac{d\varphi}{2\pi}\cos\varphi\, e^{u\cos\varphi}, \tag{27}$$

$$\eta = \frac{B}{B+\widetilde{B}}\times\frac{Bq_T^2}{2} \approx Bq_T^2/4. \tag{28}$$

Expanding

$$e^{u\cos\varphi}\cos\varphi \to \sum_{k=0}^{\infty}\frac{u^k}{k!}\cos^{k+1}\varphi \tag{29}$$

and using the following integrals [18]

$$\int_{-\pi}^{\pi}\cos^{2n+1}x\,dx = 0, \tag{30}$$

$$\int_{-\pi}^{\pi}\cos^{2n}x\,dx = \frac{\pi}{2^{n-2}}\frac{(2n-1)!}{(n-1)!n!}, \tag{31}$$

$$\int_0^\infty x^{2n+1}e^{-px^2}\,dx = \frac{n!}{2p^{n+1}}, \tag{32}$$

one arrives to

$$\Delta_{\rm NC}^{sf}(\eta) = \sum_{k=0}^{\infty}\frac{\eta^k}{(k+1)!} = \frac{e^\eta-1}{\eta}. \tag{33}$$

Thus,

$$\delta_C^h(t) = \delta_C(t) - \frac{\alpha B}{B+\widetilde{B}}\,\frac{e^\eta-1}{\eta}. \tag{34}$$

In Ref. [6], it was pointed out that hadronic spin-flip amplitude should also include the spin-flip photon exchange, i.e. one should replace

$$\gamma_N^{sf}(b) \to \gamma_N^{sf}(b) + i\chi_C^{sf}(b)\gamma_N^{nf}(b) \tag{35}$$

in Eq. (19). Using Eq. (26), one readily finds

$$\begin{aligned}&\frac{i}{2\pi}\int d^2b e^{i\vec{q}_T\vec{b}}\,\chi_C^{sf}(b)\gamma_N^{nf}(b)\\ &\quad= \frac{\kappa_p\left[\Phi_N(q_T)-\Phi_N^{sf}(q_T)\right]}{2m_p}(\vec{n}\vec{q}_T)\hat{f}_N(q_T).\end{aligned} \tag{36}$$

Thus, replacement (35) can be interpreted as an effective alteration of the hadronic spin-flip parameter

$$r_5 \to r_5 + i(i+\rho)\Delta_\gamma \approx r_5 - \Delta_\gamma, \tag{37}$$

$$\Delta_\gamma = \frac{\alpha\kappa_p B}{2(B+\widetilde{B})} \approx 0.003. \tag{38}$$

For the corrected r_5, Coulomb phase $\Phi_{\rm NC}^{sf}(t)$ is the same as in Eq. (26).

IV. SUMMARY

Adapting the technique developed in Ref. [10], the Coulomb phase shifts in the elastic pp spin-flip amplitude interferences $\widetilde{\phi}_{sf}\phi_{nf}^*$ (25) and $\phi_{sf}\widetilde{\phi}_{nf}^*$ (34) were calculated.

Small difference, $\delta_C^h - \delta_C \sim -\alpha/2$, was found for the CNI scattering $|t| \lesssim 0.1\,\text{GeV}^2$ ($\eta < 0.3$). Since $|R_5| \lesssim 0.02$, such a discrepansy can be neglected in expression (8) for $A_{\rm N}(t)$.

Thus, we can agree with the Coulomb phases approximation

$$\delta_C^{em} = \delta_C^h = \delta_C = -\alpha\times\left[\ln\frac{(B+\widetilde{B})|t|}{2}+\gamma\right] \tag{39}$$

suggested in Ref. [14].

Evaluating δ_C^{em} and δ_C^h, we did not distinguish between non-flip B and spin-flip B_{sf} slopes as well as between $\widetilde{B}_{sf}$ and

$$\widetilde{B}_{sf} = \left(r_E^2 + r_M^2\right)/3, \tag{40}$$

where $r_M = 0.851\pm0.026\,\text{fm}$ [19] is magnetic radius of a proton.

Using explicit expressions for $\Phi_C(t)$ and $\Phi_N(t)$ [10] one can derive

$$\delta_C^{em}(t,\widetilde{B}_{sf},B) = \delta_C(t,\widetilde{B},B) + \mathcal{O}(|\widetilde{B}_{sf}-\widetilde{B}|t), \tag{41}$$

$$\begin{aligned}\delta_C^h(t,\widetilde{B},B_{sf}) = \delta_C(t,\widetilde{B},B) - \frac{\alpha B_{sf}}{B_{sf}+\widetilde{B}}\\ + \alpha\ln\frac{B+\widetilde{B}}{B_{sf}+\widetilde{B}} + \mathcal{O}(|B_{sf}-B|t).\end{aligned} \tag{42}$$

For a reasonable ratio B_{sf}/B between the spin-flip and non-flip slopes [20], the related corrections to δ_C^{em} and δ_C^h can be neglected in the experimental data analysis.

Spin-flip photon exchange (35) results in the correction

$$\Delta\mathrm{Re}\, r_5 = -\Delta_\gamma \approx -0.003 \tag{43}$$

to the value of the hadronic spin-flip parameter r_5. The correction found is about triple of the experimental accuracy for R_5 in the HJET measurements [15]. Thus, the published values of R_5 [15] should be corrected,

$$\mathrm{Re}\, r_5 = \mathrm{Re}\, r_5^{\mathrm{meas}} + \Delta_\gamma \tag{44}$$

This is especially important for the Regge fit of the hadronic spin-flip amplitudes.

It is interesting to note that an absorptive correction,

$$a_{sf} = \alpha B/(B + \widetilde{B}_{sf}), \tag{45}$$

to the electromagnetic spin-flip form factor, $\mathcal{F}_{pp}^{sf}(t) \to \mathcal{F}_{pp}^{sf}(t) \times (1 + a_{sf}\, t/t_c)$, results [5] in the same effective alteration of r_5 as shown in Eq. (43).

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