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Three dimensional coherent electron cooling without a bypass

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This note considers the possibility of a coherent electron cooling system for the Electron Ion Collider that does not include a bypass for the ions. A detailed design is not included. It is simply assumed that appropriate density fluctuations in the kicker region can be obtained and the implications of those modulations are considered.

INTRODUCTION

The Electron Ion Collider (EIC) requires strong hadron cooling (SHC) to reach luminosities of $10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$. Without cooling, emittance growth times due to intrabeam scattering are as small as two hours. The current scenario involves coherent electron cooling with a microbunching amplifier. This design requires a magnetic bypass for the ions. This bypass is both complicated and expensive and a design that allowed the electrons and ions to copropagate throughout the cooling section would be preferable. This note assumes a best case scenario for a new electron amplifier and tests the efficacy of direct transverse kicks from the electron density fluctuations.

THE MODEL

The scenario is shown in 1. Here, and through most of the note, we work in the comoving frame where the average momentum of the ion bunch is zero.

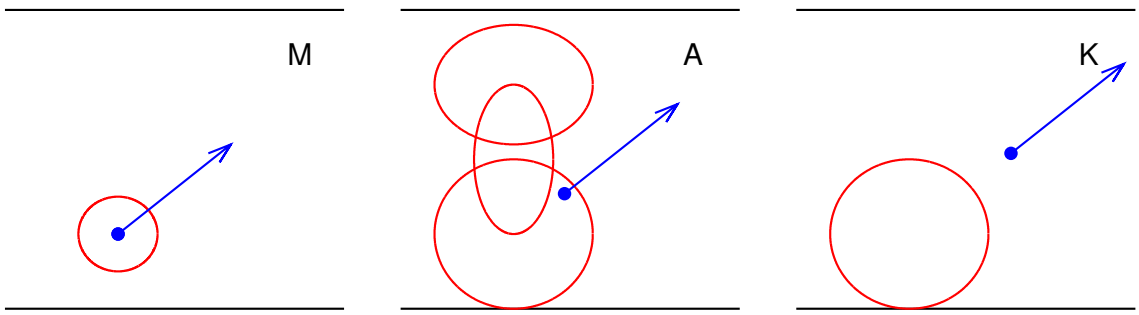


FIG. 1: Diagram of the cooling scenario in the comoving frame. The horizontal black lines indicate the one sigma contours of the electron bunch, the blue dot is the ion with its velocity indicated by the blue arrow. The red contours illustrate the extent of the electron fluctuation seeded by the ion. In the modulator (M) the ion imprints a perturbation on the electron bunch. In the amplifier (A) the electrons are focused and the signal is amplified. Note that any sort of focusing will cause the perturbation to move transversely by a significant amount. The effect of the focusing on the ions is neglected. In the kicker (K) the perturbation has grown to the point it can provide a substantial kick to the ions. Note that the betatron phase advance of the electrons between M an K is a multiple of 2π.

To simplify the calculations assume the electron density perturbation in K is spherically symmetric and model the density perturbation as

$$\delta\rho_e(\mathbf{r}) = \delta\rho_0\left[\exp\left(-\frac{r^2}{2\sigma_1^2}\right) - \frac{\sigma_1^3}{\sigma_2^3}\exp\left(-\frac{r^2}{2\sigma_2^2}\right)\right]. \tag{1}$$

Note that $\int \delta\rho_e d^3r = 0$ as required by charge conservation. Any damaging effects due to the bandwidth, dispersion *etc.* of the amplifier are neglected. This is a strong assumption. The electric field generated by the perturbation is

spherically symmetric and for $r \lesssim \sigma_1$ it is given by

$$\mathbf{E} = \frac{\delta\rho_0(1-\sigma_1^3/\sigma_2^3)}{3\epsilon_0}\mathbf{r} + O(r^3) \equiv C_E\mathbf{r} + O(r^3) \tag{2}$$

The ion's three dimensional velocity in the comoving frame is $\mathbf{v}$. The impulse approximation is used to obtain the kick. Between the center of M and the center of K the ion travels a distance

$$\Delta\mathbf{r} = \frac{2L_A + L_M + L_K}{2\gamma c}\mathbf{v},$$

where the lengths L_A *etc.* are measured in the lab frame and the factor of γ accounts for time dilation. We assume the beams in the lab frame are relativistic and only keep leading order terms in the Lorentz transformations. The offset $\Delta\mathbf{r}$ leads to the ion experiencing an electric field $\mathbf{E} = C_E\Delta\mathbf{r}$. This field is applied for a time $\Delta t = L_K/\gamma c$. The momentum impulse is then

$$M\Delta\mathbf{v} = Q\mathbf{E}\Delta t = QC_E\frac{2L_A + L_M + L_K}{2\gamma c}\frac{L_K}{\gamma c}\mathbf{v}, \tag{3}$$

where M and Q are the mass and charge of the ion. Since everything its spherically symmetric the cooling rates are equal in all three dimensions. The kick in (3) happens once per turn and only when an ion interacts with the electron bunch. This happens for a fraction of the time σ_{ze}/σ_{zi}, where σ_{ze} and σ_{zi} are the rms bunch lengths of the electrons and ions, respectively. While it is irrelvant for the ratio, both lengths are measured in the comoving frame as stated previously. The average velocity impulse for a single ion traversal of the cooling section is then

$$\Delta\mathbf{v} = C_E\frac{Q}{M}\frac{\sigma_{ze}}{\sigma_{zi}}\frac{2L_A + L_M + L_K}{2\gamma c}\frac{L_K}{\gamma c}\mathbf{v} \equiv g\mathbf{v}. \tag{4}$$

Consider horizontal motion in the lab frame. We can model the cooling as a thin element with $\Delta x' = -gx'$. As long as the tune is not too close to an integer the betatron amplitude decays as $A_x(n) \propto \exp(-gn/2)$ with turn number n. The inverse emittance cooling time is then

$$\frac{1}{\tau_c} = C_E\frac{Q}{M}\frac{1}{T_0}\frac{\sigma_{ze}}{\sigma_{zi}}\frac{2L_A + L_M + L_K}{2\gamma c}\frac{L_K}{\gamma c}, \tag{5}$$

with T_0 the revolution period and C_E given by equation (2).

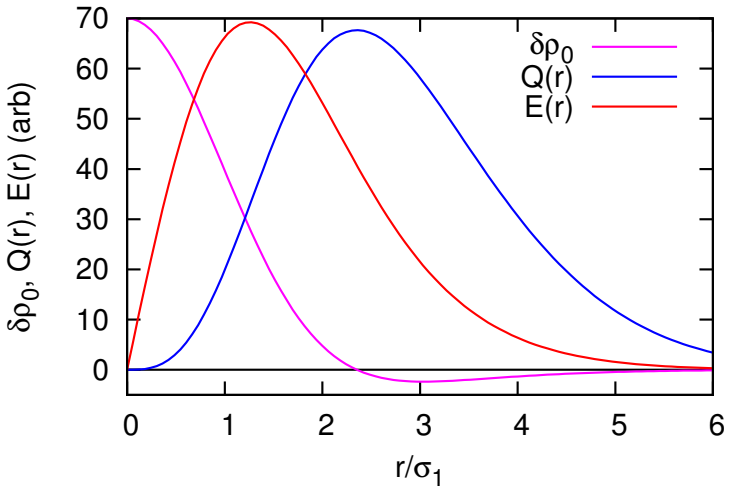


FIG. 2: Density pertubation as a function of radius for $\sigma_2 = 2\sigma_1$; enclosed charge within volume r, $Q(r)$; resulting radial electric field, $E(r)$.

With the cooling rate of equation (5) we need to know the maximum value of C_E, which is proportional to $\delta\rho_0$. The primary consideration is maintaing the linearity of the electron amplifier [1, 2]. For simplicity assume $r_2 = 2r_1$.

Relevant quantities are shown in Figure 2. We go back to the comoving frame and approximate the total density perturbation of the electron bunch as the linear superposition of density perturbations from all ions and electrons

$$\delta\rho_{tot}(\mathbf{r}) = \sum_{k=0}^{N_e} \delta\rho_e(\mathbf{r} - \mathbf{r}_{ek}) + \sum_{k=0}^{N_i} \delta\rho_e(\mathbf{r} - \mathbf{r}_{ik}), \tag{6}$$

where $\mathbf{r}_{ek}$ and $\mathbf{r}_{ik}$ denote the three dimensional locations, in the center of the modulator, of the electrons and ions, respectively. There are N_i ions and N_e electrons and the expression for $\delta\rho_{tot}$ is only valid well within the electron bunch. We neglect variations in $\delta\rho_0$ with macroscopic location within the electron bunch.

To estimate the maximum value of $\delta\rho_0$ we focus on the center of the electron bunch and use the parameter M defined via

$$\langle\delta\rho_{tot}^2(0)\rangle = \frac{1}{M}\rho_e^2(0), \tag{7}$$

where the angular brackets denote an ensemble average and $\rho_e(0)$ is the charge density at the center of the electron bunch. To estimate the ensemble average treat the electrons and ions as identical, independently distributed, random variables. Then

$$\langle\delta\rho_{tot}^2(0)\rangle = \int \delta\rho_e^2(\mathbf{r}) \left(N_e P_e(\mathbf{r}) + N_i P_i(\mathbf{r})\right) d^3r \approx \left(N_e P_e(0) + N_i P_i(0)\right) \int \delta\rho_e^2(\mathbf{r}) d^3r, \tag{8}$$

where $P_e(\mathbf{r})$ and $P_i(\mathbf{r})$ are the single particle distribution functions of the electrons and ions, respectively. The approximation assumes σ_1 and σ_2, the length scales of $\delta\rho_e$, are small compared to any of the distribution widths. With equation (1) the integral is straightforward.

$$\int \delta\rho_e^2(\mathbf{r}) d^3r = \delta\rho_0^2 \sigma_1^3 \pi^{3/2} \left(1 + \lambda^{-3} - \frac{2^{5/2}}{(1+\lambda^2)^{3/2}}\right) \equiv \delta\rho_0^2 V_{eff},$$

where we have defined $\lambda = \sigma_2/\sigma_1$ and V_{eff}, the effective volume of the perturbation.

Now, $\rho_e(0) = eN_eP_e(0)$ with e the charge of the electron. So

$$\delta\rho_0 = \frac{eN_eP_e(0)}{\sqrt{MV_{eff}[N_eP_e(0) + N_iP_i(0)]}}. \tag{9}$$

For a numerical estimate we use the parameters in Table I. The emittance cooling time of 1.4 hours is less than the 2 hour growth times predicted for IBS. . The parameter $\delta\rho_0 V_{eff}/e$ is an estimate of the number of electrons contributing to the kick of a single ion. The horizontal emittance of the protons is $\epsilon_x = 10$ nm. Coupled with $\beta_x = 200$ m this implies an angular spread of $\sigma(x') = 7 \times 10^{-6}$ and a typical motion between M and K of $\delta x/\sigma_1 = 1.6$. One sigma particles in x are at 1.6 on the horizontal axis of Figure 2. This is well outside the linear cooling region so the 1.4 hour cooling time estimate is too short. A nonlinear analysis is required to get a reliable estimate.

Consider horizontal motion in the lab frame. For small amplitudes we have $\Delta x' = -gx'$ where g is defined in equation (4). We have $g = T_0/\tau_c$. For larger values of x'

$$\Delta x' = -g\frac{E(L_{eff}x')}{L_{eff}dE/dx_0} \equiv -gf(x'), \tag{10}$$

where $E(x)$ is the horizontal electric field plotted in Figure 2, $L_{eff} = L_A + (L_M + L_K)/2$ is the effective distance between the modulator and the kicker, and dE/dx_0 is the derivative of E with respect to x at $x = 0$.

The change in the mean square angular spread on one traversal of the cooling system is

$$\Delta\langle x'^2\rangle = -2g\langle x'f(x')\rangle = -2g\int P_\theta(x')x'f(x')dx', \tag{11}$$

where $P_\theta(x')$ is the angular probability distribution function for the ions. In the linear approximation this is just

$$\Delta\langle x'^2\rangle = -2g\langle x'^2\rangle,$$

The nonlinear emittance cooling time $\tau_{c,nl}$ is related to the linear cooling time by

$$\frac{\tau_c}{\tau_{c,nl}} = \frac{\int P_\theta(x')x'f(x')dx'}{\int P_\theta(x')x'^2dx'}$$

Taking a Gaussian distribution for the ions and using the parameters in Table 1, $\tau_{c,nl} = 4.\tau_c = 5.6$ hours. The last concern is heating due to the kicks from the other coherent kicks created by the other ions and electrons [3]. We simply state it is irrelevantly small.

TABLE I: Cooling parameters for protons at peak luminosity. Entries above the double horizontal line are inputs and those below are outputs. Simulations by W. Bergan suggest $M = 10$ is a reasonable lower limit.

parameter	value
γ	293
$L_M = L_A = L_K$	40m
β_x, β_y	200m
σ_{xi}	1.4mm
σ_{yi}	0.47mm
σ_{zi}/γ	7 cm
N_i	7×10^{10}
$\sigma_{xe} = \sigma_{ye}$	1.4mm
σ_{ze}/γ	1cm
N_e	6.25×10^9 (1 nC)
M	10
$\sigma_1 = \sigma_2/2$	0.35mm $= \sigma_{xi}/4$
τ_c	1.4 hours
$\delta\rho_0 V_{eff}/e$	13.3

CONCLUSIONS

A first pass estimate of transverse cooling without using a bypass was made. An emittance cooling time of 5.6 hours was found. A cooling system without a bypass is probably cheaper and less prone to jitter and similar effects than a cooling system with a bypass. It is also less tuneable. However, this all relies on a sufficiently robust amplifier, which is by no means a certainty. Multiparticle simulation codes exist that can fully model these effects.

[1] G. Stupakov, MS. Zolotorev FEL2013 p132
[2] VN. Litvinenko, C-A/AP/480 2013
[3] M. Blaskiewicz, 'Cooling of High-Energy Hadron Beams', Annu. Rev.Nucl. Part. Sci. Vol 64, p299 (2014).