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Nominal oxygen parameters for RHIC Run 21

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In addition to gold ions, oxygen ions have been provided for oxygen-oxygen collisions in RHIC during the FY21 running period. The relevant parameters are summarized in this document. Details on how the parameters are calculated can be found in references [1] and [2].

1 Ion mass and energy

The mass-energy equivalents of the O8+ and O6+ ions are

$$mc^2 = 14.8950805330 \ \text{GeV} \ \underline{\text{O8+}} \tag{1}$$

and

$$mc^2 = 14.8961025309 \ \text{GeV} \ \underline{\text{O6+}}. \tag{2}$$

The energy per nucleon is

$$E/A = mc^2\gamma/A \tag{3}$$

and the kinetic energy per nucleon is

$$W/A = mc^2(\gamma - 1)/A \tag{4}$$

where A is the number of nucleons.

2 EBIS O6+ in Booster

At injection:

1. Revolution frequency $f = 96.640$ kHz
2. $4f = 386.560$ kHz
3. $B\rho = 0.539803510111$ Tm
4. $B = 389.311324509$ Gauss
5. $W/A = 1.97575052327$ MeV per nucleon
6. Inflector $V = 25.288$ kV

On merge porch:

1. $f = 553.000$ kHz
2. $B\rho = 3.32096941490$ Tm
3. $B = 2.39511410606$ kG
4. $W/A = 72.0706566891$ MeV per nucleon

At extraction:

1. $f = 960.000$ kHz
2. $B\rho = 7.01060241802$ Tm
3. $B = 5.06028370932$ kG
4. $W/A = 288.808684661$ MeV per nucleon
5. Bunch width $\mathcal{W} = 134$ ns. Here the longitudinal emittance of the merged bunch is taken to be 0.10 eV-s per nucleon. The bunch is assumed to be sitting in a stationary harmonic $h = 1$ bucket at 25 kV.

3 EBIS Au32+ in Booster (for comparision)

At injection:

1. Revolution frequency $f = 96.640$ kHz
2. $4f = 386.560$ kHz
3. $B\rho = 1.24651719998$ Tm
4. $B = 898.999826895$ Gauss
5. $W/A = 1.97627401907$ MeV per nucleon
6. Inflector $V = 58.396$ kV

On merge porch (for 12 transfers to AGS):

1. $f = 553.000$ kHz
2. $B\rho = 7.66880062604$ Tm
3. $B = 5.53081051382$ kG
4. $W/A = 72.0897525649$ MeV per nucleon

At extraction:

1. $f = 658.910$ kHz
2. $B\rho = 9.46202808578$ Tm
3. $B = 6.82973355563$ kG
4. $W/A = 107.758798130$ MeV per nucleon
5. Bunch width $\mathcal{W} = 274$ ns

4 O8+ in AGS with 6 to 3 to 1 merge

At injection with 6 to 3 merge:

1. Revolution frequency $f = 960/4 = 240$ kHz
2. $18f = 4.320$ MHz (Standard RF cavities)
3. $9f = 2.160$ MHz (Standard RF cavities)
4. $T = 1/f$
5. $T/18 = 231.481481$ ns
6. $8T/18 = 1851.852$ ns
   This, minus the bunch width, is the gap available for the AGS injection kicker [3].
7. $W/A = 288.788869956$ MeV per nucleon
8. $B\rho = 5.25759107381$ Tm
9. $B = 615.799088672$ Gauss

On 3 to 1 merge and squeeze porch:

1. $9f = 2.349$ MHz (Standard RF cavities)
2. $6f = 1.566$ MHz (KL cavity)
3. $3f = 0.783$ MHz (L10 cavity)
4. $W/A = 377.424655157$ MeV per nucleon
5. $B\rho = 6.13312066149$ Tm
6. $B = 718.346113465$ Gauss

At extraction:

1. $9f = 3.33313546773$ MHz
2. $B\rho = 81.11378003$ Tm
3. $B = 9471.79665265$ Gauss
4. $W/A = 11.2632945341$ GeV per nucleon
5. $\gamma = 13.0988075322$

5 Au77+ in AGS (for comparison)

At injection with 12 to 6 merge:

1. Revolution frequency $f = 163.125$ kHz
2. $24f = 3.915$ MHz (Standard RF cavities)
3. $12f = 1.9575$ MHz (Standard RF cavities)
4. $T = 1/f$
5. $T/24 = 255.427841635$ ns
6. $8T/24 = 2043$ ns
7. $W/A = 105.291998331$ MeV per nucleon
8. $B\rho = 3.88434102815$ Tm
9. $B = 454.956201737$ Gauss

On 6 to 2 merge and squeeze porch:

1. $12f = 2.349$ MHz (Standard RF cavities)
2. $8f = 1.566$ MHz (KL cavity)
3. $4f = 0.783$ MHz (L10 cavity)
4. $W/A = 164.485536147$ MeV per nucleon
5. $B\rho = 4.92742448406$ Tm
6. $B = 577.128092350$ Gauss

At extraction:

1. $12f = 4.43700723632$ MHz
2. $B\rho = 83.2210113689$ Tm
3. $B = 9717.86170763$ Gauss
4. $W/A = 8.86486800852$ GeV per nucleon
5. $\gamma = 10.5204666071$

6 O8+ in RHIC at injection

The nominal O8+ ion paramters are

1. $W/A = 11.2632945341$ GeV per nucleon
2. $E/A = 12.1942370674$ GeV per nucleon
3. $B\rho = 81.1137800300$ Tm
4. $\gamma = 13.0988075322$
5. $hf = 9.35616973399$ MHz, $h = 120$
6. $3hf = 28.06850920197$ MHz
7. AGS $hf = 4.44418062365$ MHz, $h = 12$.

7 O8+ in RHIC at store

The nominal O8+ ion paramters are

1. $E/A = 100$ GeV per nucleon
2. $B\rho = 667.099281304$ Tm
3. $\gamma = 107.418016066$
4. $hf = 28.14944344115$ MHz, $h = 360$
5. $7hf = 197.046104088$ MHz.

8 Nominal machine radii [1]

The nominal orbit radius in RHIC is

$$R_R = 3833.845181/(2\pi) \text{ meters.} \tag{5}$$

At AGS extraction the nominal radius is

$$R_A = (4/19)R_R. \tag{6}$$

At AGS injection the nominal radius is

$$R_A = 128.4526 \text{ meters.} \tag{7}$$

At Booster extraction the nominal radius is one fourth the nominal radius at AGS injection. At Booster injection the nominal radius is

$$R_B = 201.780/(2\pi) \text{ meters.} \tag{8}$$

9 Energy loss in the BTA stripping foil

The stripper used for oxygen ions in the BTA transfer line is an aluminum foil with surface density (as measured by Peter Thieberger)

$$\rho d = 4.50 \text{ mg/cm}^2. \tag{9}$$

We consider two revolution frequencies in Booster at extraction and estimate for each the energy loss in the foil.

For oxygen ions extracted from Booster at revolution frequency

$$\underline{f = 740 \text{ kHz}} \tag{10}$$

the kinetic energy of a proton that has the same velocity as the ion is

$$W_p = 143.75 \text{ MeV.} \tag{11}$$

The rate of energy loss of a proton passing through the foil with kinetic energy W_p is

$$-\frac{dE_p}{dx} = 4.388 \text{ MeV cm}^2/\text{g.} \tag{12}$$

The rate of energy loss of the ion in the foil is obtained by scaling the Bethe-Bloch result for protons. This gives

$$-\frac{dE}{dx} = -Q^2 \frac{dE_p}{dx} \tag{13}$$

and, for $Q = 8$,

$$-\frac{dE}{dx} = 280.832 \text{ MeV cm}^2/\text{g}. \tag{14}$$

Multiplying this by the foil surface density gives energy loss

$$\Delta E_{740} = 0.07898 \ \text{ MeV per nucleon.} \tag{15}$$

The resulting revolution frequency of the ion in AGS at injection is then

$$f_A = (739.835/4) \text{ kHz} \tag{16}$$

which very close to the nominal

$$f_A = 740/4 \ \text{ kHz.} \tag{17}$$

For oxygen ions extracted from Booster at revolution frequency

$$\underline{f = 960 \text{ kHz}} \tag{18}$$

we have

$$W_p = 291.06 \text{ MeV} \tag{19}$$

$$-\frac{dE_p}{dx} = 2.819 \text{ MeV cm}^2/\text{g} \tag{20}$$

and

$$-\frac{dE}{dx} = 180.416 \text{ MeV cm}^2/\text{g}. \tag{21}$$

This gives energy loss

$$\Delta E_{960} = 0.05074 \ \text{ MeV per nucleon} \tag{22}$$

and revolution frequency

$$f_A = (959.944/4) \text{ kHz} \tag{23}$$

in AGS at injection. This is again very close to the nominal

$$f_A = 960/4 \ \text{ kHz.} \tag{24}$$

10 Heating and cooling in the foil

We consider first the temperature rise in the aluminum foil assuming no cooling by heat flow or radiation.

The energy deposited as N ions travel a small distance d in the foil is

$$E = -N \frac{dE}{dx} \rho d \tag{25}$$

where ρ is the density of the foil material. As shown in the previous section

$$-\frac{dE}{dx} = 280.832 \text{ MeV cm}^2/\text{g} \tag{26}$$

and

$$-\frac{dE}{dx} = 180.416 \text{ MeV cm}^2/\text{g} \tag{27}$$

for oxygen ions extracted from Booster at revolution frequencies 740 kHz and 960 kHz respectively.

If the ions are incident on foil surface area A then the energy is deposited in mass

$$M = \rho A d. \tag{28}$$

The resulting temperature increase (assuming no heat flow or radiation) is

$$\Delta T = \frac{E}{cM} = -\frac{N}{cA}\frac{dE}{dx} \tag{29}$$

where c is the heat capacity of the foil material. Note that the factor ρd cancels out when (25) is divided by (28). The heat capacity of aluminum is

$$c = 0.897 \text{ J/(gK)}. \tag{30}$$

As an upper limit we take

$$N = 200 \times 10^9 \tag{31}$$

oxygen ions incident on the foil per AGS cycle. As in [4], we take

$$A = 0.5 \text{ cm}^2 \tag{32}$$

and using

$$1 \text{ eV} = 1.602\,176\,634 \times 10^{-19} \text{ Joules} \tag{33}$$

we obtain temperature increases

$$\Delta T_{740} = 20.0643 \text{ K}, \quad \Delta T_{960} = 12.8900 \text{ K} \tag{34}$$

for oxygen ions extracted from Booster at revolution frequencies 740 kHz and 960 kHz respectively.

Now allowing the foil to cool radiatively after each energy deposition and carrying out the analysis described in [4], we find that an equilibrium is reached in which the foil temperature repeatedly peaks at a temperature T_H and cools to a temperature T_C. As in [4], we assume that 8 Booster loads of $N/8$ ions are extracted into the BTA line per AGS cycle. The Booster cycle and supercycle periods are taken to be 267 and 5600 ms respectively. The resulting peak and cooled temperatures are

$$T_H = 420.655 \text{ K}, \quad T_C = 407.303 \text{ K} \tag{35}$$

and

$$T_H = 388.372 \text{ K}, \quad T_C = 379.791 \text{ K} \tag{36}$$

for oxygen ions extracted from Booster at revolution frequencies 740 kHz and 960 kHz respectively. Figure 1 shows the time evolution of the foil temperatures for the two extraction frequencies. The temperatures stay well below the melting point of aluminum.

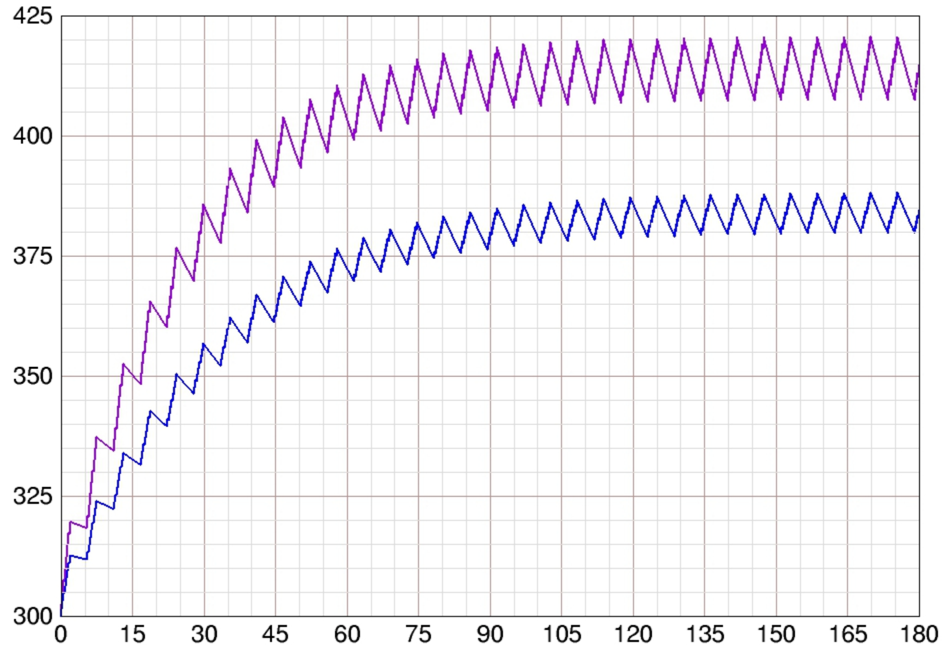

Figure 1: Aluminum foil temperature over 32 supercycles with $A = 0.50$ cm^2 and $N = 200\text{e}9$ oxygen ions incident on the foil per supercycle. The horizontal axis gives the time in seconds. The vertical axis gives the temperature in degrees K. The melting point of aluminum is 933.47 K. The violet (upper) and blue (lower) traces show the temperature for ions extracted from Booster at revolution frequencies 740 kHz and 960 kHz respectively. In each trace an equilibrium is reached in which the temperature repeatedly peaks at a temperature T_H and cools to a temperature T_C. These are 421 and 407 K for the upper curve and 388 and 380 K for the lower curve. The Booster and supercycle periods are 267 and 5600 ms respectively.

11 Rate of oxygen ion energy loss in copper

We consider O8+ ions traveling in the copper absorber of the AGS beam dump at extraction energy. The kinetic energy of the ion at extraction is

$$W/A = 11.2632945341 \ \text{GeV/nucleon}. \tag{37}$$

The kinetic energy of a proton that has the same velocity as the ion is

$$W_p = 11.3519733245 \ \text{GeV}. \tag{38}$$

The rate of energy loss of a proton traveling in copper with kinetic energy W_p is [5]

$$-\frac{dE_p}{dx} = 1.57 \ \text{MeV cm}^2/\text{g}. \tag{39}$$

The rate of energy loss of the O8+ ion traveling in copper is obtained by scaling the Bethe-Bloch result for protons [6]. This gives

$$-\frac{dE}{dx} = -Q^2 \frac{dE_p}{dx} \tag{40}$$

and, for $Q = 8$,

$$-\frac{dE}{dx} = 0.101 \ \text{GeV cm}^2/\text{g}. \tag{41}$$

This is just 1.055% of the rate

$$-\frac{dE}{dx} = 9.5737 \ \text{GeV cm}^2/\text{g} \tag{42}$$

for Au79+ ions in copper [7] with kinetic energy

$$W/A = 8.8649 \ \text{GeV/nucleon}. \tag{43}$$

Since we have safely run with

$$N = 6 \times 10^9 \tag{44}$$

Au79+ ions put into the dump per AGS cycle at this energy, one might conclude that we could safely run with as many as

$$N = (9.5737/0.101) \times 6 \times 10^9 = 569 \times 10^9 \tag{45}$$

O8+ ions per AGS cycle at energy (37). However, as discussed in [8], we must take into account the temperature increase given by

$$\Delta T = -\frac{N}{c\text{A}} \frac{dE}{dx} \tag{46}$$

where

$$c = 0.385 \ \text{J/(gK)} \tag{47}$$

is the heat capacity of copper and A is area upon which the ions are incident. The area A is estimated to be an order of magnitude smaller for oxygen ions than it is for gold ions owing to the fact that the Au77+ ions can be stripped to Au79+ upsteam of the dump thereby increasing the area of incidence on the copper absorber [9]. That means that the number in (45) should be reduced by an order of magnitude to

$$N = 57 \times 10^9 \tag{48}$$

O8+ ions per AGS cycle.

12 Energy deposited in the dump per AGS cycle

The kinetic energy of a single O8+ ion circulating in AGS at extraction is

$$W/A = 11.2633 \ \text{GeV per nucleon.} \tag{49}$$

Multiplying by the number of nucleons ($A = 16$) and converting to Joules we have

$$W = 28.873 \ \text{nJ.} \tag{50}$$

If N oxygen ions are put into the dump per AGS cycle then the total energy deposited is NW. For

$$N = 60 \times 10^9 \tag{51}$$

we have

$$NW = 1732.4 \ \text{Joules} \tag{52}$$

per AGS cycle. This is to be compared with

$$NW = 1678.8 \ \text{Joules} \tag{53}$$

obtained in [10] for

$$N = 6 \times 10^9 \tag{54}$$

Au79+ ions per AGS cycle at

$$W/A = 8.8649 \ \text{GeV per nucleon.} \tag{55}$$

Since we have safely run with gold ions under these conditions, we can conclude that it is safe to operate AGS with the numbers given in (49) and (51) for O8+ ions.

References

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[6] W.R. Leo, "Techniques for Nuclear and Particle Physics Experiments", Second Revised Edition, Springer-Verlag, 1994, pp. 24–28.

[7] C.J. Gardner, "FY2020–21 parameters for Gold ions in Booster, AGS, and RHIC," C-A/AP/Note 639, February 2021, Section 31.

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[9] C.J. Gardner, L.A. Ahrens, and P. Thieberger, "Notes on Dumping Gold Beam in the AGS," C-A/AP/Note 396, August 2010.

[10] C.J. Gardner, "FY2020–21 parameters for Gold ions in Booster, AGS, and RHIC," C-A/AP/Note 639, February 2021, Section 33.