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Longitudinal space charge kick in Coherent Electron Cooling

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EIC strong hadron cooling

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Abstract

In recent EIC Coherent electron Cooling (CeC) design, we adopted 591 MHz SRF LINAC to accelerate electron beam to certain energy(150 MeV, 54.1 MeV and 22.3 MeV). To reduce the energy spread caused by the RF curvature,a short bunch is preferred and results in high peak current. The longitudinal space charge kick will cause the energy spread increasing and lengthen the bunch length when through the R_{56} elements. This slippage of modulated microbunches may misalign with hadrons providing imprinting at kicker section. In this note, we will estimate this effect and discuss possible solution.

1 Longitudinal electric field calculation

A round Gaussian beam(both longitudinal and transverse are Gaussian distribution)'s current density is

$$J_z = \nu\rho(r,z,t) = \frac{I_{peak}}{2\pi\sigma_r^2}e^{-\frac{r^2}{2\sigma_r^2}-\frac{(z-vt)^2}{2\sigma_z^2}} \tag{1.1}$$

where z is particle coordinate in lab frame,v is beam's velocity, σ_r and σ_z are standard deviation of transverse and longitudinal distribution and $I_{peak}$is the peak current. When the $\gamma > b/\sigma_z$, where b is radius of the beam pipe. In the rest frame, the transverse electric field is approximately **?**

$$E_r = \frac{Z_0 I_{peak}}{2\pi\gamma r}(1-e^{-r^2/2\sigma_r^2})e^{-z^2/2(\gamma\sigma_z)^2} \tag{1.2}$$

where Z_0 is free space impedance, 377 Ohm.z and r are the rest frame coordinate. Now place the beam in the center of the pipe and pipe is grounded. The electrostatic potential and longitudinal electric field are

$$\Phi(r,z) = \int_r^b E_r dr \tag{1.3}$$

$$E_z = -\frac{\partial\Phi}{\partial z} \tag{1.4}$$

Considering the beam pipe and assuming $\gamma > 3b/\sigma_z$ **?**, in lab frame, we get

$$\langle E_z\rangle = \frac{Z}{n}\frac{z-vt}{2\pi\sigma_z^2}I_{peak}e^{-\frac{(z-vt)^2}{2\sigma_z^2}} \tag{1.5}$$

where,

$$\frac{Z}{n} = \frac{Z_0}{\beta\gamma^2}ln(\frac{b}{1.5\sigma_r}) \tag{1.6}$$

We calculate the longitudinal space charge force F at position of 1 σ_z, $z-vt=\sigma_z$, the F unit in Eqn **??** is eV/m.

$$F = q\langle E_z\rangle = q\frac{e^{-1/2}}{2\pi}\frac{Z_0 I_{peak}}{\beta\gamma^2\sigma_z}ln(\frac{b}{1.5\sigma_r}) = 0.0965\frac{Z_0 I_{peak} q}{\beta\gamma^2\sigma_z}ln(\frac{b}{1.5\sigma_r}) \tag{1.7}$$

2 Micro-bunch slippage in R_{56}

The energy spread of electron bunch can change the bunch length by passing R_{56} elements such as Chicane, drift or dogleg. In microbunch plasma enhanced amplifier CeC**?**, the amplification section includes three Chicane/doglegs and two drift section as shown in figure **??**.

The slippage of the microbunch at σ_z can be calculated by

$$\delta L = \frac{F\cdot L}{E}R_{56} \tag{2.1}$$

$F\cdot L$ is the energy spread after electron bunch drifts length of $L.E$ is kinetic energy.

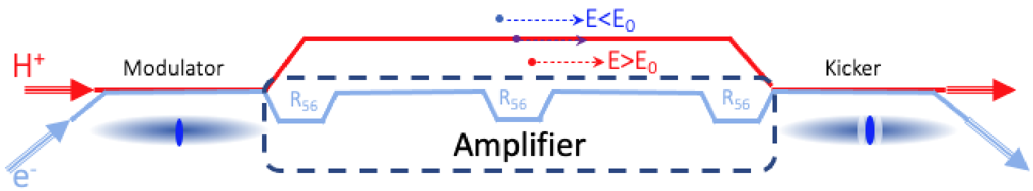

Figure 1: Schematic drawing of the CeC layout.Amplifier section is labeled in dash frame

3 Estimate microbunch slippage in EIC strong hadron cooler

In EIC design, we are going to cool hadrons in three different energies by CeC type strong hadron cooler.**?** The optimal cooling parameters and correspond hadron energy are listed in In EIC pCDR, the strong

Table 1: EIC CeC parameters

Parameter	Value		
Hadron gamma	44.7	107	294
Electron energy [MeV]	22.3	54.1	150
Electron R_{56} [cm]	-1.26	-0.652	-0.68
Modulator, Kicker length [m]	30	40	50
Amplification length [m]	46	44	96

hadron cooling amplification uses three-dipoles chicane($R56 < 0$) to convert the energy modulation to density modulation. The energy spread increase linearly in drift. Because the beam size in modulator, amplification and kicker are different about factor of 5, we evaluate the slippage in these sections independently.
In modulator section:

$$\delta L_{mod} = \int_0^{L_{mod}} \frac{F \cdot l}{E} \frac{1}{\gamma^2} dl = \frac{F \cdot L_{mod}^2}{2E} \frac{1}{\gamma^2} \tag{3.1}$$

In amplification section:

$$\delta L_{amp} = \int_0^{L_{amp}} \left(\frac{F(\beta_{mod})L_{mod}}{E} + \frac{F(\beta_{amp})l}{E}\right)\frac{1}{\gamma^2} dl + 3 \times R_{56}\left(\frac{F(\beta_{mod}) \cdot L_{mod}}{E} + \frac{F(\beta_{amp}) \cdot L_{amp}}{2E}\right) \tag{3.2}$$

In kicker section:

$$\delta L_{kick} = \int_0^{L_{kick}} \left(\frac{F(\beta_{mod})L_{mod}}{E} + \frac{F(\beta_{amp})L_{amp}}{E} + \frac{F(\beta_{kick})l}{E}\right)\frac{1}{\gamma^2} dl \tag{3.3}$$

Then, total slippage is $\delta L = \delta L_{mod} + \delta L_{amp} + \delta L_{kick}$. Using the parameters from Table **??** and Eqn. (3.1-3.3), the calculated the total slippage are listed in Table **??**.

Table 2: Slippage evaluation in different energies

Gamma (Electron Energy)	δL_{drift}	$\delta L_{chicane}$	δL_{total}
294(150 MeV)	18 nm	160 nm	178 nm
107(54.1 MeV)	2.5 um	2.79 um	5.29 um
44.7(22.3 MeV)	71 um	33 um	104 um

Considering the bandwidth is scale of um, the cases of $\gamma = 107, 44.7$ are in danger range.

4 Proposed solution

There are several ways to reduce longitudinal space charge kick induced microbunch slippage by either reducing the drift length or control the sign of chicane/dogleg type R_{56} elements.
For example:

- Reduce the length of modulator section. The length of modulator section can be shorter than kicker length. Recently, the design shows reduce the modulator length to 20 meters can help to reduced saturation as well. It is still in development. We will not discuss here.
- Reduce the amplification section beam size by adding more focusing lens. That can reduce the plasma wavelength and result in short amplification length. However, it will increase cost of focusing magnets. We will not discuss here as well.
- Reverse the two of R_{56} elements sign in amplification section. It can compensate the slippage in somewhat.

In the following discussion, we are focusing on the 3rd method. When three of R_{56} elements have different value in amplification section, the δL_{R56} (the 2nd term of 3.2) can be rewritten as

$$\delta L_{R56} = R_{56\_1}\left(\frac{F(\beta_{mod})\cdot L_{mod}}{E}\right) + R_{56\_2}\left(\frac{F(\beta_{mod})\cdot L_{mod}}{E} + \frac{F(\beta_{amp})\cdot L_{amp}}{2E}\right) + R_{56\_3}\left(\frac{F(\beta_{mod})\cdot L_{mod}}{E} + \frac{F(\beta_{amp})\cdot L_{amp}}{E}\right) \tag{4.1}$$

By replacing the three-dipoles chicane by two-dipoles dogleg, the R56 sign can be reversed. To avoid anti-cooling, $R_{56_1}R_{56_2}R_{56_3} < 0$. Here negative R_{56} means the high momentum particles move forward. Therefore, two of chicanes are replaced by dogleg as shown in Figure ??.

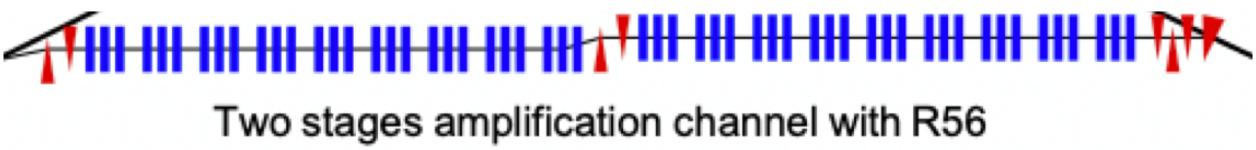

Figure 2: Proposed amplification arrangement

We evaluated the slippage at σ_z again and listed at Table ??

Table 3: EIC CeC parameters

Gamma (Electron Energy)	δL_{drift}	$\delta L_{R_{56}}$	δL_{total}
294(150 MeV)	18 nm	-53 nm	-35 nm
107(54.1 MeV)	2.5 um	-0.93 um	1.57 um
44.7(22.3 MeV)*	71 um	-69.1 um	1.9 um

* For the case of gamma=44.7, the R56 of dogleg was optimized to get small slippage and still get cooling time less than 2hrs. The selected R56 are -0.86 cm, 3.2 cm and 3.2 cm.

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