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Effect of hadron-electron focusing in EIC low energy coolers

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Effect of Hadron-Electron Focusing in EIC Low Energy Coolers

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Abstract

A promising approach to achieving the high luminosity in the Electron Ion Collider (EIC) [1] is to apply a bunched electron cooling to the protons at low energy. The most feasible option for such a cooler is a non-magnetized bunched e-cooling. Among many factors affecting the e-bunch angular spread in the cooling section (CS) of the EIC low energy cooler (LEC), the focusing induced on electrons by the space charge of the p-beam presents unique challenges. In this note we both derive analytical formulas to estimate this effect and perform numerical simulations of hadron-electron focusing. The ultimate goals of this exercise are to set a framework for prompt estimates of the p-e focusing under various LEC design parameters and to check the feasibility of non-magnetized approach for the EIC coolers.

1 Introduction

One of the scenarios of achieving the luminosity of 10^{34} cm^{-2}s^{-1} in the EIC is to pre-cool protons for 30 min to the required transverse emittance at injection energy of 25 GeV and to replace the colliding p-beam once per 45 minutes with the pre-cooled protons accelerated to the collision energy. It is also feasible to utilize the same electron cooler for the case of EIC operations with 41 GeV protons both as a pre-cooler at 25 GeV energy and as a cooler counteracting IBS-driven emittance growth during ten hours of collisions.

The LEReC [2, 3, 4, 5] experience proves that bunched electron cooling with non-magnetized e-beam is an effective approach to relativistic electron coolers, which can be scaled to any energy.

Among many factors contributing to the angular spread of the e-bunches in the CS of non-magnetized coolers, the hadron-electron focusing stands

out as a uniquely challenging one. The space charge of hadron bunches co-traveling with electrons in the CS provide a constant nonlinear focusing on the e-bunch. This focusing can result in substantial growth of the e-bunch angular spread and it can not be counteracted by lumped focusing elements, such as short solenoidal modules installed every 3 m throughout the LEReC CS.

Although the relativistic γ-factor even for the 25 GeV cooler ($\gamma = 27$) is more than 5 times higher than the γ-factor of the LEReC at its highest operational energy ($\gamma = 5$), the requirements to the e-p relative angles in the EIC LEC are much stricter than the LEReC angular spread requirements. Therefore, in spite of the space charge effect being inversely proportional to γ^3, the p-e focusing might be a problem for the EIC coolers.

If the hadron-electron focusing for the chosen parameters of the cooler results in intolerable growth of electrons' angles then the only option for mitigating this effect is to employ a continues CS solenoidal field, typically of a few hundred Gauss, matched by the field on the cathode [6, 7]. This approach leads to the cooling with weakly magnetized electron beam and it was successfully employed at Fermilab electron cooler [8].

While we considered an option of weakly magnetized cooling for the EIC LEC [9, 10] and found it to be manageable, our goal is to find such design parameters for the LEC that no beam magnetization is required.

Below we will derive analytical formulas allowing for prompt estimates of the angular spread driven by hadron-electron focusing. We will perform numerical simulations of this effect. Finally, we will show that for the proper choice of the LEC parameters e-beam magnetization is not required, hence, concluding that the LEReC-style cooler is a feasible option for the EIC LEC.

2 LEC Parameters

Simulation of the cooling process performed with Beta-Cool code showed [11] that the overall rms angular spread throughout the LEC cooling section must be $\sigma_{\theta tot} = 30 - 40$ μrad. Other relevant electron and proton bunch parameters in the cooling section (CS) of both 45 GeV and 25 GeV EIC coolers are given in Table 1.

The p-bunch parameters for 25 GeV cooler in Table 1 are given for the beginning of the cooling process. At the end of the cooling the vertical emittance of the protons must be reduced by a factor of four, which results in final $\sigma_{py1} = 1.9$ mm.

The longitudinal distribution of the p-bunch at the end of the cooling in the 25 GeV LEC will be flattened, because the two e-bunches are placed

Parameter	**25 GeV**	**41 GeV**
CS length (L_{CS}), m	130	130
number of e-bunches per p-bunch	2	1
electron bunch charge (Q_e), C	2.5	1.3
proton bunch charge (Q_p), C	45	4.2
relativistic γ-factor	27	44
p-bunch transverse size (σ_p), mm	3.8	3
e-bunch transverse size (σ_e), mm	3.6	3
e-bunch thermal angular spread ($\sigma_{\theta th}$), urad	20	20
p-bunch length (σ_{pz}), cm	60	7.5
e-bunch length (σ_{ez}), cm	5	3

Table 1: Parameters of 25 GeV and 41 GeV coolers.

longitudinally of-center of the p-bunch. Cooling simulations show that for the chosen parameters at the end of the cooling the current of the longitudinal p-slice overlapped with the e-bunches will be about 6 A.

The longitudinal distribution of the e-bunch can be Gaussian (with σ_{ez} listed in Table 1) or it can be made uniform.

3 Analytical formulas

Let us assume that both the e-bunch and p-bunch have an axially symmetric transverse distribution. This corresponds to the actual conditions at the beginning of the cooling (at 25 GeV). At the end of the cooling though the p-bunch has a flattened transverse distribution. Therefore, our approximation will lead to an overestimate of the p-e focusing at the end of the cooling process since we are overestimating the transverse density of the protons by a factor of $\sigma_{p0}/\sigma_{py1} = 2$.

Next, let us assume that both electrons and protons have a uniform transverse distribution with respective radii of $a_e = 2\sigma_e$ and $a_p = 2\sigma_p$.

The r-equation of a longitudinal slice of e-bunch with current I_e probing p-slice with current I_p under our assumptions (and for $r \leq a_p$) can be written as [12]:

$$r'' = K_e \left(\frac{r_0^2}{a_{e0}^2} \frac{1}{r} - \frac{I_p}{I_e} \frac{r}{a_p^2} \right) - kr + \frac{\varepsilon^2 r_0^4}{\sigma_{e0}^4} \frac{1}{r^3} \tag{1}$$

where, generalized perviance $K_e = \frac{2I_e}{I_A \beta^3 \gamma^3}$, I_A is Alfven current, external focusing k is assumed to be zero, $r = r_0$ at the entrance to the CS and $a_{e0} = 2\sigma_{e0}$ is the radius of the e-bunch at the entrance to the CS.

Under near-equilibrium conditions we can represent r as $r(s) = r_0 + \rho(s)$ with $\rho/r_0 \ll 1 \; \forall \; s$ in the CS. Expanding the right hand side of (1) in Taylor series, keeping the first degree of ρ/r_0 and solving the obtained differential equation we get:

$$\begin{cases} r &= r_0 + r_0 \frac{f}{\kappa} \left(1 - \cos(\sqrt{\kappa} \cdot s)\right) \\ \theta &= r_0 \frac{f}{\sqrt{\kappa}} \sin(\sqrt{\kappa} \cdot s) \\ f &= \frac{K_e}{I_e} \frac{I_e a_p^2 - I_p a_{e0}^2}{(a_{e0} a_p)^2} + \frac{\varepsilon^2}{\sigma_{e0}^4} \\ \kappa &= \frac{K_e}{I_e} \frac{I_e a_p^2 + I_p a_{e0}^2}{(a_{e0} a_p)^2} + \frac{3\varepsilon^2}{\sigma_{e0}^4} \end{cases} \tag{2}$$

Here $\theta = r'$ is the correlated angle of r-layer and we assumed that at the entrance to the CS $\theta(0) = 0$.

Averaging θ^2 over transverse distribution and over the length of the cooling section (noticing that for a wide range of the LEC parameters $\sqrt{\kappa} L_{CS} \geq \pi$) we get the following approximation for the quadrature of the mean correlated angular spread driven by the p-e focusing:

$$\sigma_\theta^2 = \frac{a_{e0}^2}{8} \frac{f^2}{\kappa} \tag{3}$$

If the e-bunch has a uniform longitudinal distribution and its length (as compared to σ_{pz}) is small enough to assume that the slice current I_p is constant over the length of the e-bunch, then (3) solves the problem of estimating the e-bunch angular spread resulting from hadron-electron focusing. On the other hand, if the e-bunch has a Gaussian distribution, or if it is not short in comparison to p-bunch, then (3) must be convoluted with e-bunch distribution function $p(z) = \frac{1}{\sqrt{2\pi}\sigma_{ez}} \exp(-\frac{z^2}{2\sigma_{ez}^2})$ taking into account $f(I_e(z), I_p(z))$ and $\kappa(I_e(z), I_p(z))$ dependence. The required numerical integration can be easily carried out in any one of standard scientific applications.

For the case of substantial difference in a_p and a_e which is relevant for the end of cooling process in the 25 GeV LEC, the near-equilibrium condition is broken, nonetheless, as it will be shown below, the equations derived under an assumption of near-equilibrium still give reasonable results.

As a simple estimate, when $a_p < a_e$, one can consider only the angular spread of the portion of e-bunch physically overlapping with the protons in the cooling section.

Averaging angles (2) we obtain the formula for the quadrature of the mean correlated angular spread of electrons in $a_p < a_e$ case:

$$\sigma_\theta^2 = \frac{f^2}{\kappa} \frac{a_p^4}{8 a_{e0}^2} \tag{4}$$

3.1 25 GeV cooler

Applying (3) to the 25 GeV LEC at the start of the cooling process we see that the optimal longitudinal placement of the e-bunches is at $\approx \pm\sigma_{pz}$. Figure 1 shows dependence of the total angular spread on longitudinal position of the e-bunch for e-bunch with Gaussian and uniform (with total length of 12.5 cm) distributions.

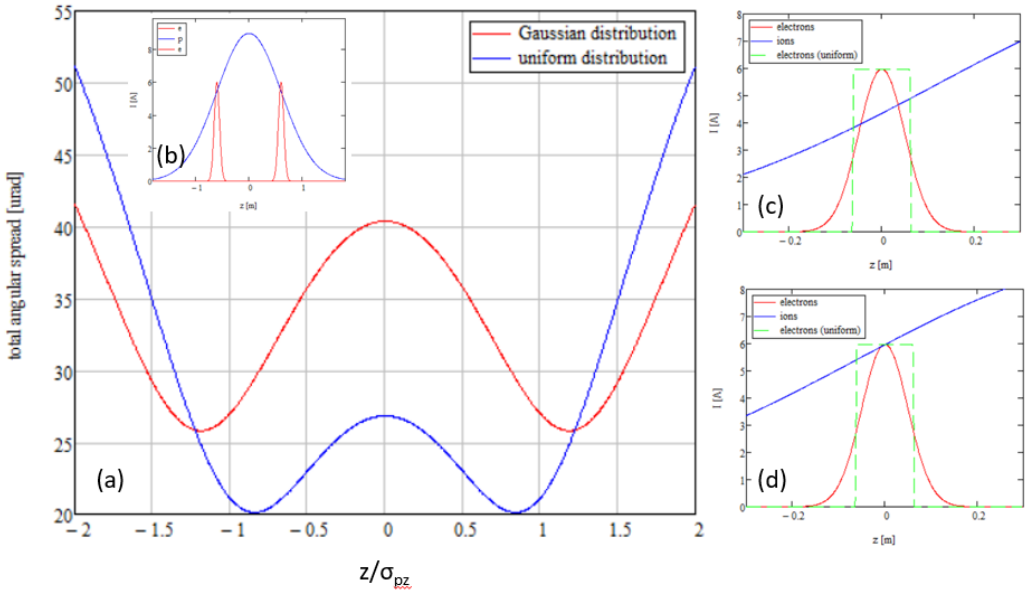


Figure 1: (a) Total rms angular spread of the electron bunch with Gaussian (red) and uniform (blue) longitudinal distributions depending on the e-bunch position with respect to the p-bunch. (b) Two electron bunches overlapping with the proton bunch. (c) Electron bunch placed at $-1.15\sigma_{pz}$ - optimal for Gaussian e-bunch. (d) Electron bunch placed at $-0.9\sigma_{pz}$ - optimal for uniform e-bunch.

Next, we place the two e-bunches at $\pm\sigma_{pz}$ and apply (4) to find the correlated angular spread (driven by p-e focusing) of the useful portion of e-bunch throughout the cooling process in the 25 GeV LEC. We see (Fig. 2) that the angles stay small even if the p-bunch core is severely over-cooled to $0.25\sigma_{p0}$.

3.2 41 GeV cooler

Application of (3) to the 41 GeV LEC shows (Fig. 3) that the electron bunch can be placed anywhere within $\pm 2\sigma_{pz}$ of the p-bunch. This result is true for both Gaussian and uniform longitudinal distribution of the e-bunch.

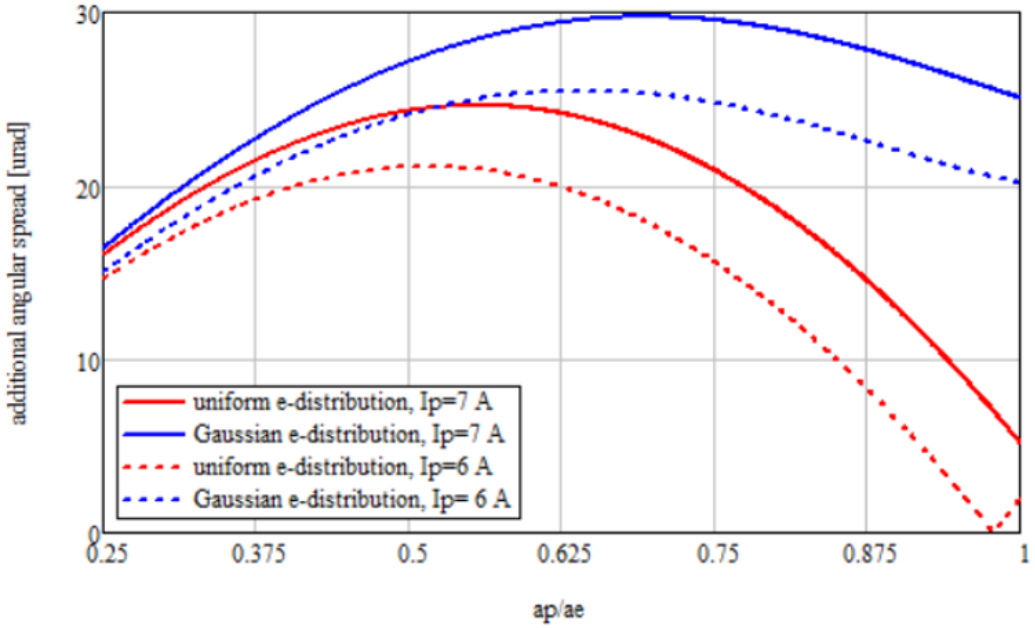


Figure 2: Correlated (caused by p-e focusing) rms angular spread of the electron bunch with Gaussian (blue) and uniform (red) longitudinal distributions depending on a_p/a_e. The solid lines are calculated for the p-slice probed by e-bunch with $I_p = 7$ A. The dash lines are for $I_p = 6$ A.

4 Simulations of proton-electron focusing

In reality both the electron and the proton bunches in the cooling section will have essentially non-uniform transverse distributions, which will result in strongly non-linear space charge focusing of the electron bunch.

Our approach to fast simulations of this effect is described in details in [12]. A dedicated simulation code based on the described approach was benchmarked with experimental data in the LEReC, where the hadron-electron focusing is a very strong and critically important effect.

To check the analytical formulas derived in the previous section we first compare their results to simulations done for near-equilibrium conditions. Figure 4 shows rms envelope and the rms angular spread of the e-bunch both simulated and calculated from (2) for near-equilibrium parameters.

The total rms angular spread in the CS obtained in simulations presented in Fig. 4 is 20.1 μrad, while the angular spread obtained from (3) is 20.4 μrad. This confirms that, as one expects, for near-equilibrium parameters the derived analytical formulas give extremely good summing-up of the hadron-electron focusing effects.

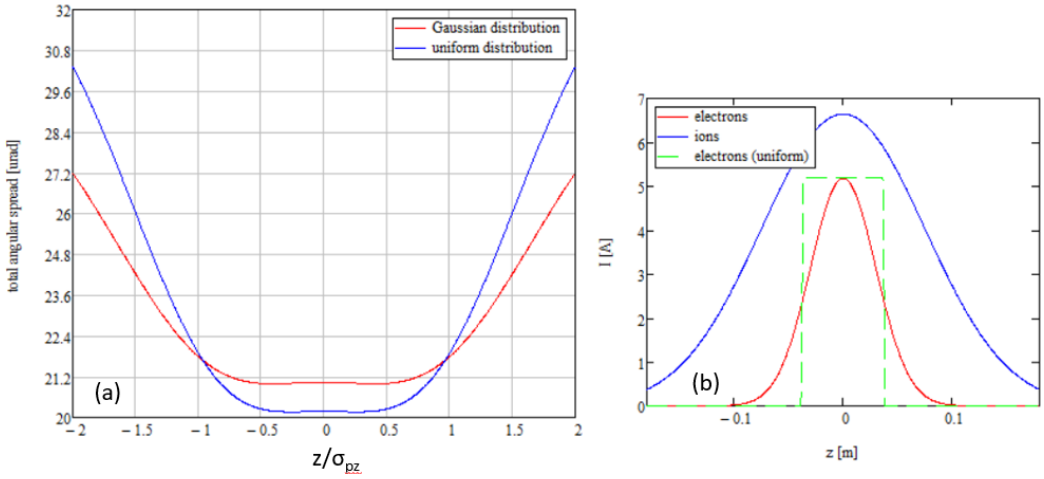


Figure 3: (a) Total rms angular spread of the electron bunch with Gaussian (red) and uniform (blue) longitudinal distributions depending on the e-bunch position with respect to the p-bunch. (b) Electron bunch centered with respect to the p-bunch.

Comparison of analytic formulas to numerical simulations when two beams' parameters are far from equilibrium show that analytical expressions still give a reasonable approximation even for the strong hadron-electron focusing effect. Figure 5 shows simulations and analytical calculations of electron beam envelope and angular spread for the case of $I_e = 3$ A and $I_p = 6$ A.

The total rms angular spread in the CS obtained in simulations presented in Fig. 5 is 42 μrad, while the angular spread obtained from (3) is 30 μrad.

There are two important implications of this result. First, the averaging of angular spread (3) over longitudinal distribution of e- and p- bunches for the case of "not-short" and/or longitudinally nonuniform e-bunch gives a good estimate of the expected total angular spread, even though the near-equilibrium conditions are realized only for central slices of the e-bunch. Second, formulas (3) and (4) give a good first approximation of the expected angular spread throughout the whole cooling process for the case of the actively cooled p-beam (i.e. the beam with shrinking emittance).

4.1 25 GeV Cooler

We performed a set of simulations of proton-electron focusing effect in the LEC CS for proton bunch size changing from $\sigma_{p0} = 3.8$ mm to $\sigma_p = 0.5\sigma_{p0}$. Figure 6 shows the expected angular spread of the e-bunch throughout the whole cooling process. The simulations were performed for the proton slice

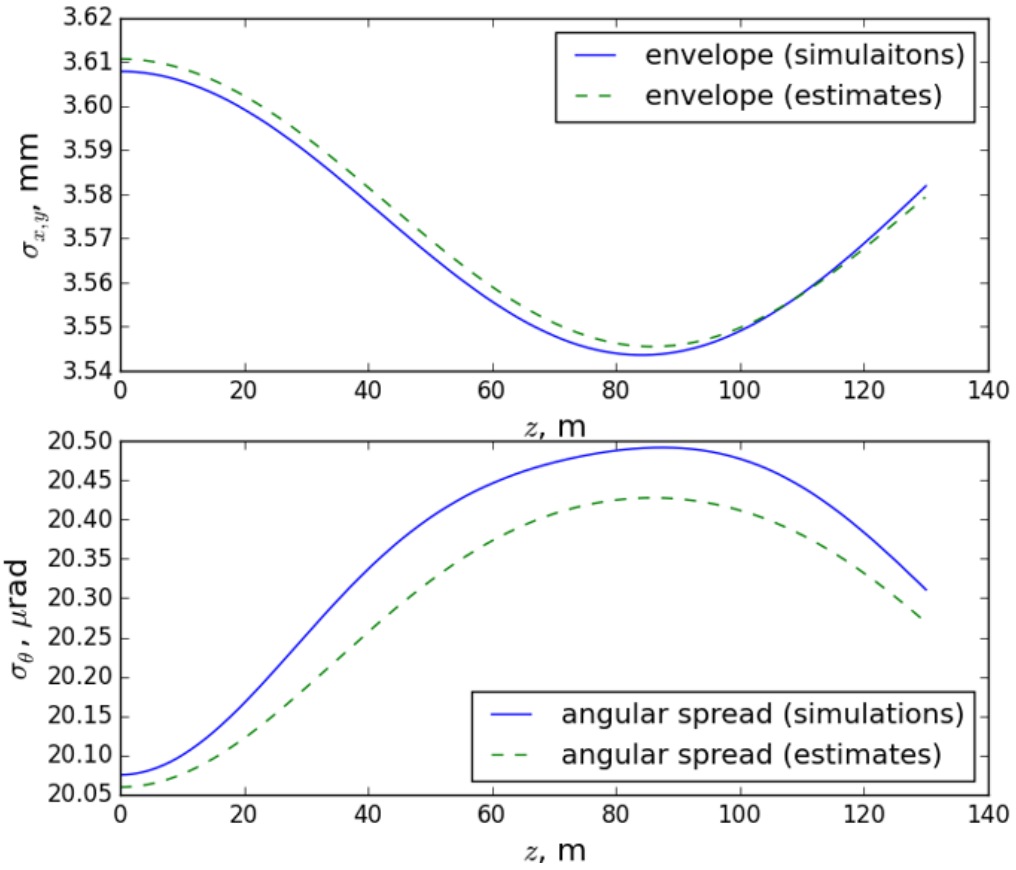


Figure 4: Simulations (solid blue line) and formula (2) (green dash line) of electron bunch envelope and rms angular spread for the case of $I_e = 5.5$ A and $I_p = 6$ A. The simulations are done for Gaussian transverse distribution of both e- and p- bunches with $\sigma_e = \sigma_p = 3.6$ mm. Analytical calculations are done for uniform transverse distributions with $a_e = 2\sigma_e$ and $a_p = 2\sigma_p$.

current of $I_p = 6$, which results in overestimating e-bunch σ_θ for the larger part of duration of the cooling.

As one can see, the e-bunch angular spread is satisfactory through the whole cooling process and on average is ≈ 30 μrad.

4.2 41 GeV Cooler

For the 41 GeV LEC we simulate the effect of proton-electron focusing for the e-bunch centered (longitudinally) with respect to the p-bunch (see Fig. 7). As analytical formulas predict, the expected angular spread due to the p-e focusing is small and the total angular spread of the e-bunch in the CS

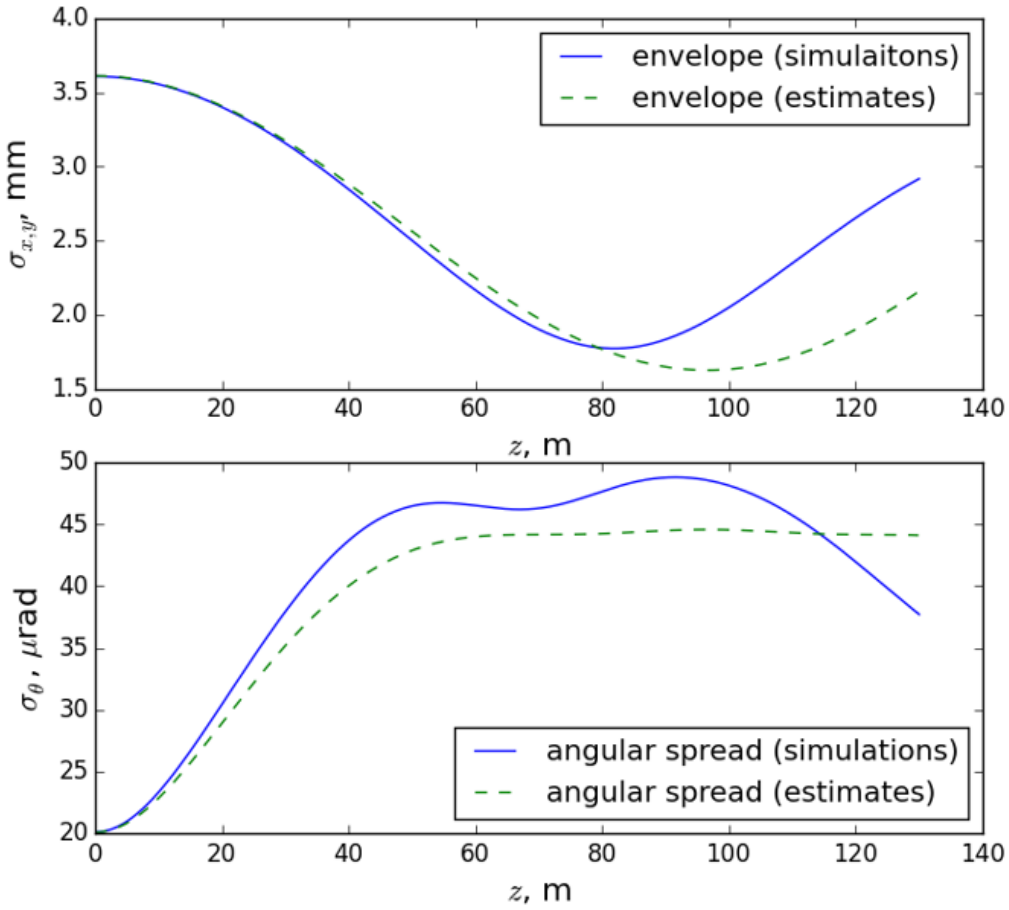


Figure 5: Simulations (solid blue line) and formula (2) (green dash line) of electron bunch envelope and rms angular spread for the case of $I_e = 3$ A and $I_p = 6$ A. The simulations are done for Gaussian transverse distribution of both e- and p- bunches with $\sigma_e = \sigma_p = 3.6$ mm. Analytical calculations are done for uniform transverse distributions with $a_e = 2\sigma_e$ and $a_p = 2\sigma_p$.

is well within our tolerances ($\sigma_{\theta tot} = 20.5$ μrad).

5 Conclusion

We considered the effect of proton-electron focusing in the cooling sections of the EIC low energy coolers.

We derived simple analytical formulas, which are useful for quick evaluation of cooler parameters required to keep a low angular spread driven by hadron-electron focusing.

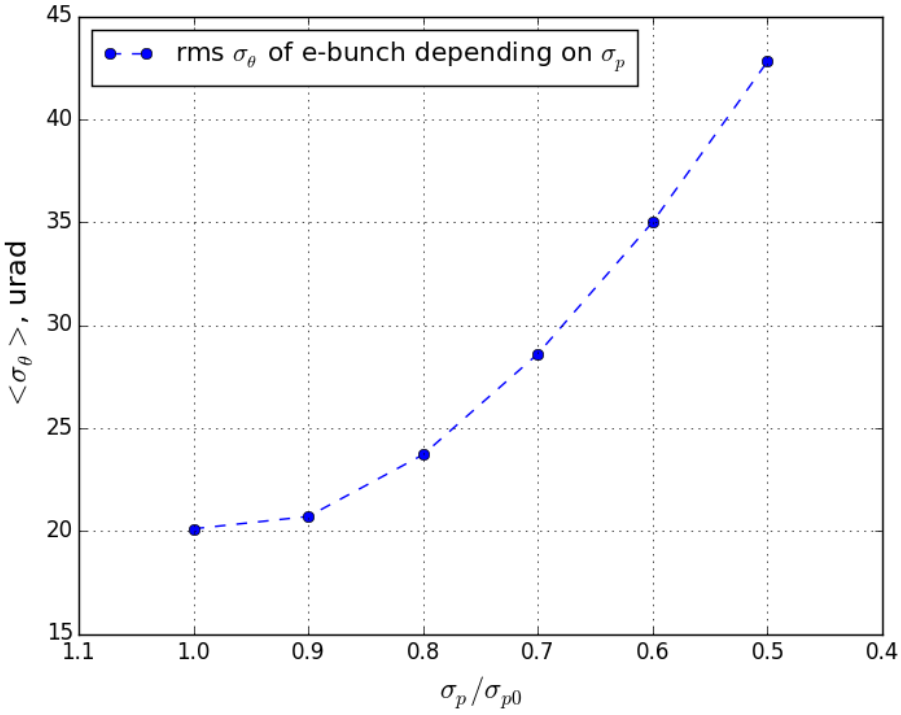


Figure 6: Dependence of the rms angular spread of the whole e-bunch on the size of the p-bunch (shrinking throughout the cooling process).

We performed numerical simulations of the p-e focusing for the optimized LEC parameters and compared the obtained results to analytical estimates.

We concluded that both 25 GeV and 41 GeV coolers can be operated without weak magnetization of electron beam.

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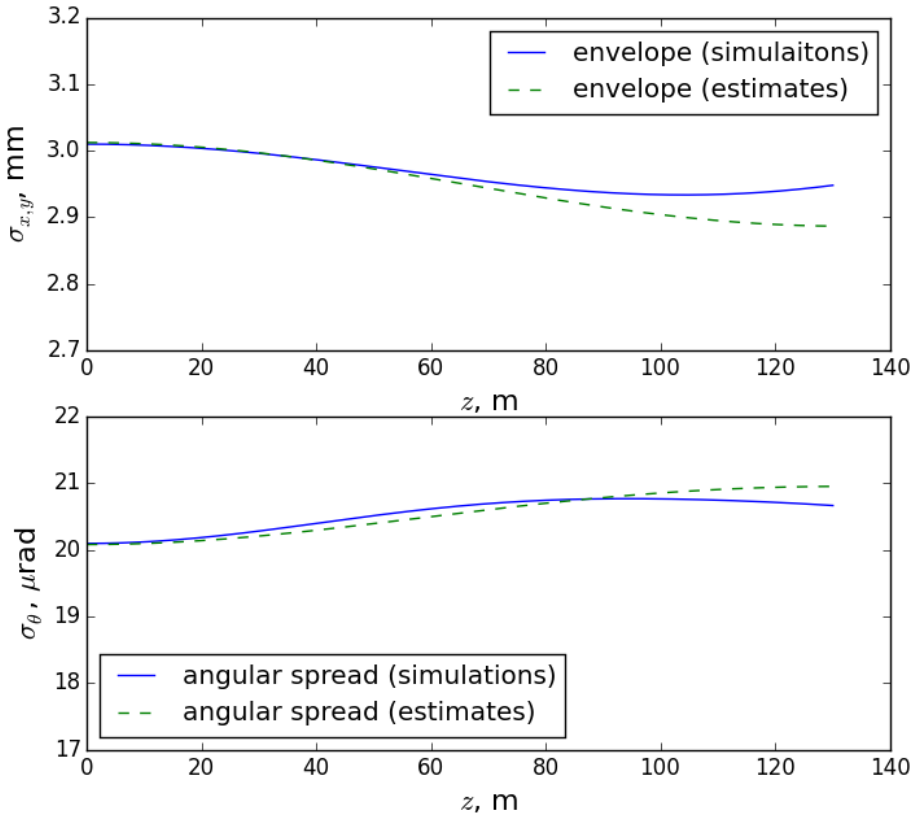


Figure 7: Simulations (solid blue line) and formula (2) (green dash line) of electron bunch envelope and angular spread in 41 GeV cooler for $I_e = 5.2$ A and $I_p = 6.6$ A (the most "extreme" case - central slice of the e-bunch with uniform longitudinal distribution as shown in Fig. 3). The simulations are done for Gaussian transverse distribution of both e- and p- bunches with $\sigma_e = \sigma_p = 3$ mm. Analytical calculations are done for uniform transverse distributions with $a_e = 2\sigma_e$ and $a_p = 2\sigma_p$.

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