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EIC low energy cooler: Requirements to uniformity of cathode solenoidal field

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EIC Low Energy Cooler: Requirements to Uniformity of Cathode Solenoidal Field

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Abstract

In this note we estimate the requirements to the uniformity of the cathode solenoidal field magnetizing the electron beam in the EIC low energy cooler. If this field is nonuniform over the size of the electron beam at the cathode then the extra angular spread of the e-beam related to the canonical angular momentum will not be canceled out completely by the matched solenoidal field of the cooling section (CS). Our goal is to keep the resulting additional angular spread below critical value $\sigma_\theta = 10$ μrad.

1 Assumed parameters

In this note we are assuming that the transverse profile of the e-beam in the CS can be approximated by a circular uniform distribution with radius $a_1 = 6.4$ mm. The requirement to the CS magnetic field is $B_1 = 230$ G [1]. Assuming that the e-beam size on the cathode is two times smaller than the beam size in the CS, $a_0 = 3.2$ mm, we get the requirement for the strength of the cathode field [2]:

$$B_0 = B_1 \frac{a_1^2}{a_0^2} = 920\text{G} \tag{1}$$

We farther assume that the LEReC-type gun is utilized for our cooler. The most feasible way to magnetize the beam on the cathode of such a gun is with the fringe field of a solenoid installed right at the gun exit. For our estimates we will assume that the distance from the edge of the solenoid to the cathode is 10 cm.

The longitudinal component of the fringe field will inevitably be nonuniform over the size of electron beam on the cathode. Below we estimate the tolerable level of this non-uniformity.

2 Requirements to field uniformity

Busch's theorem states that for the axially-symmetric system [2] the canonical angular momentum along the electron trajectories is conserved:

$$M = \gamma\beta m_e c r^2 \varphi' - \frac{e}{2\pi}\Psi = \text{const} \tag{2}$$

Here the magnetic flux encircled by an electron with radius r: $\Psi = 2\pi \int_0^r B(\tilde{r})\tilde{r}d\tilde{r}$.

The canonical angular momentum at the cathode is solely determined by the solenoidal field:

$$M_0 = e\int_0^{r_0} B(\tilde{r})\tilde{r}d\tilde{r} \tag{3}$$

In the cooling section, assuming the uniformity of the solenoidal field ($B_1 = \text{const}\ \forall r_1 \le a_1$):

$$M_1 = \gamma\beta m_e c r_1^2 \varphi_1' - \frac{eB_1 r_1^2}{2} \tag{4}$$

Noticing that the rotational angles associated with M are given by $\theta = \varphi' r$ and introducing the magnetic rigidity $B\rho = \frac{\beta\gamma m_e c}{e}$, we obtain from (2-4):

$$\theta = \frac{1}{B\rho}\cdot\frac{1}{r_1}\left(\int_0^{r_0} B(\tilde{r})\tilde{r}d\tilde{r} - \frac{B_1 r_1^2}{2}\right) \tag{5}$$

Let us represent the cathode field as $B(r) = B_0 + \delta B_0(r)$, where B_0 satisfies (1) and $\delta B_0(r)$ is the nonuniform part of the cathode field. We will assume that the field non-uniformity can be approximated by $|\delta B_0(r)| = b\cdot r/a_0$. Then, assuming paraxial motion of the electron beam through the transport beamline:

$$\theta = \frac{b}{B\rho}\cdot\frac{r_0^2}{3a_1} \tag{6}$$

Averaging (6) over the beam's transverse distribution we get the following condition for tolerable non-uniformity of cathode solenoidal field:

$$b \le 3\sqrt{3}B\rho\cdot\sigma_\theta\cdot\frac{a_1}{a_0^2} \tag{7}$$

For 13.6 MeV e-beam $B\rho = 47048$ G·cm. Hence, the tolerable field non-uniformity $b \approx 15$ G or about 1.6% of the design cathode field.

Of course, for a given non-uniformity ($B(r) = B_0 + b\cdot r/a_0$) the rms uncompensated angular spread can be found as:

$$\sigma_\theta = \frac{b}{3\sqrt{3}B\rho}\cdot\frac{a_0^2}{a_1} \tag{8}$$

3 Gun solenoid considerations

A convenient approximation for longitudinal component (B_z) of the field of solenoid with radius R and length L, valid whenever $r \leq R$ and accurate to 1% in this range, is given by [3]:

$$B_z = \frac{\mu_0}{4}\frac{NI}{L}\left[m(1+2k_1)\frac{2\phi}{\pi} + \left(\frac{m^2(1-m)}{4} + 2k_1\right)\sin\phi\right]_{s=z-L/2}^{s=z+L/2} \qquad (9)$$

Here $m = \frac{1-k_1}{1+k_1}$, $k_1 = \sqrt{1-k(s)^2}$, $k(s) = \sqrt{\frac{4Rr}{s^2+(R+r)^2}}$, $\phi(s) = \arctan\left(\frac{s}{R-r}\right)$ and z is the distance from solenoid center to the cathode.

The larger is R/L ratio the larger is B_z non-uniformity at the cathode. On other hand, large R/L reduces the peak solenoid field corresponding to the required 920 G field at the cathode. To optimize the gun solenoid parameters one can first find approximate values for solenoid length and radius using analytic formulas (7) and (9). Next, one must substitute (9) dependence into (5) and numerically integrate the convolution of the obtained result with the beam transverse distribution function. Although, for realistic parameters the non-uniformity predicted by (9) is rather small.

Indeed, performing the described analysis for our beam parameters we obtained the solenoid of length $L = 5$ cm, radius $R = 20$ cm and with the peak field of 1.5 kG. Such a solenoid produces 920 G field on the cathode located 10 cm from the solenoid edge. The resulting cathode field easily satisfies non-uniformity requirements. For such a field the uncompensated rotational angular spread in the CS is just 0.8 μrad. It is also worth noticing that formula (8) worked perfectly for the considered case as Fig. 1 and Fig. 2 demonstrate.

Figure 1 shows the dependence of the longitudinal component of the solenoidal at the cathode on the radial displacement from the axis of the solenoid.

Figure 2 shows the dependence of the uncompensated rotational angle in the CS for the electron, which had a particular radial coordinate at the cathode.

4 Conclusions

We derived formulas (7) and (8) for estimating the tolerable non-uniformity of the solenoidal field at the cathode for the electron coolers utilizing magnetized electron beam.

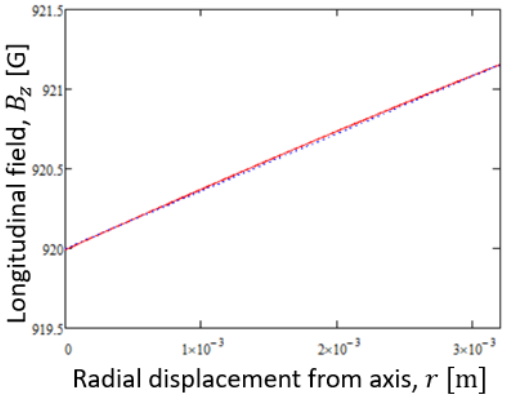


Figure 1: Longitudinal component B_z of the solenoidal field on the cathode depending on the radial displacement r. The red solid line represents the precise analytical formula (9), the blue dashed line is a simple linear model of non-uniformity.

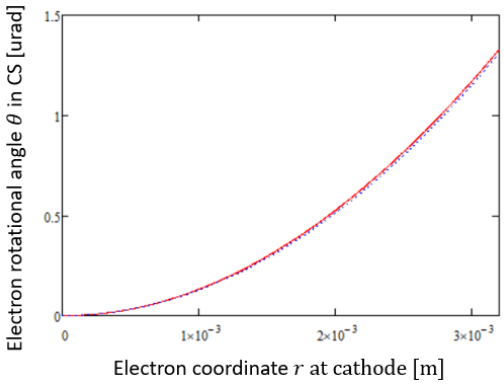


Figure 2: Longitudinal component B_z of the solenoidal field on the cathode depending on the radial displacement r. The red solid line represents the result of substituting field given by precise analytical formula (9) into (5), the blue dashed line is an angle calculated for a simple linear model of non-uniformity.

We applied the derived formulas to check whether the field produced on the cathode by the gun solenoid is satisfactory for the beam magnetization in the electron cooler.

We concluded that non-uniformity of the field produce by gun solenoid with realistic dimensions is small enough to be ignored for the purposes of design of the EIC low energy cooler.

References

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[3] E. Callaghan, S. Maslen, The Magnetic Field of a Finite Solenoid, NASA TN D-465 (1960).