BNL-216176-2020-TECH

EIC/71;EIC-ADD-TN-002

Requirements to the cooling section solenoidal field in the EIC low energy cooler

S. Seletskiy

April 2020

Electron-Ion Collider

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC), Nuclear Physics (NP) (SC-26)

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EIC TECHNICAL NOTE	NUMBER: **EIC-ADD-TN-002**
AUTHORS: Sergei Seletskiy, Dmitri Kayran	DATE: 04/13/2020
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S. Seletskiy, D. Kayran

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Abstract

In this note we estimate the requirements to the continuous solenoidal field in the cooling section (CS) of the EIC low energy cooler. This field must be strong enough to effectively counteract the focusing induced on electrons by the space charge of the proton bunch in the EIC CS. That is, the resulting angular spread must not exceed 10 μrad.

1 Assumed parameters

We consider the central longitudinal slice of the e-bunch co-traveling with the central longitudinal slice of the p-bunch. Electron and proton beam parameters in the cooling section (CS) are given in Table 1.

Parameter	**Value**
CS length (L_{CS}), m	120
electron bunch peak current (I_e), A	1.5
proton bunch peak current (I_p), A	7
relativistic γ-factor	27.6
geometric electron bunch emittance (ε), mm·mrad	0.1
rms transverse size of the proton bunch (σ_p), mm	4.5

Table 1: Assumed electron and proton beam parameters.

In this note we assume the uniform transverse distribution for both beams with the proton bunch transverse size $a_p = \sqrt{2}\sigma_p = 6.3$ mm and with the initial electron bunch radius at the entrance to the CS $a_e(0) = a_p$.

We assume that although the cooling itself is not necessarily magnetized, the electron bunch is magnetized and that the field on the cathode is well matched to the CS field. That is, we are assuming that the e-beam "rotational velocities" associated with its canonical angular momentum are effectively canceled out as the beam enters the CS.

The longitudinal field (B_z) in the CS is assumed to be constant over the length of the CS and uniform across the beam profile.

Finally, in this note, we ignore the effect of the thermal emittance on the beam angles.

2 Analysis

Under our assumptions the effect of the space charge is given by:

$$r'' = r\left(\frac{K_e}{a_e(s)^2} - \frac{K_p}{a_p^2}\right) \tag{1}$$

Here the coefficients $K_{e,p} = \frac{2I_{e,p}}{I_A\gamma^3\beta^3}$, where Alfven current $I_A = 17$ kA.

Since we are looking for the equilibrium conditions it is reasonable to assume for this exercise the constant $a_e(s) = a_p$.

The two forces acting on the electron at radius $r^2 = x^2 + y^2$ are the radial force represented by (1) and the Larmor force normal to the present transverse velocity of the electron.

Equation of motion under these conditions can be conveniently written as:

$$\begin{cases} \xi' = \theta \\ \theta' = \frac{K_e - K_p}{a_p^2}\xi - \frac{iB_z}{B\rho}\theta \end{cases} \tag{2}$$

where $\xi = x + iy$ and $\theta = \theta_x + i\theta_y$.

From (2), with initial conditions $\xi(0) = \xi_0$ and $\theta(0) = 0$ we get:

$$\xi = e^{-\frac{iks}{2}}\xi_0\left[\cos\left(\frac{\sqrt{k^2+4K}}{2}s\right) + \frac{ik}{\sqrt{k^2+4K}}\sin\left(\frac{\sqrt{k^2+4K}}{2}s\right)\right] \tag{3}$$

$$\theta = -e^{-\frac{iks}{2}}\frac{2K\xi_0}{\sqrt{k^2+4K}}\sin\left(\frac{\sqrt{k^2+4K}}{2}s\right) \tag{4}$$

where $K = \frac{K_p - K_e}{a_p^2}$ and $k = \frac{B_z}{B\rho}$.

Considering the envelope electron (i.e. the sample particle with $\xi_0\xi_0^* = a_p^2$ experiencing the strongest space charge focusing) and introducing the notation $\theta_{cr} = 10$ μrad, we get for the critical solenoidal field B_{cr}:

$$B_{cr} \geq B\rho\sqrt{\frac{4K^2a_p^2}{\theta_{cr}^2} - 4K} = 460\text{G} \tag{5}$$

Noticing that (4) holds true for any electron within the beam we can average the angles over the uniformly distributed beam. Since there are many Larmor oscillations over the length of the CS we can farther average the angles over the CS. Then, the critical field keeping the rms angles of the e-beam below θ_{cr} is given by:

$$B_{crRMS} \geq B\rho\sqrt{\frac{K^2a_p^2}{\theta_{cr}^2} - 4K} = 230\text{G} \tag{6}$$

For the parameters of our model $B_{crRMS} = 230$ G. Figures 1 and 2 show respectively the sample particle trajectory and the angle ($\sqrt{\theta\theta^*}$) evolution of the sample particle in the CS in field $B_z = 230$ G.

As one can see from Fig. 1, our assumption for $a_e(s) = \text{const} = a_p$ is well justified.

The wavelength of the oscillation and the amplitude of oscillation of r are also important for the cooling. To get a simple formula for r-oscillations we write r as $r(s) = r_0 + \rho(s)$, therefore, $\rho(s) = \sqrt{\xi\xi^*} - r_0$. Then, from (3) we get:

$$\rho \approx \frac{2r_0K}{k^2+4K}\sin^2\left(\frac{\sqrt{k^2+4K}}{2}s\right) \tag{7}$$

The wavelength of the oscillation is apparently given by:

$$\lambda = \frac{2\pi}{\sqrt{k^2+4K}} \tag{8}$$

For our parameters and for $B_z = 230$ G $\lambda = 12.9$ m.

The amplitude of r-oscillations of the envelope particle is given by:

$$A = \frac{2a_pK}{k^2+4K} \tag{9}$$

For $B_z = 230$ G $A = 40$ μm for the envelope particle and $A = 30$ μm for the rms particle.

Figure 3 shows the closeup of radius oscillations of the envelope and the rms particles and Fig. 4 shows the projection of the CS trajectory of these particles on the $x - y$ plane.

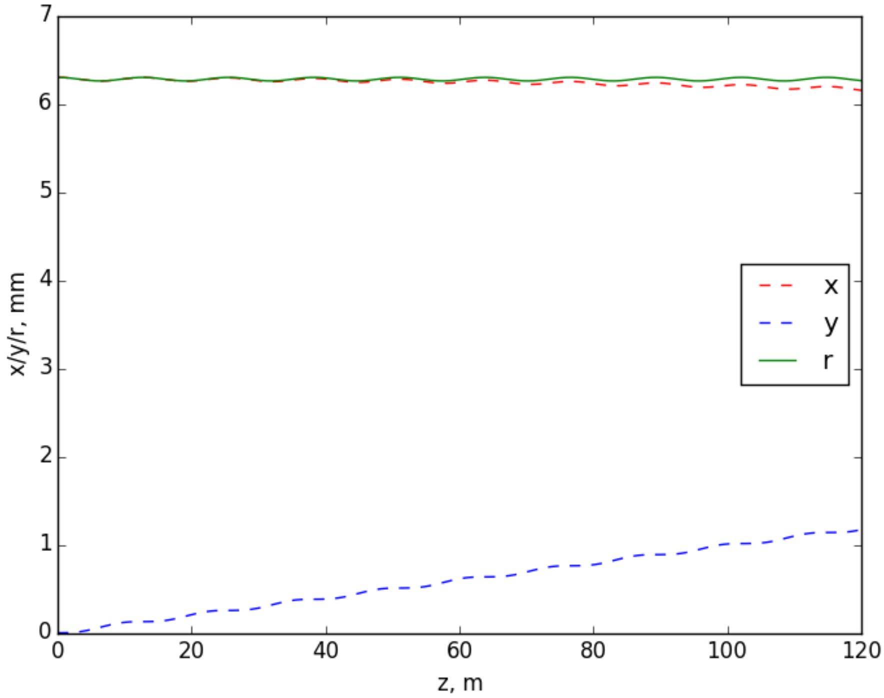

Figure 1: Motion of the envelope sample particle of the weakly magnetized beam in the CS. The CS field $B_z = 230$ G.

3 Conclusion

We derived simple analytical formula (6), which estimate the CS magnetic field needed to counteract the space charge focusing in the weakly magnetized electron coolers. Formulas (8) and (9) respectively give the wavelength and the amplitude of r-oscillations for the beam in the crossed solenoidal and radial space charge fields.

For the parameters of the EIC low energy cooler the CS magnetic field keeping the overall rms angular spread below 10 μrad must be ≥ 230 G. For such a field the wavelength of r-oscillations is 12.9 m and the peak to peak amplitude of r-oscillations of the rms electron is 30 μm.

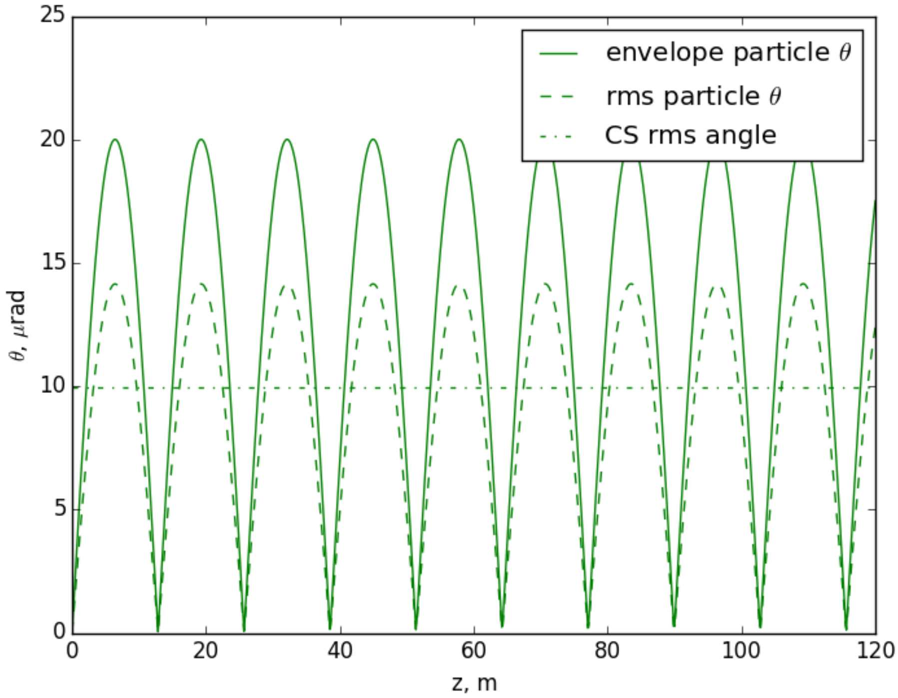

Figure 2: Angle of the envelope (solid line) and the rms (dashed line) sample particles of the weakly magnetized beam in the CS. The CS field $B_z = 230$ G.

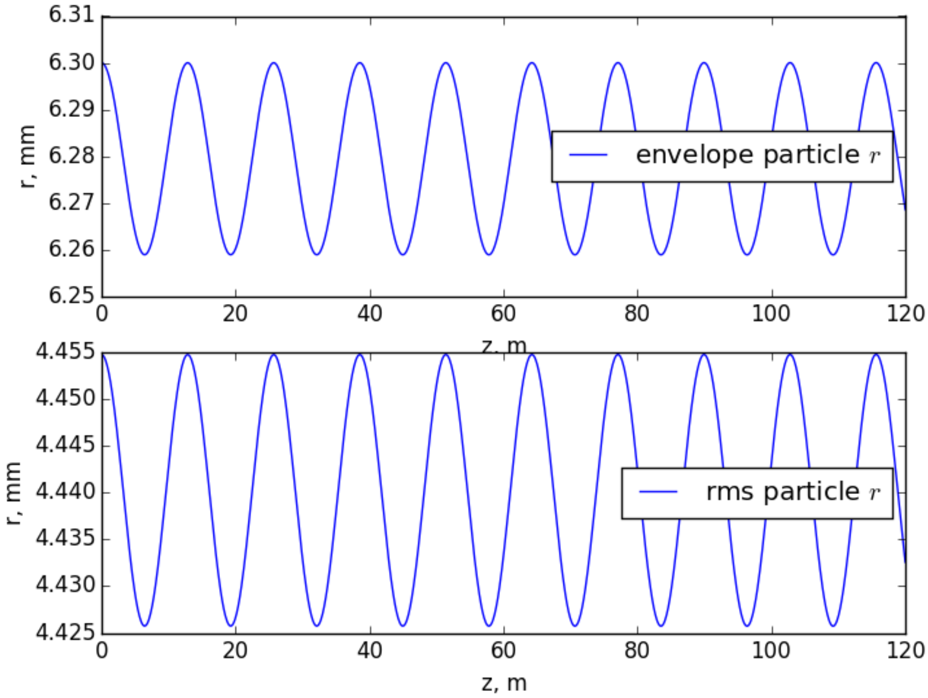

Figure 3: r-oscillations of the envelope and the rms particles.

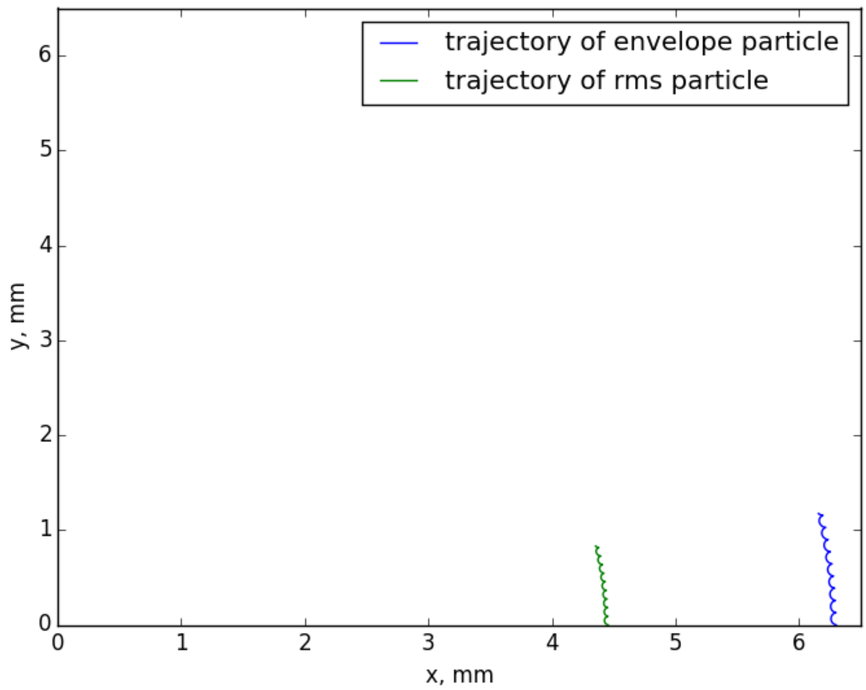

Figure 4: Projection of the CS trajectory of the envelope and the rms particles on the $x - y$ plane.