

Estimating longitudinal emittance near transition energy in the AGS

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September 2019

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Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC), Nuclear Physics (NP) (SC-26)

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Estimating Longitudinal Emittance near Transition Energy in the AGS

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9-3-2019

1. Introduction

In order to obtain accurate longitudinal emittance (ϵ) measurements in the AGS when the energy where the measurement is made is near transition energy it was realized that an especially accurate value for the slip factor (η) at that time in the cycle is required.¹ This is because η , which equals $1/\gamma_t^2 - 1/\gamma^2$, is close to zero near transition and $\sqrt{|\eta|}$ appears in the denominator of the equation used to calculate ϵ .² Measuring γ , the Lorentz factor at the energy the measurement is made, is relatively straightforward providing the orbit circumference (C) is known but measuring γ_t (γ at transition) at that time in the cycle is not.

γ_t is directly related to the momentum compaction (α) which to zeroth order depends on the strength of the horizontal quadrupole focusing field and therefore the horizontal tune (Q_h). It also depends on the orbit radius (ΔR) because the focusing strength changes when the beam energy changes (for a constant B field) and because the horizontal focusing field has components that are functions of ΔR . In particular, the sextupole field or horizontal chromaticity (ξ_h) causes a focusing field that depends on ΔR . Since these three parameters (Q_h , R , and ξ_h) vary throughout the cycle it is not enough to find γ_t by measuring γ when the transition phase jump occurs as their values are likely to be different there than the time in the cycle where ϵ is measured.

Take for example measuring the extraction flattop ϵ for the 7.3 GeV cycle where $\gamma=7.85$. The value for γ_t , used for the ϵ calculation in Bbat, is 8.50. If γ_t was instead 8.60 then the ϵ calculation will give an answer that's a factor of 1.078 too high, and if it were 8.40 it will be a factor of 0.94 too low. So, if the actual ϵ was 0.70 eVs then the calculated ϵ would be 0.755 eVs for $\gamma_t=8.40$ and 0.658 eVs for $\gamma_t=8.60$. From the measurements taken this run it appears that γ_t can easily vary over this range depending on what values Q_h , ξ_h , and ΔR have. This note will attempt to develop a way involving these 3 variables to estimate what γ_t is at the time in cycle that the ϵ measurement is taken so a more reasonable value for η can be used in the calculation.

2. The Method Used to Measure γ_t at the Transition Phase Jump

In general, the method used to measure γ_t is, with the γ_t jump quads off, to adjust the transition phase jump to minimize bunch shape oscillations after it occurs. Gold beam was used, and the bunch intensity, no higher than $2.7e9$ ions or so, is low enough that the lack of a γ_t (quad)

¹ See for example, [K. Zeno, "AGS Longitudinal emittance measurements for RHIC low energy gold runs" C-A/AP/615](#), pgs. 5-6.

² Equation (2.76) on page 45 of D.A. Edwards and M.A. Syphers, "An Introduction to the Physics of High Energy Accelerators", John Wiley and Sons, Inc, 1993. The equation as it appears in the book is:

$$S = \frac{\pi(v/c)(\Delta\phi)^2}{\omega_{rf}} \sqrt{\frac{eV \cdot E_s}{2\pi h |\eta|}}$$

where S is ϵ , (v/c) is the relativistic β , $\Delta\phi$ is the "maximal extent of the presumed small oscillations in $\Delta\phi$ ", ω_{rf} is the angular Rf frequency, e is the electron charge, E_s is the energy of the synchronous particle, V is the Rf voltage, h is the Rf harmonic, and η is the slip factor.

jump does not appear to complicate matters. The measurements were made on the 9.8 GeV cycle (AGS user 1) since that is the only setup used this year where the beam passes through Transition. These oscillations are viewed on the envelope of the wall current monitor (WCM) signal and what setting of the phase jump minimizes them is judged by eye.

The revolution frequency (f_{rev}) was found by measuring the frequency of the Rev Tick signal on a scope, or the f_{rev} delivered by the Rf system through GPM ($F_{\text{rev_System}}$) at the time of the phase jump (Figure 1). Although one needs to be careful in measuring it through a GPM, the agreement between these 2 methods was typically quite good. The value obtained from the GPM does not have any noise although it is only sampled every millisecond and its value at the exact time of the phase jump needs to be interpolated. The scope measurement has significant noise (± 50 -100 Hz), but despite that f_{rev} can be measured well enough that a typical uncertainty in γ_t from the measurement is perhaps about ± 0.0044 .³ It rarely disagrees with the GPM value by more than a few Hz out of 370000. The scope measurement can also be averaged over many cycles, and although the noise level goes down it is not clear that this reduction results in a significantly more accurate measurement, so most measurements were made without averaging.

The phase jump is delayed by 28100 μs from the *arf.transition.gt* gauss event. The time that event occurs was found through AGS TimeLineDisplay and 28100 μs was added to it to find the time from At0 that the phase jump occurs. Finding the exact time from At0 that it occurs is not necessary though because when it does occur there is a large spike in the Rev Tick frequency measurement. Its frequency is measured immediately prior to that, and that value is used for f_{rev} at the phase jump.

To find the optimal timing for the phase jump the value of *arf.transition.gt* was scanned. On either side of the optimal timing the bunch shape oscillations begin to increase, and the optimal timing is not only where the bunch shape oscillations are minimized but also roughly equidistant from where the oscillations have clearly become worse. It was found that a change in field of ± 25 gauss from the optimal time clearly produced greater than the minimal amount of bunch shape oscillations. This range in the B field corresponds to a range in γ_t of ± 0.027 , but this is not the uncertainty in the optimal timing of the phase jump.⁴ That uncertainty is much less because the timing is (roughly) set so that it's in the middle of that range. On several occasions

³ See [Jul 10 1548 and 1649 entries in Booster-AGS-EBIS 2019 elog](#). In the first entry f_{rev} is measured as 368808 Hz, and the 2nd entry a value of f that is obviously too high is measured (368817 Hz). The difference of 9 Hz would correspond to γ changing from 8.403 to 8.412, a difference of 0.009. But the actual uncertainty is likely less than this because the measurement is not made when f_{rev} is obviously too high or too low. There were 5 sets of f_{rev} measurements and within each set the conditions were the same: (8.4029,8.4032,8.408), (8.44,8.449), (8.499,8.791,8.496,8.500), (8.440,8.456), (8.491,8.496). The most that any of the individual measurements vary from the average of that set is by 0.008 and the standard deviation of each measurement's difference from the average of that set is 0.0044. I will use this standard deviation as representative of the typical uncertainty due to the f_{rev} measurement.

⁴ See [Jul 10 1542 through 1544 entries in Booster-AGS-EBIS 2019 elog](#). A difference of ± 500 gauss clock counts is ± 25 g with a field around 7.8 kG this corresponds to a change in γ of ± 0.027 .

the phase jump was optimized for the same setup but on different days (without recourse to an archive) and the value arrived at was typically within ± 5 gauss so I would say that this range reflects the typical uncertainty in the optimal timing and corresponds to a range in γ_t of ± 0.0053 .

AGSOrbitDisplay was used to find ΔR , and since the BPMs are all at positions where the dispersion has its average value, the displacement from zero of the average of the BPMs position is nominally the same as the orbit's displacement from R_0 . Under normal running conditions this average, ΔR , was +7 mm. The uncertainty in the ΔR measurement is on the order of a few tenths of a millimeter, but to be conservative let's say it's ± 1 mm.

γ is calculated first by finding $\beta = 2\pi(R_0 + \Delta R)f_{rev}/c$, where R_0 is the radius of the “design” orbit, 128.4526 m, and then plugging that into $\gamma = 1/\sqrt{1 - \beta^2}$.⁵ It's not clear to me what the uncertainty in the design radius is, but it's least significant digit is 10^{-4} m. If it were off by 10 times that (± 1 mm) that would change the value of γ (or γ_t) by about ± 0.004 . Note that an error of ± 1 mm in the ΔR measurement from AGSOrbitDisplay would also change γ by ± 0.004 .

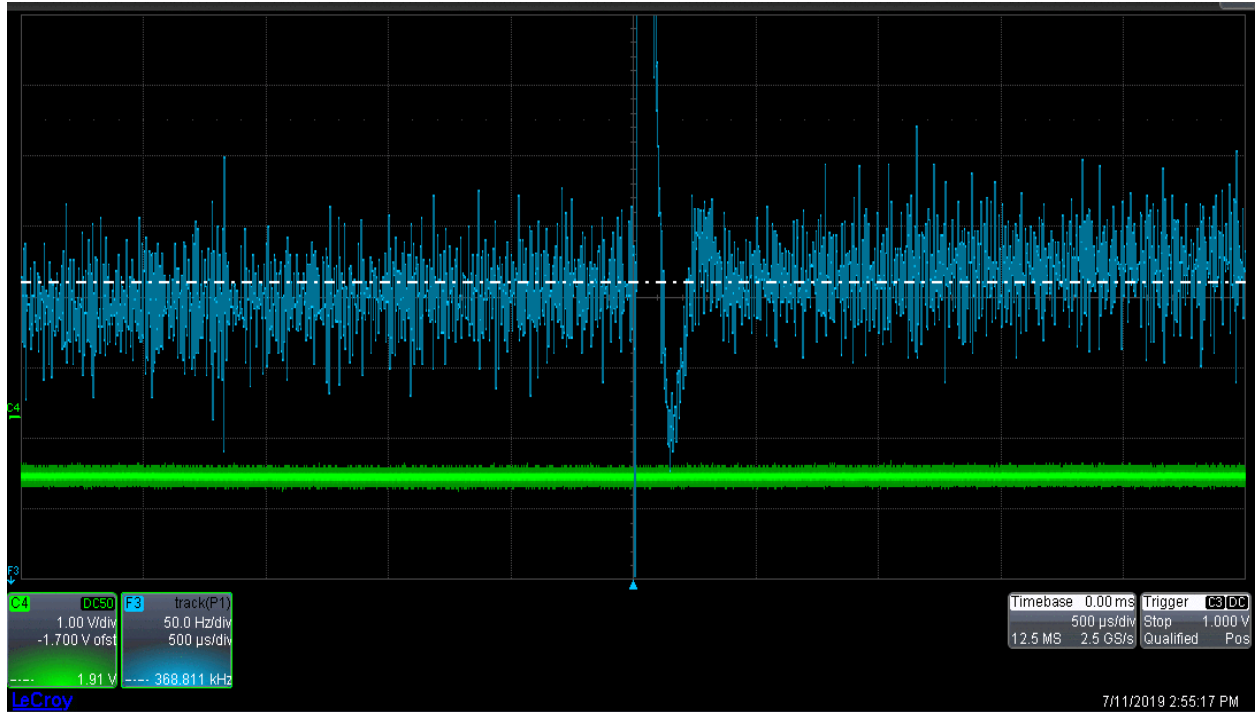


Figure 1: A typical scope measurement to find f_{rev} at the phase jump. The scope measures the time interval between each rising edge of the rev tick signal, inverts that making it a frequency and plots it as a function of time (blue trace). Note the spike in the frequency where the trigger is. This corresponds to the phase jump. In this case the sweep speed is 500 $\mu\text{s}/\text{div}$ and the frequency is 368.811 kHz as indicated in the F3 icon at the bottom of the display. Also note that the vertical gain is displayed there too and is 50 Hz/div.

⁵ This value for R_0 , 128.4526 m, is from C.J. Gardner, [“FY2016 Parameters for Gold Ions in Booster, AGS, and RHIC”](#), Sept. 7, 2016. Pg. 5 equation 34.

To estimate a typical uncertainty in γ_t that would result from these 4 independent sources of error I add them together in quadrature to find

$$\delta(\gamma_t) = \pm\sqrt{0.0044^2 + 0.0053^2 + 0.004^2 + 0.004^2} = \pm 0.009.$$

3. γ_t Measurement with Tune Quad and Chromaticity Sextupole currents near Zero and R Near Ro.

On July 9, the ‘bare machine’ γ_t was measured. The radius (ΔR) was shifted from its running value of +7 mm to 0 mm using radial steering, and the currents in the tune and chromaticity supplies were set to approximately zero.⁶ After the phase jump was optimized (*arf.transition.gt* = 155050 gcc) it occurred at At0+3740.488 ms where f_{rev} is 368808.2 Hz according to F_{rev_System} . Using $R_o=128.4527$ m ($C_o=807.092$ m) this corresponds to a γ of 8.4032 which is equated with γ_t .

On July 10 the measurement was made again. In this case, after optimizing the phase jump (*arf.transition.gt*=154900 at At0+3740.674 ms) f_{rev} using the rev tick on a scope was 368808 Hz for $\gamma_t=8.4029$.⁷ Note that these measurements were made a day apart, the phase jump was optimized for each case, and f_{rev} was measured in different ways (GPM vs. scope). On July 11 the same measurement was made again. This time the same phase jump timing was used and f_{rev} from the scope was 368811 Hz for $\gamma_t=8.408$.⁸

4. Measuring γ_t Under Normal Running Conditions

When the beam crosses transition the γ_t jump quads are on, Q_h is set to 8.765 (H. quad I=126A), Q_v to 8.71 (V quad I=-7A), ξ_h is set to -1.28 (H. sext. I=150A and V. sext. I=35A). On July 9 the pulsing for the γ_t jump quads was turned off and the phase jump was optimized (*arf.transition.gt*=155100 gcc). f_{rev} measured using F_{rev_System} was 368811 Hz at the phase jump time (3742.068 ms). Using $R=R_o+0.007m=128.4597$ results in $\gamma_t=8.440$.⁹

On July 12 γ_t was measured again under the same conditions.¹⁰ In this case *arf.transition.gt* was 155160 gcc, and the phase jump happened at 3742.031 ms. f_{rev} from the rev tick was 368821 Hz which gives $\gamma_t=8.456$. Using F_{rev_System} gives an f_{rev} of 368817.0 Hz which gives $\gamma_t=8.449$.

⁶ The currents in the tune quads were just roughly zeroed. The actual currents in this state were: H. quad I=+5A, V. quad I=-8 A, H sextupole I=+10A, and V sextupole I=+30A. See [Booster-AGS-EBIS 2019 elog Jul 9](#) 1815 and 1816 entries. In this case $Q_h \sim 8.70$ and $\xi_h \sim -2.8$

⁷ See [Booster-AGS-EBIS 2019 Jul 10](#) entries from 1540 to 1448.

⁸ See [Booster-AGS-EBIS 2019 Jul 11](#) entries from 1452 to 1456.

⁹ See [Booster-AGS-EBIS 2019 elog July 9](#) entries from 1734 to 1738.

¹⁰ See [Booster-AGS-EBIS 2019 elog for July 12](#) entries from 1507 to 1519.

5. γ_t with Nominal Tunes and Chromaticities but with $\Delta R=0$

This measurement was performed on July 10.¹¹ After the phase jump was optimized *arf.transition.gt* was 156600 gcc which corresponded to a phase jump time of 3749.123 ms. Without averaging f_{rev} from the rev tick was 368868 Hz and when an average over 12 cycles was taken it was 368863 Hz. Using the former value for f_{rev} gives $\gamma_t=8.499$ and using the latter value gives $\gamma_t=8.491$.

On July 11 this measurement was repeated.¹² After the phase jump was optimized *arf.transition.gt* was 156600 gcc and the phase jump happened at 3749.166 ms. f_{rev} from the scope was 368866 Hz and from Frev_System it was 368869 Hz corresponding to γ_t of 8.496 and 8.500, respectively. The value 8.4935, which is the average of 8.496 and 8.491, will be used for γ_t in this case.

6. Measuring the Change in γ_t associated with a Change in Q_h at $\Delta R=0$

This measurement was performed on July 10.¹³ After the phase jump was optimized with nominal tunes and chromaticities *arf.transition.gt* was 156650 gcc. This is essentially the same value that was found earlier that day for this state (156600 gcc) so f_{rev} was not measured again and the value found earlier for this state (368863 Hz for the 12 cycle average) is used. Then the Q_h setting was lowered by 0.05 from 8.765 to 8.715 and the phase jump was optimized. In this state *arf.transition.gt* is set to 155800 gcc and the phase jump happened at 3745.129 ms. Using the rev tick f_{rev} at the time of the phase jump was 368840 Hz for one cycle and 368836 Hz for a 5 cycle average. An f_{rev} of 368836 Hz corresponds to a γ_t of 8.447.

Assuming the change in γ_t with change in Q_h is linear it is possible to estimate what γ_t will be for different values of Q_h . Specifically, $\Delta\gamma_t/\Delta Q_h=(8.491-8.447)/0.05=0.93$, so for every +0.01 unit change in Q_h , γ_t increases by $(0.01)(\Delta\gamma_t/\Delta Q_h)=0.0093$. To estimate the error in this estimate, recall that $\delta(\gamma_t)$ is 0.009 and assume that it is equally likely that the measured value of γ_t is greater or less than the actual value for it by $\delta(\gamma_t)$ for both $Q_h=8.715$ and 8.765. The standard deviation of the values obtained for $\Delta\gamma_t/\Delta Q_h$ in each of the 4 possible cases is 0.255 so let's say $\Delta\gamma_t/\Delta Q_h=0.880\pm0.255$

The same exercise was performed with Q_v instead of Q_h and if there was an effect it was too small to see.

¹¹ See [Booster-AGS-EBIS 2019 elog for July 10](#) entries from 1527 to 1537.

¹² See [Booster-AGS-EBIS 2019 elog for July 11](#) entries from 1435 to 1439.

¹³ See [Booster-AGS-EBIS 2019 elog for July 10](#) entries from 1721 to 1735.

7. The Effect of Changing the Chromaticity on γ_t with $\Delta R=0$

These measurements were taken on July 11.¹⁴ First the currents in the Tune and Chromaticity supplies were roughly zeroed (corresponding to settings of $\xi_h=-2.8$ and $\xi_v=0.3$). With optimized phase jump timing *arf.transition.gt* is 154900 gcc and γ_t from the rev tick was 8.407 (this measurement has already been mentioned previously).¹⁵ Then the nominal chromaticity currents were loaded (corresponding to $\xi_h=-1.3$ and $\xi_v=0.5$ settings and H. sext. I of 153A and V. sext. I of 36A). The phase jump was optimized (155500 gcc, 3743.705 ms) and f_{rev} was 368832 Hz for $\gamma_t=8.441$.¹⁶ Another measurement was made about half an hour later (155450 gcc, 3743.496 ms) and f_{rev} was 368829 Hz for $\gamma_t=8.436$.¹⁷

Then the setting for ξ_h was raised by 1 unit, the phase jump was optimized (155600, 368833 Hz), and f_{rev} was measured from the rev tick (368833 Hz) which gives a value for γ_t of 8.443.¹⁸ The difference in γ_t between the initial ξ_h and ξ_h shifted by +1 unit is quite small and may be in the noise. However, the bunch shape oscillations did seem to change a bit even though the phase jump timing was barely changed between the 2 cases (155500 vs. 155600gcc). Naively, one might not expect to see any effect on γ_t from the horizontal chromaticity if $\Delta R=0$. What would have been more to the point would be to change ξ_h with a none zero ΔR . There was no noticeable effect on the bunch shape oscillations when ξ_v was changed.

8. The Effect of Changing ΔR on γ_t with Nominal Running Tunes and Chromaticities

γ_t was measured twice with nominal running tunes and chromaticities and $\Delta R=+7$ mm (section 4). The values obtained for γ_t , using f_{rev} from the rev tick measurement, were 8.440 and 8.456 and the average of these 2 values is 8.448. γ_t was also measured twice at $\Delta R=0$ mm and with these tune and chromaticity settings and using the rev tick. The values obtained were 8.491 and 8.496, for an average of 8.4935 (section 5).

The fact that γ_t depends on ΔR is due to ξ_h and, at least for the ξ_h setting these measurements were made at a relation between the 2 can be found. That is, $\Delta\gamma_t/\Delta(\Delta R)=(8.448-8.4935)/7$ mm= -0.0065/mm so for every +1 mm change in ΔR , γ_t decreases by -0.0065. The uncertainty in this measurement can be found in the same way it was found for $\Delta\gamma_t/\Delta Q_h$ which gives $\Delta\gamma_t/\Delta(\Delta R)= -0.0065 \pm 0.0018/\text{mm}$.

¹⁴ See [Booster-AGS-EBIS 2019 elog for July 11](#) entries from 1453 to 1559

¹⁵ Ibid. entries 1453-1455

¹⁶ Ibid. entries 1458-1503

¹⁷ Ibid. entries 1525-1529

¹⁸ Ibid. entries 1553-1556

9. Preliminary Estimate for γ_t in the 7.3 GeV ϵ Calculation

In the case where ξ_h is the same as the case here an estimate for γ_t can be made. For example, for the 7.3 GeV cycle Q_h was set to 8.657 and ΔR was about +5 mm on the extraction flattop.¹⁹ If only the effect of Q_h is considered, γ_t would be $8.4935 + (0.88)(8.657 - 8.765) = 8.393$. Now, the ξ_h setting when $\Delta\gamma_t/\Delta(\Delta R)$ was found is not close to where it was on the 7.3 GeV flattop (0 units vs. -1.30). However, if I treat it as if it were the same then γ_t would equal $8.393 + (-0.0065 \times 5) = 8.361$. Using Bbrat with γ_t set to 8.361 instead of 8.50 changes the measured ϵ on the 7.3 GeV flattop from 0.693 to 0.858 eVs which is 24% higher. If I use only the estimate from Q_h (8.393) I get 0.813 eV-s.²⁰

Alternately, if say Q_h on the flattop were 8.89, $\xi_h = -1.3$, and $\Delta R = -5$ mm then the change in γ_t from the higher Q_h would be +0.116 and the change from ΔR would be +0.032 making $\gamma_t = 8.642$ and resulting in an ϵ of 0.583 eVs which is 16% lower than its value if $\gamma_t = 8.50$ is used.

10. Momentum Compaction

It may be that increasing ξ_h from the value where the ΔR dependence was measured reduces or even reverses the sign of that dependence. As mentioned, it would have been easy enough to measure this, but I didn't.

According to the AGS OpticsControl program a Horizontal sextupole current (I_{HS}) of about 265 A with no current in the vertical sextupoles will produce $\xi_h = 0$. Model predictions show that as I_{HS} is increased the ΔR dependence reverses somewhere between an I_{HS} of 121 and 190 A (presumably with no current in the vertical sextupoles since they were not mentioned in this reference).²¹ The model data is actually in terms of $\delta = (p_1 - p_0)/p_0$ where p_1 is the momentum (p) at ΔR and p_0 is p at R_0 . δ is related to ΔR , at a constant B field, by the equation $\delta = \gamma_t^2 (\Delta R / R_0)$. Whether γ_t increases or decreases with increasing ΔR depends on the value of α_1 , the 1st order component of the momentum compaction (α) via the equation,

$$\gamma_t(\delta) = \gamma_{t0} [1 - (\alpha_1 + 0.5 - \alpha_0/2)\delta + O(\delta^2)]$$

¹⁹ For Q_h see Tue#05071820_AGS_2 tune control archive at 4300 ms (where bunch lengths were measured). For ξ_h data see the Tue#05071820_AGS_2 chromcontrol archive where $\xi_h = 0$ (about 250A in the horizontal string and 55A in the vertical). For the extraction ΔR see [Booster-AGS-EBIS 2019 Feb 15](#) 1743 entry by John Morris.

²⁰ See [Booster-AGS-EBIS 2019 elog May 7 2033](#) entry which shows the ϵ measurement of 0.693 eVs using Bbat. The average of 10 bunch length measurements is used for the bunch length and the Rf voltage is found by measuring the synchrotron frequency. The bbrat programs takes γ_t as an input. Both programs are found in startup under "Specialist Tools".

²¹ See Figure 1 in [C. Ankenbrandt et al., "Bunching near transition in the BNL AGS", Physical Review Special Topics – Accelerators and Beams, Volume 1, 030101 \(1998\).](#)

where γ_{t0} is γ_t when $\delta=0$, or equivalently $\Delta R=0$, $\alpha_0 = 1/\gamma_{t0}^2$ is the zeroth order momentum compaction factor, and $O(\delta^2)$ is the δ^2 term.²² Note that, if the δ^2 term is ignored, when $\alpha_1 < (\alpha_0/2)-0.5$ then $\gamma_t(\delta) > \gamma_{t0}$ for a positive δ and that α_0 is only $1/8.5^2 = 0.007$.

It is straightforward to find α_1 (if the δ^2 term is ignored) from the data for the case where $\xi_h=-1.3$ (section 8) since γ_{t0} ($\Delta R=0$) was 8.4935 ± 0.009 and for $\Delta R=+7$ mm γ_t was 8.448 ± 0.009 . For $\Delta R=+7$ mm, $\delta=\gamma_t^2(\Delta R/R_0)=3.93e-3$. Plugging these values into the equation and using the same ‘error handling’ method used for the Q_h and ΔR dependence one finds that $\alpha_1=0.87 \pm 0.38$.

There are measurements of α_1 at 2 different values of I_{HS} around transition energy from 1998. Presumably they were taken with no current in the vertical sextupoles and what ultimately matters is ξ_h . For $I_{HS}=0$ A α_1 was 7.2 ± 1.5 and for $I_{HS}=100$ A α_1 was 3.5 ± 1.5 .²³ For the case measured here ξ_h was -1.3 and to produce that using only horizontal sextupole current requires about 141 A (instead of 153 A) according to the model AGSOpticsControl uses.²⁴

Fitting these three (I_{HS} , α_1) data points to a line one can estimate α_1 when I_{HS} is 265A from the linear fit and the value is -4.2 (see Figure 2). γ_t can be found for this value of α_1 from the equation for $\gamma_t(\delta)$ but instead of a δ corresponding to a ΔR of $+7$ mm a δ corresponding to the extraction radius of $+5$ mm for the 7.3 cycle is used, $\delta(5\text{mm}) = (8.5^2)(0.005/128.4526) = 2.81e-3$, to find that $\gamma_t=8.582$. This value is without considering the effect that Q_h has on γ_t . The estimate with a $Q_h=8.657$ is $8.582 + (0.88)(8.657-8.765)=8.487$, which after all this is not very far from the value Bbat uses (8.50) and only changes the calculated ε by 1.8% taking it from 0.693 to 0.705 eVs.

To estimate the uncertainty in α_1 with $I_{HS}=265$ A, a linear fit is performed on the following 9 points: (I_{HS}, α_1) = (0,7.2), (0,7.2+1.5), (0,7.2-1.5), (100,3.5), (100,3.5+1.5), (100,3.5-1.5), (141,0.87), (141,0.87+0.38), and (141,0.87-0.38) and the standard error from the fit, 1.22, is used so that α_1 when I_{HS} is 265A is estimated to be -4.2 ± 1.2 . The uncertainty in γ_t obtained from the $\gamma_t(\delta)$ formula arising from the uncertainty in α_1 is ± 0.03 and the uncertainty due to $\delta(\gamma_t) = \pm 0.009$ is independent of that so they can be added in quadrature to obtain ± 0.031 .

The uncertainty in γ_t associated with the estimate when Q_h is changed from 8.765 to 8.657 is ± 0.027 . Since this uncertainty is independent of the other the two can be summed in quadrature, which gives $\sqrt{0.027^2 + 0.031^2} = 0.041$. So, the estimate for γ_t on the 7.3 GeV flattop is then 8.487 ± 0.041 which, with a central value of 0.705 eVs, corresponds to a range in ε from 0.668 to 0.748 eVs or approximately 0.705 ± 0.04 eVs.

²² Ibid. This is equation (3) on page 030101-1

²³ Ibid. See pg. 30101-3

²⁴ Of course, 1998 is a long time ago and α_1 may behave differently now than it did then. As far as I know the configuration of the horizontal sextupoles hasn't changed since then.

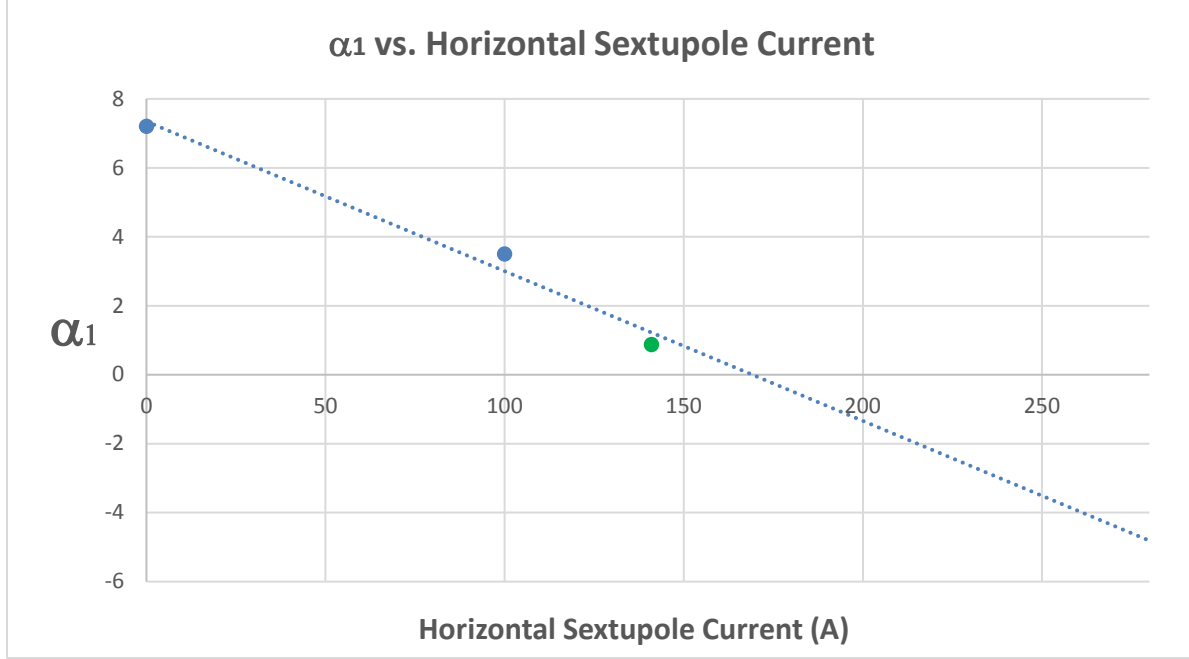


Figure 2: α_1 as a function of the horizontal sextupole current with no vertical sextupole current. The 2 blue dots are the 1998 data, the green dot is from the data in Section 8. The dotted line is the linear fit to the 3 data points which is $\alpha_1 = -0.0434(I_{HS}) + 7.3446$.

As a further complication η also has a higher order term that depends explicitly on α_1 ,

$$\eta(\delta) \approx 1/\gamma_t^2 - 1/\gamma^2 + \left(1/\gamma_{t0}^2\right)(\alpha_1 + 1.5\beta)\delta + O(\delta^2)$$

or

$$\eta(\delta) \approx \eta_0 + \left(1/\gamma_{t0}^2\right)(\alpha_1 + 1.5\beta)\delta + O(\delta^2)$$

where $\eta_0 = 1/\gamma_t^2 - 1/\gamma^2 = -2.36e-3$ ($\gamma_t = 8.487$), $\beta = 2\pi(R_0 + \Delta R)f_{rev}/c = 0.99185$ for $\Delta R = +5\text{mm}$ and $f_{rev} = 368405\text{ Hz}$ ($\gamma = 7.8470$), $\alpha_1 = -4.2$ and $\delta = 2.81e-3$.²⁵ In the latter equation the η_0 term is just the normal (zeroth order) slip factor, the second term is small ($-1.11e-4$) and only becomes important when η_0 is small, and the δ^2 term will be ignored. In this case the 2nd term changes $\eta(\delta)$ from $-2.36e-3$ to $-2.47e-3$. Since $\sqrt{|\eta(\delta)|}$ is in the denominator in the ε calculation this will make the calculated ε smaller by a factor of $\sqrt{2.36/2.47} = 0.977$, which takes the central value from 0.705 to 0.689 eVs. Since the second term is much smaller than η_0 this adjustment does not change the uncertainties significantly.

²⁵ Ibid. This is equation (4) on page 030101-1.

Now, all this may seem like it wasn't worth the trouble since the difference between the result from the initial ε calculation (0.693 eVs) and this result (0.689 \pm 0.04 eVs) is very small. But this was not known beforehand and, as shown in section 9, this result could have been very different depending on what Q_h and ξ_h were set to. The attempt made here to quantify the effect the measurement uncertainties have indicate that they are about $\pm 6\%$, which is still significant.

11. Sources of Error in this ε Measurement other than from η

This note has focused on finding an accurate value for γ_t for use in the ε calculation and not on the other details of a typical ε measurement, but a description of the errors or uncertainties arising from those seems relevant.

The Rf voltage and bunch length are required as inputs to the calculation, which assumes that the maximal extent of oscillations in phase ($\Delta\hat{\phi}$) are small relative to the size of the bucket (see footnote 2). In the case considered here this condition is met reasonably well. In practice the bunch length in nanoseconds is measured and input into the program (either Bbat or Bbrat), and for this case the length was 35.0 ns which corresponds to a $\Delta\hat{\phi}$ of about 23° (out of 180°).

The Rf voltage is obtained from a measurement of the synchrotron frequency (f_{synch}). Although that measurement appears to be quite accurate²⁶, in converting it to an Rf voltage it's assumed that this is f_{synch} near the center of the bucket.²⁷ In other words, in order to have confidence that the conversion is accurate the bunch should be small relative to the bucket. With a bucket area determined by this f_{synch} of about 21.7 eVs the bunch fills only about 3.2% of the bucket.²⁸

A bunch length measurement consists of measuring the length of the first half of the bunch with the WCM and multiplying it by 2 since the 2nd half of the bunch appears to be longer due to the frequency response of the WCM signal and because the bunch should be symmetric.²⁹ There were 10 measurements of the bunch length made and their mean and σ was 35.0 \pm 1.7 ns. The variation in ε due to a $\pm\sigma$ variation in the bunch length is ± 0.07 eVs. The bunch length measurement varies for 2 reasons, the first is measurement error, and the second is because the bunch length varies from cycle to cycle.

²⁶ See [Booster-AGS-EBIS 2019 elog May 7](#) 2030 entry

²⁷ The Rf voltage obtained this way also depends on γ_t . If γ_t were 8.4 (61.5 kV), the bucket area would be 25.2 eVs instead of 21.7 eVs (53.0 kV) for a γ_t of 8.5. Even though the ε is also different in these 2 cases the amount of the bucket that's filled is similar since as the bucket area increases the ε calculated for a given bunch length does as well.

²⁸ Whether or not the bunch is small relative to the bucket does depend strongly on the measured f_{synch} since if the Rf voltage used is lower than it actually is the calculated ε will be lower than it actually is and the amount of the bucket filled by the bunch will not change much.

²⁹ See [Booster-AGS-EBIS 2019 elog May 7](#) 20:14 entry where the length measurements are shown.

As regards the former, the length is not measured using a computer algorithm, but is measured by eye using a scope so there is a subjective element to it. There is also the sampling rate of the scope and noise on the signal. Since it is only the length of the first half of the pulse that is measured, whatever error there is there is doubled for the full length. From experience with these measurements, I would say the overall measurement uncertainty for this data is about $\pm 0.75\text{ns}$ (corresponding to $\pm 0.03\text{ eVs}$). Aside from the bunch to bunch variation, there is also the potential for a bias, which may lead to an average bunch length that depends on the person doing the measurement. It is also true that if the signal gain is increased, that the measured length will tend to increase.³⁰

The measured bunch length also varies because the bunch length really does vary. Nominally there would be 3 main reasons for this, the first is that the AGS injection matching varies from Booster transfer to Booster transfer by a noticeable amount because the Booster extraction field varies. It also varies because the quality of the bunch extracted from the Booster varies from transfer to transfer. The 3rd reason is that there are typically some bunch shape oscillations on the flattop so the length of the bunch when it's measured varies depending on whether it's taken closer to the peak or the valley.³¹ In the latter case the ϵ of the bunches need not be varying.

The effect on the final bunch length (or ϵ) due to the first 2 reasons is smaller than it would be otherwise though because after the AGS merges, when 'rebucketing' occurs into the Rf harmonic used for acceleration, beam that exceeds a certain ϵ winds up in the baby bunches and does not contribute to the ϵ of the main bunch. During the 7.3 GeV run the accelerating harmonic was changed from 12 to 10. When the harmonic is 10 there is more room in the acceleration bucket at 'rebucketing' and the effect of larger bunches at AGS injection translates more to larger ϵ bunches on the flattop.³² The measurements in this note were made when the harmonic was 10.

It's hard to separate the measured length variations from measurement errors and from actual variations. However, the object of this note is not to determine the variation in ϵ from

³⁰ An attempt to quantify this effect was done last year. The bunch length was measured for 2 gain settings, with one setting the bunch amplitude was about 5.5 divisions (4 measurements) and in the other case it was about 2.3 divisions (7 measurements). In the higher gain case, the measured bunch length was 3.6% longer, which would result in an ϵ that is 7.4% larger. See page 3 of [K. Zeno, "AGS Longitudinal emittance measurements for RHIC low energy gold runs" C-A/AP/615](#). The measurements made for the analysis in this note had bunch amplitudes of about 2 divisions.

³¹ For these measurements, judging from the f_{synch} elog entry, the variation in bunch length from peak to valley was about 3%. Note however that the amplitude of those oscillations varies from cycle to cycle and sometimes they're barely visible. When measuring f_{synch} I typically wait for a cycle that has particularly large oscillations since then it is easier to measure. See [Booster-AGS-EBIS 2019 elog May 7](#) 2030 entry

³² At one point during h=10 running the energy match was detuned by 70 Hz in f_{rev} . The flattop ϵ was measured in both cases and was 18% larger when detuned (0.77 vs. 0.65 eVs). See [Booster-AGS-EBIS 2019 elog May 8](#) entries from 1816 to 1943.

bunch to bunch. It seems reasonable that the average of 10 measurements would give a representative value for the length, and that any random errors related to the measurement itself would largely cancel out. The error associated with any bias, or the gain setting, though would remain.

12. Conclusions

This investigation was motivated by the desire to make an accurate ε measurement in the AGS on the 7.3 GeV extraction flat-top. It was realized, since this energy is rather close to transition (~ 7.9 GeV), that a more accurate value for γ_t than the one typically used (8.50) was needed. If nothing else, the data and analysis in this note indicates, depending on the values of the set Q_h , set ξ_h , and ΔR , that γ_t may easily vary over a range that could introduce an error in the ε calculation of as much as $\pm 18\%$ (see section 9).

After the above analysis it looks like the error in the ε calculation introduced by these variables may be reduced to about $\pm 6\%$ by making appropriate adjustments to the value of γ_t used in the calculation. However, whether this uncertainty is $\pm 6\%$ or $\pm 18\%$ it pertains only to accuracy of the measurements described here and not to how those measurements are used to arrive at an estimate for γ_t (or η). For example, if the Q_h or ξ_h dependence of γ_t is far from linear, the error in the estimate will be greater even though the uncertainties in the measurements have not changed.

It also looks like the initial 7.3 GeV ε measurement (0.693 eVs) happens to be close to the value obtained after the effects on γ_t of the set Q_h , set ξ_h , and ΔR are estimated (0.689 ± 0.04 eVs).

Now that I've gone through this analysis and can see better what's required to accurately estimate γ_t it's evident that the estimate could be improved with more or at least different data. In particular, some data from 1998 was used together with a minimal amount of data from last run to find the dependence of α_1 on ξ_h . It would be preferable to rely on recent data than on data that is 20 years old if only because the machine may have changed since then. To do that, measurements of γ_t for several different values of ξ_h , at $\Delta R=0$ and one or more ΔR values are desirable. Ideally, one of those ΔR values should be close to its value at extraction (typically 4 to 6 mm) since α_1 may not be approximated well as a linear function of ξ_h . The Q_h data could also be improved by measuring γ_t at several different values of Q_h , one of which is the set Q_h at extraction, instead of only two.

Instead of making all these measurements to better describe the γ_t dependence it might be more practical, because it could be easier and more to the point, to just measure γ_t with the set values of Q_h , ξ_h , and ΔR that are used in the extraction setup in question.

It might also be possible to forego all this by measuring γ_t more directly, say by measuring f_{rev} at different radii at the time in the cycle the ε measurement is made (typically on the flattop near extraction) although that method may be fraught with its own problems.