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The dipole corrector magnets for the FFAG beam line of the CBETA accelerator

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DISCLAIMER

The dipole corrector magnets for the FFAG beam line of the CBETA accelerator[1]

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Abstract

The Cornell Brookhaven Experimental Test Accelerator (CBETA) [1] is a novel, and the first of its kind electron accelerator to combine two remarkable concepts, the Energy Recovery Linac (ERL) [2] and the Fixed Field Alternating Gradient (FFAG) concept [3]. The accelerator has been built at Cornell University in collaboration with Brookhaven National Laboratory (BNL) and it is at its commissioning stage. The FFAG beam line consist of two arcs sections and a straight section, with all sections comprised of single cells, each cell made of two Halbach type of magnets. The cell in the arcs consists of one focusing quadrupole and a combined function magnet which is comprised of a dipole and a defocusing quadrupole multipole. The cells in the straight section consist of a focusing and a defocusing quadrupoles. Each focusing quadrupole has a window-frame dipole corrector magnet generating vertical dipole field and the other magnet has a corrector generating horizontal dipole field. This technical note reports on the mechanical design and the electromagnetic properties of these window frame corrector magnets.

Keywords: FFAG, ERL

1. Introduction

The CBETA electron accelerator [1] is unique of its kind to combine two remarkable concepts, the Energy Recovery Linac (ERL) concept and the Fixed Field Alternating Gradient (FFAG) concept. Fig. 1 is a top view of the CBETA accelerator. A 6 MeV electron bunch generated from the injector (IN) shown in Fig. 1 is injected to the ERL (LA) to increase the kinetic

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energy of the electron bunch by 36 MeV and subsequently enters the "42 MeV" line of the four-line splitter section (SX) of the accelerator.

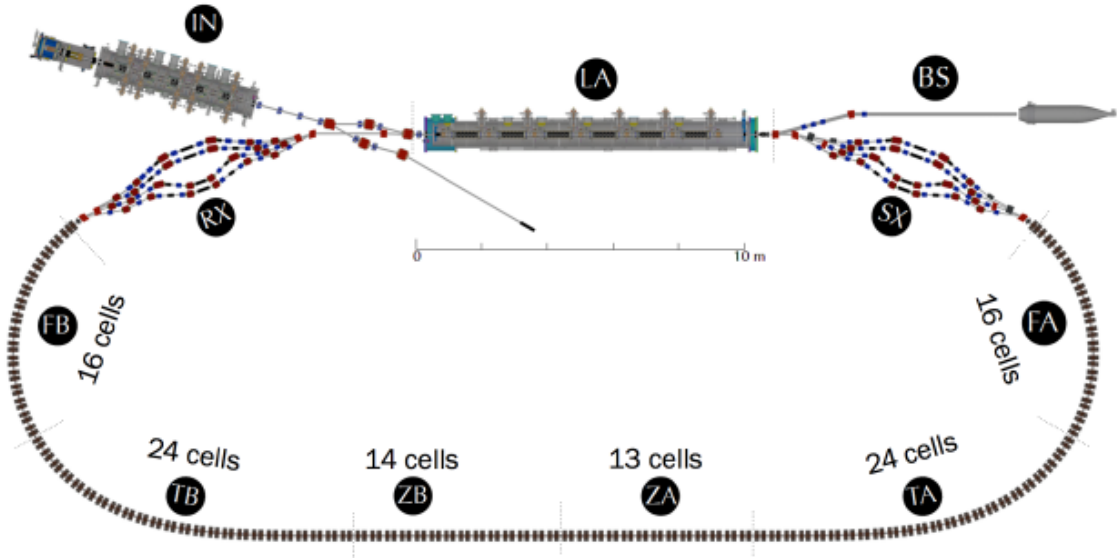

Figure 1: Layout of the CBETA accelerator. The section labeled (IN) is the 6 MeV electron injector into the CBETA accelerator. The section labeled (LA) is the ERL, the sections labeled (FA), (TA), (ZA), (ZB), (TB), and (FB) are the FFAG sections which accommodate four recirculating electron bunches in the energy range from 42 MeV to 150 MeV. The sections (SX) and (RX) are the splitter and combiner sections respectively each comprised of four lines to transport the electron bunches with energies 42, 78, 114, and 150 MeV respectively.

The 42 MeV beam line of the splitter (SX) transports the electron bunches to the FFAG-sections (FA, TA, ZA, ZB, TB, EB) and the combiner (RX) section, recirculate the bunches to the entrance of the ERL for the bunches to receive an additional 36 MeV at the exit of the ERL. The bunches attains their final energy of 150 MeV after two additional recirculations in the FFAG. Subsequently, by changing the path length of the 150 MeV bunch, the phase of the electron bunches with respect to the RF accelerating field of the ERL changes by 180^o and the energy of the electron bunches is reduced by 36 MeV each time the bunches transverse the the ERL, for their energy to be reduced to 6 MeV after four recirculations through the ERL. The 6 MeV electron bunches are dumped in the designated electron dump (BS). It is for the remarkable property of the FFAG that electron bunches with energy range from 42 MeV to 150 MeV can be transported by the single FFAG

transport line which consists of the sections (FA), (TA), (ZA), (ZB), (TB), and (FB) shown in Fig. 1.

The FFAG transport line consists of 107 cells each comprised of two Halbach type permanent magnets. The cell in the arcs (FA), (TA), (TB), and (FB) consist of a focusing quadrupole (QF) and a combined function magnet (BD) which is a combination of a dipole multipole, and a defocusing quadrupole multipole. Each cell in the straight section (ZA), (ZB), consist of a focusing quadrupole (QF) and a defocusing quadrupole. Fig. 2 is a perspective view of three consecutive FFAG arc-cells of the CBETA. Each QF and DB magnet of the cell has a window frame iron core with a coil generating a vertical dipole field for the QF magnet and an horizontal dipole field for the BD magnet. These window frame magnets act as dipole corrector magnets. This paper

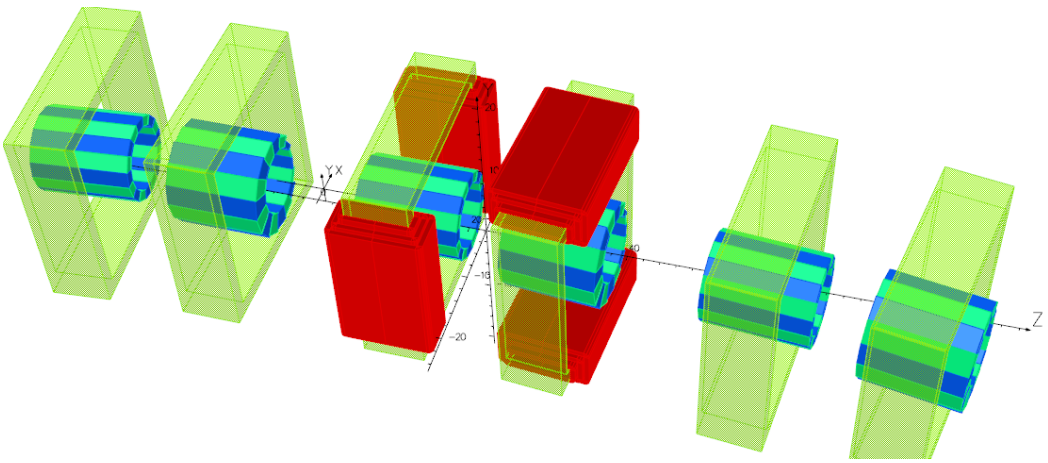

Figure 2: A perspective view of three consecutive cells of the FFAG arcs. Each cell consists of two magnets, one of a focusing quadrupole (QF) and one of a combine function (BD) with dipole and quadrupole multipoles. Each magnet has a window frame magnet acting as a corrector.

provides information on the mechanical and the electromagnetic properties of the window frame magnet. The effect of the dipole corrector magnet on the magnetic multipoles of a Halbach magnet is discussed and calculations and experimental results are presented. The extend of the dipole field of each corrector magnet on the neighboring magnets is also studied and presented.

2. Description of the mechanical design of the window frame magnet

Mechanical drawings of the window frame magnet are showing in Figs. 3 and 4. The dimensions in the drawings are in inches. Table 1 lists the

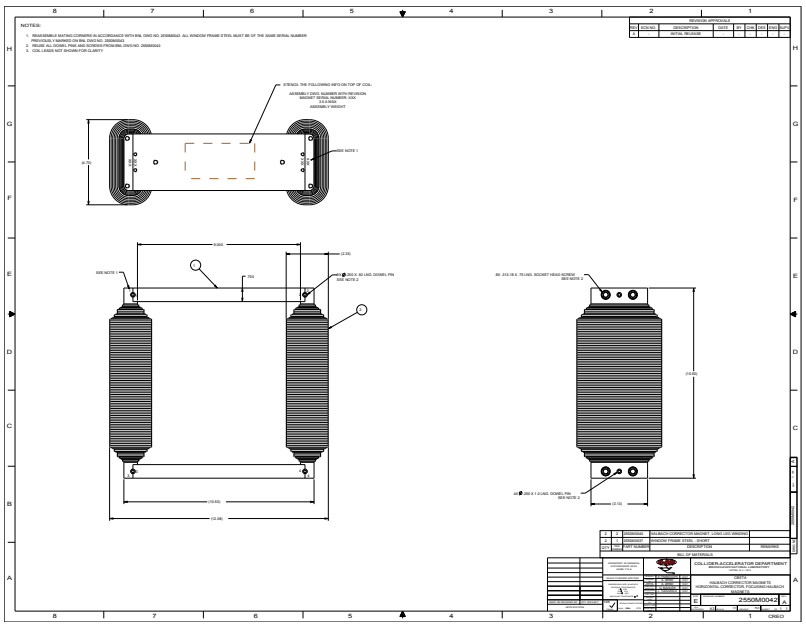

Figure 3: A mechanical drawing of the corrector showing the dimensions in inches of the iron core of the window frame magnet.

dimensions in units of cm of the square window frame iron core. The thickness (label "Thick" in Table 1) of the iron frame has been chosen for the iron not to saturate when the corrector is excited at its maximum field. The inner width (W) and height (H) of the frame have been chosen to allow space for the placement of an aluminum block shown in Fig. 5 around the magnet for keeping the temperature of the permanent magnet constant. The length of the iron frame has been chosen for the coil of the corrector magnet not to extend into the drift space between the Halbach type of magnets.

2.1. The coil of the dipole corrector magnet

The constrain on the maximum field generated by the corrector and the optimum power supply to be used to power the magnet, defines the AWG

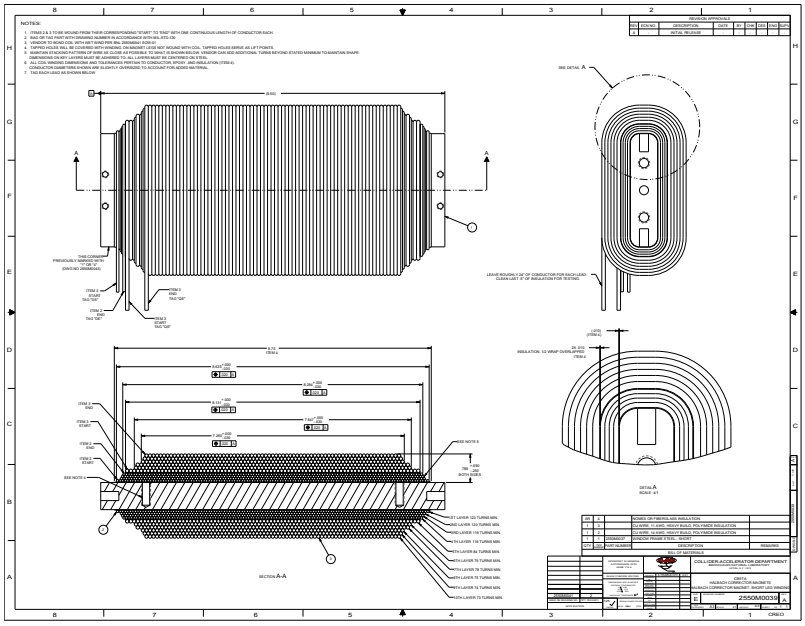

Figure 4: A drawing of the window frame magnet showing the number of copper-wire-turns wound in one of the sides of the frame to make the coils. Dimensions in the drawing are in inches. The maximum current running through the wire of the coils is 2.29 A

gauge and the turns of the conductors which make the magnet's air cooled coils.

Fig. 4 is a another drawing of the magnet which shows that there are 10 layers of copper conductors for each coil and lists the number of turns of the copper conductors per layer. The first four inner layers of each coil are made of copper conductor gauge 14 and the rest six layers of copper conductor gauge 11. Both wire conductors have heavy built polyimide insulation. Initially the corrector magnet was designed to generate a dipole field by connecting in series the coil made of the four inner layers to that of the opposite side of the frame, and also generate a quadrupole field by "appropriately" connecting the outer-six-layers-coil of one side, with the corresponding coil placed on the other three sides of the frame. Table 2 lists the number of layers and the turns per layer wound around each side of the iron window frame. The gauge of the copper wire conductors is also listed in Table 2. For completion we introduce Table 3 which lists the inductance of the window frame magnet when various combinations of the coils are used to generate Normal and Skew dipoles and Normal quadrupole. In Table 3 the meaning of

symbols are: N (Normal) S (Skew) V (Vertical) H (Horizontal). To generate a Skew quadrupole the window frame magnet should be rotated by 45^o. In this case four coils, one in each side of the window frame magnet have to be used to generate either normal or skew dipole or a combination of both.
APPENDIX B describes possible coil-connection of a window frame magnet that generates dipole and quadrupole fields as well as a combination of of both multipoles as described above. In addition APPENDIX B describes possible use of floating power supplies to generate a combination of Normal Dipole, Skew Dipole, and Normal Quadrupole multipoles. In the CBETA setup the coil made of wire gauge 14 is connected in series to that of gauge 11 and this combined coil is connected in series with the coil on the opposite side of the window frame to make a dipole corrector, with the maximum current running through the coil's conductors at 2.29 A. The coils which generate the vertical dipole field (Horizontal-corrector) are placed on the vertical sides of the window frame and those which generate the horizontal field (Vertical-correctors) are placed on the horizontal sides of the window frame.

Table 1: Dimensions of the square window frame iron of the corrector magnet.

Length [cm]	Inner(W/H) [cm]	Out(W/H) [cm]	Thick [cm]
7.95	22.86	26.67	1.905

Table 2: The number of copper-wire turns in each of the layers of the coils. The wire gauge of the layers 1 to 4 is AWG=14 the rest layers have wire gauge of AWG=11

Layer #	1	2	3	4	5	6	7	8	9	10	Total turns
# of turns	123	120	119	118	84	79	78	75	74	73	914
Wire AWG	14	14	14	14	11	11	11	11	11	11	

Res. [Ω]	Induct. [H]	I_{max} [A]
2.61	0.680	2.29

Table 3: Inductance of the window frame magnet based on the coils used to generate a combination of dipole and quadrupole corrector

Multipole	Coils	Layers/side	Turns/side	Inductance [H]
N-Dipole	V-Side	1 to 4	480	0.192
N+S-Dipole	V+H-Side	1 to 4	480	0.377
N-Dipole	V-Side	1 to 10	914	0.680
N-Quad	V+H-Side	5 to 10	434	0.129
N+S-Dip N-Quad	V+H-Side	1-4 5-10	480 434	0.504

3. The electromagnetic properties of the dipole corrector magnet

The OPERA computer code [2] was used to calculate the electromagnetic properties of the window frame dipole corrector magnet. Because the cross section of the magnet's iron is well defined (window frame) there was no need to shape the magnet's pole faces, therefore only 3D calculations were performed to calculate the multipoles of the magnet. These multipoles include the effect (interference) of the iron frames of the neighboring magnets. Fig. 6 is a perspective view of the B_y field on a rectangular patch on the median plane. This B_y field is generated by the single window frame magnet with the coils as shown in Fig. 6. The 2D view of the B_y field shown in Fig. 6 is generated by the window frame magnet having the iron's permeability and the current in the conductors of the coils is 2.29 A. The rest of the window frames shown in the figure have permeability μ=1. The extend of the B_y field in Fig. 6 has to be compared with the extend of the field generated by the same magnet but with the neighboring frames of the corrector magnets having the same permeability of the iron as the corrector magnet with the coils. The extend of the B_y field shown in Fig. 7 is less than that shown in Fig. 6 because the iron frames in Fig. 7 have the permeability of magnetic iron and act as field clamps.

Although Figs. 6 and 7 show some details of the dipole field generated by the window frame corrector magnet, like the existence of some sextupole component of the dipole field, a better way to view the variation of the B_y component of the field as a function of the z distance measured from the center of the corrector magnet is shown in Fig. 8. The rectangles on the figure are the iron frames of the corrector magnets. The black curve corresponds to

the B_y field with all the window frames having permeability μ=1 except the window frame which generates the field that has the permeability of magnetic iron. The red curve corresponds to the B_y field with all the window frames having permeability of magnetic iron. Notice the red curve is not symmetric with respect to the center of the magnet at z=0, because the neighboring iron frames are not place symmetrically with the frame of the central magnet. An important information which can be used in beam optics calculations is the strength of the multipoles generated by the field of the corrector. These multipoles are generated by a Fourier expansion of the radial components of the field at a radious R= 1 cm and are plotted as a function of the the distance from the center of the corrector magnet in Fig. 9. The integrated strength of each multipole is shown in Table 4. The existance of a quadrupole multipole in a magnet with dipole symmetry is explained by the asymmetric placement of the neighboring window frame iron cores which affect the otherwise dipole symmetric field of the corrector. In Fig. 9 the values of the quadrupole and sextupole multipoles have been multiplied by 100 to be made visible in the plot. The integrated strength of the multipoles higher than sextupole is two order of magnitude lower then the sextupole strength. It is understood that the multipoles generated by the dipole corrector may be different along the four reference trajectories of the electron bunches, but simulation studies show that this difference is of second order effect.

Table 4: The integrated multipoles of the dipole corrector at a radius R=1 cm.

Dipole [Tm]	Quadrupole [Tm]	Sextupole [Tm]
$2.1714\text{x}10^{-3}$	$1.94\text{x}10^{-6}$	$1.79\text{x}10^{-6}$

Dipole [units]	Quadrupole [units]	Sextupole [units]
10000	8.9	8.2

4. superposition of the dipole corrector field with the field of the Halbach magnet

Prior of deciding to manufacture and place the corrector magnets around the main Halbach magnets, magnetic field measurement were performed to

measure the effect of the dipole's corrector field on the main field of the permanent magnet. Although it is clear that a permanent magnet is saturated along the easy direction (μ=1), therefore any external field will be superimposed along this direction, it was not clear how a known external field applied on a different direction will change the field of the permanent magnet. Such an experimental study was conducted [3] and the results of the measurements showed that the effect of the dipole correctors excited to their maximum field on the FFAG main magnets is almost negligible. The left picture in Fig. 10 shows the permanent magnet wedges which comprise a modified quadrupole Halbach magnet and the window frame magnet surrounding the Halbach magnet. The right picture in Fig. 10 is an expanded view of the window frame magnet. The cylindrical pipe through the dipole magnet is the rotating coil which measures the integrated multipoles of the PM-Quad+WF-Dipole assembly. APPENDIX A presents the results of the magnetic measurements performed by [3] and the conclusions.

5. Appendix A: Superposition of Dipole field with the field of a Permanent Magnet Quadrupole

This section includes the results of the magnetic measurements performed by [3].

Fig. 11 shows the effect of the bare WF yoke on the normal (left) and skew (right) integrated multipoles of the PM quadrupole.

Fig. 12 shows the effect of the Horizontal Field Generated by the WF dipole which is excited at its nominal current, on the normal (left) and skew (right) integrated multipoles of the PM quadrupole.

Fig. 13 shows the effect of the Vertical field generated by the WF dipole on the normal (left) and skew (right) integrated multipoles of the PM quadrupole.

Fig. 14 shows the effect of the Horizontal and Vertical fields generated by the WF dipole on the normal (left) and skew (right) integrated multipoles of the PM quadrupole.

Fig. 15 is the summary of the measurements as provided by [3].

6. Appendix B: Generating a Dipole and a quadrupole field using the same four coils of a WF magnet

The main sections of this technical note is devoted in providing information on the mechanical and electromagnetic properties of window frame magnet which generates a normal dipole (vertical field) by exciting two coils on the opposite vertical sides of the iron frame or generate a skew dipole (horizontal field) by exciting two coils on the opposite horizontal sides of the iron frame. This appendix discuses methods on how to connect coils of the widow frame magnet to generate a normal quadrupole or a skew quadrupole and combination of quadrupole field with dipole field. To cover as many possibilities of quadrupole and dipole fields the subsections below start with the most simple cases.

6.1. Normal dipole only

Details of generating a normal dipole was discussed in the main part of this technical note and in this subsection a schematic figure is provided to explain the connection of the coils of the window fram magnet. The picture in Fig. 16 (a) is a cross section of WF magnet with two racetrack coils wound around the vertical side to generate a normal dipole field. For simplicity in all the text below this magnet will be represented by only the two inner conductors of the coils as shown in Fig. 16 (c) with the outer conductors and the WF iron omitted. Fig. 17 is a schematic diagram showing the connection of the coils to generate a normal dipole field. Note that the top and bottom coils are not connected to a power supply. The sense of the normal field (pointing up or down) is adjusted by the polarity of the power supply.

6.2. Skew dipole only

To generate a skew dipole (horizontal field) the top and bottom coils shown in Fig. 17 are used instead of the vertical coils. Otherwise the connection of the coils is identical to that of generating a normal field. The rotation of the WF magnet, which generates normal dipole field, by 90^o is equivelent to magnet which generates skew dipole field. At present the correction magnets of the CBETA FFAG line are either normal or skew dipoles.

6.3. Normal and skew dipoles of same strength and sense

Fig. 18 is a schematic diagram for the wire connection of the coils of the WF magnet which generates both normal and skew dipoles. If the coils

which generate normal field are identical to those which generate skew field, the strength of the normal dipole is the same with that of the skew dipole. Also if the of the normal field changes by changing the polarity of the power supply the sense of the skew dipole will change too. To change the sense of only one of the fields, say the normal field only, the wiring of the coils has to change.

6.4. Normal and skew dipoles of variable strength and sense

This subsection describes two ways of generating normal and skew fields of variable strength and sense.

6.4.1. Normal and skew dipoles two independent power supplies

Fig. 19 is schematic diagram showing the wire connection of the coils with two independent power supplies. Such a connection allows independent variation of the normal and skew dipole strength, as well as independent variation of the sense of the dipole fields by changing the polarity of the power supplies.

6.4.2. Normal and skew dipoles with two power supplies one floating

In applications where both normal and skew dipole fields are to maintain almost the same strength a floating power supply maybe used to vary slightly the strength of one of the fields. Fig. 20 is a schematic diagram for the wire connection of the coils of the WF magnet which generates both normal and skew dipoles but of variable strengths between the normal and the skew dipoles. The coils which generate the normal dipole field do not have to be identical to those which generate the skew dipole field. Note that there is an additional power supply which floating. This floating power supply works in conjunction with the regular power supply to adjust the strength and the sense of the normal and skew dipoles independently.

6.5. Normal quadrupole

Fig. 21 is a schematic diagram for the connections of the four identical coils of the WF magnet to generate a normal quadrupole. If the vertical coils are not identical with the horizontal ones the strength of the quadupole will correspond to the the coils with the least ampere-turns.

6.6. Combination of Normal Quadrupole and Normal or Skew Dipole

This can be accomplished in two ways as it is described in the following two subsections below.

6.6.1. Separate Quad and Dipole coils powered by separate power supplies

Fig. 22 is a schematic diagram for the connections of the Quadrupole coils (4 coils) by a power supply and the connection of the Dipole coils (2 coils) by a separate power supply. This method requires six racetrack coils and two power supplies.

6.6.2. Using floating power supplies to generate Normal Quad and Normal of Skew Dipole

6.7. Combination of Normal Quadrupole with Normal and Skew Dipole

This can be accomplished in two ways as it is described in the following two subsections below.

6.7.1. Normal Quad Normal Dipole and Skew Dipole

The Quad-coils, the Normal-Dipole-coils and the Skew-Dipole-coils are all separate and each type of coils is powered by a separate power supply. In all this method requires 8 coils and three power supplies. To reduce the sextupole component generated by the dipole coils it is recommended to wound the dipole coils right up against the iron frame.

6.7.2. Normal Quad Normal Dipole and Skew Dipole with floating PS

Fig. 24 is a schematic diagram for the wire connections of four coils and six power supplies to generate a Normal quadrupole combined with a Normal and Skew dipole field. This method requires four racetrack coils and six power supplies.

7. References

[1] ICFA BD Newsletter No. 74 http://icfa-bd.kek.jp/

[2] https://operafea.com/

[3] J. Animesh private communication

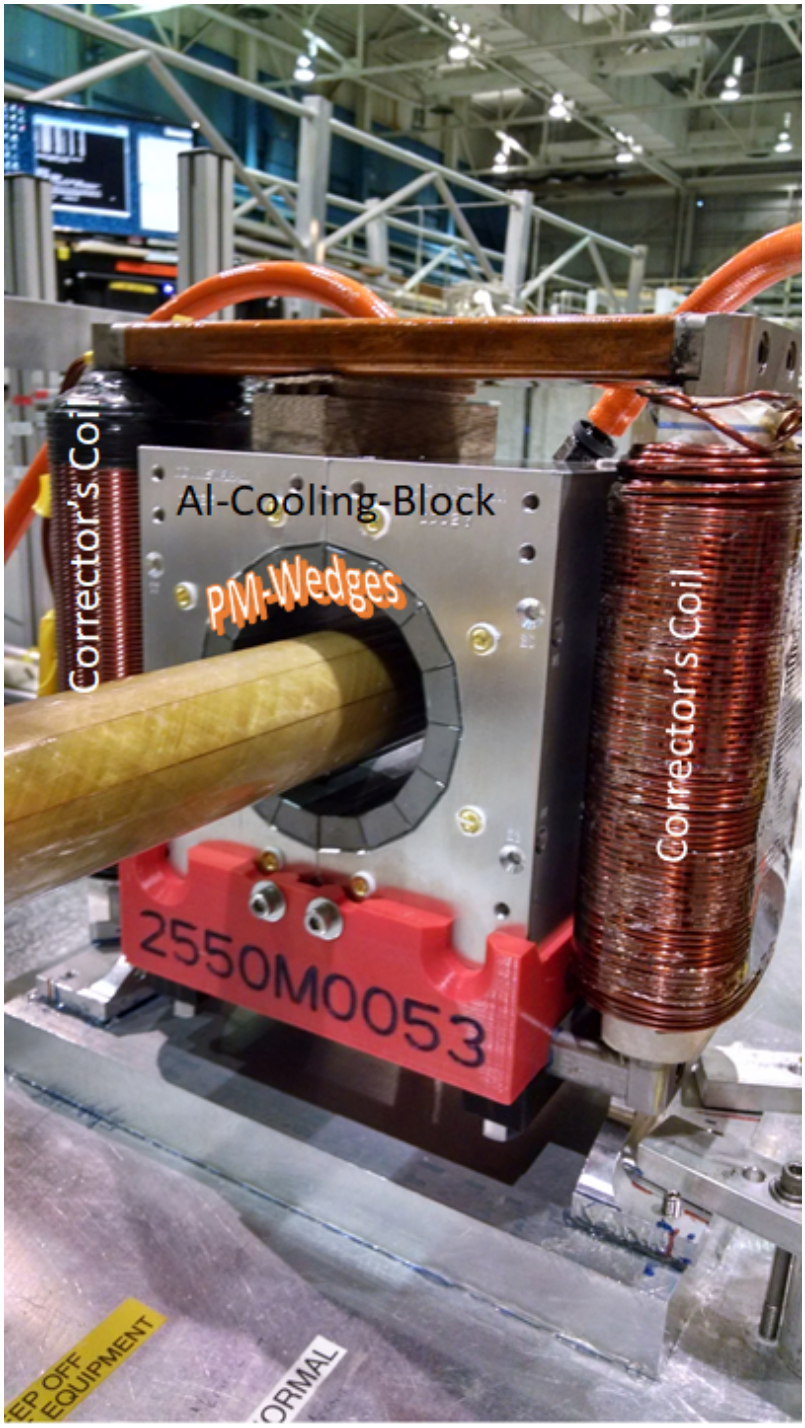

Figure 5: A picture of the quadrupole permanent magnet wedges surrounded by the Al cooling block. The coils of the dipole corrector magnet are also shown.

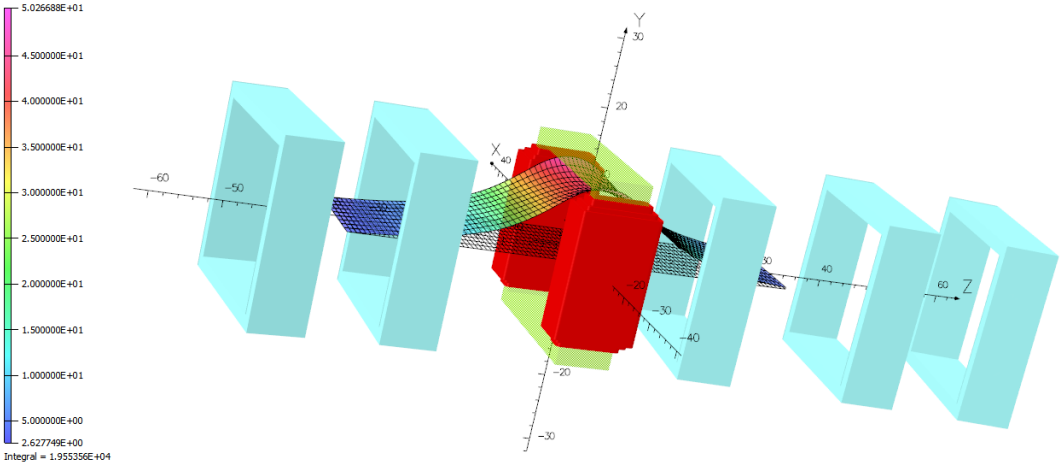

Figure 6: A perspective view of the B_y field on a rectangular patch. The iron of the neighboring window frame magnets appear in the drawing to show the extend the field of a single corrector inside the neighboring magnets. The current in the wire of the coil of the single corrector magnet is at its maximum value of 2.29 A.

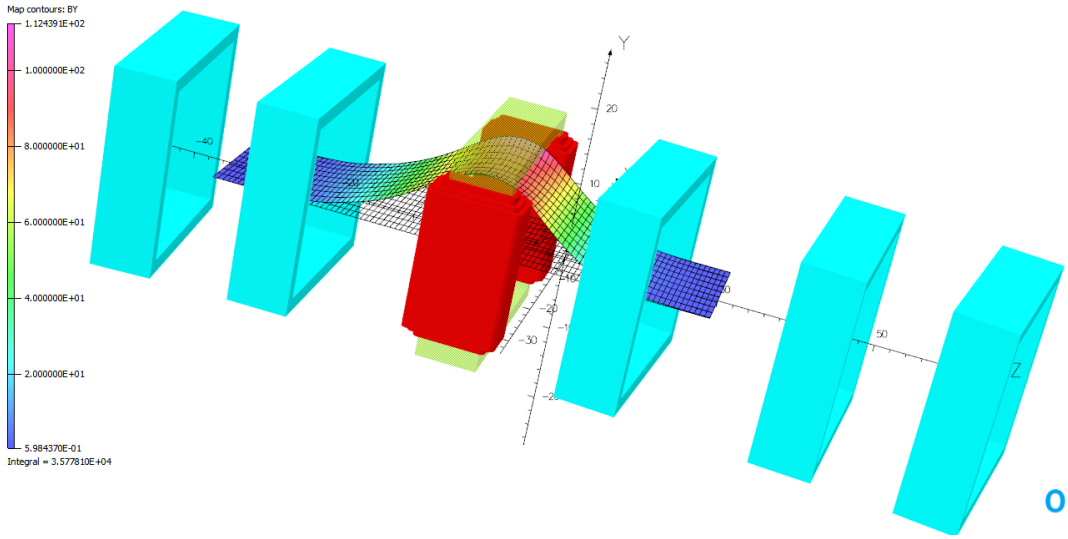

Figure 7: A perspective view of the B_y field on a rectangular patch. The iron of the neighboring window frame magnets appear in the drawing to show the extend the field of a single corrector inside the neighboring magnets. The current in the wire of the coil of the single corrector magnet is at its maximum value of 2.29 A. The extend of the field of the corrector magnet with the coils is reduced when the material of the neighboring iron frames has the same permeability as the iron of the corrector magnet with the coils.

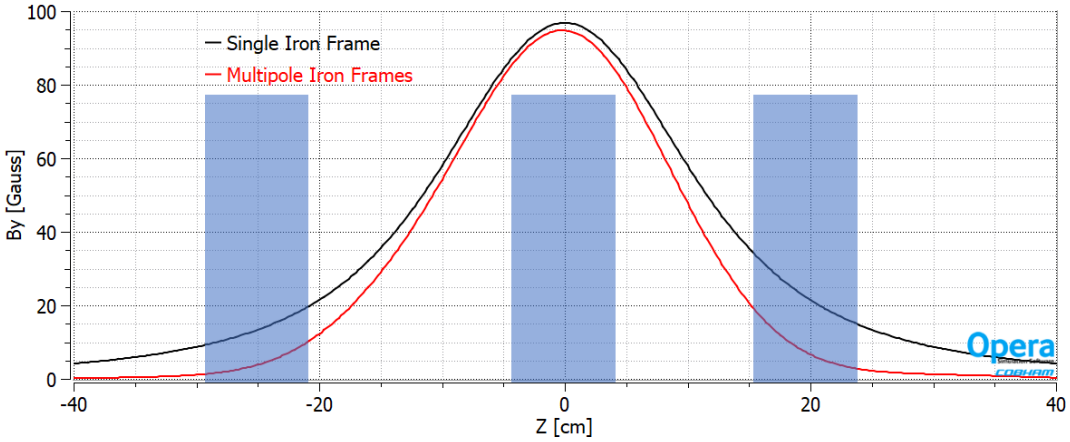

Figure 8: The B_y component of the field as a function of z distance measured from the center of the corrector magnet. The rectangles on the figure are the iron frames of the corrector magnets. The black curve corresponds to the B_y field with all the window frames having permeability μ=1 except the window frame which generates the field and has the permeability of magnetic iron. The red curve corresponds to the B_y field with all the window frames having permeability of iron. Notice the red curve is not symmetric with respect to the center of the magnet because the neighboring iron frames are not place symmetrically to the central one.

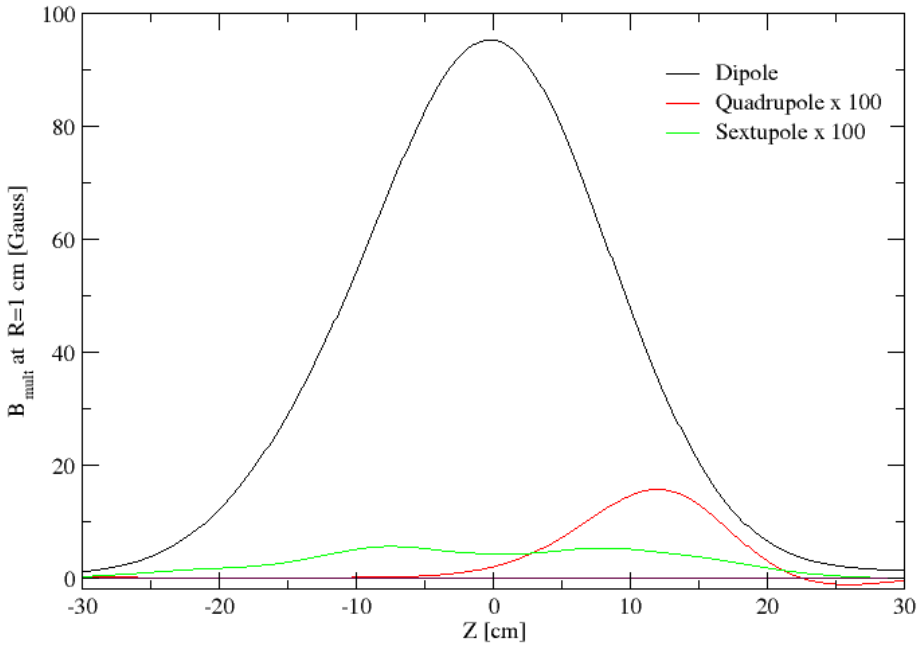

Figure 9: The dipole, quadrupole and sextupole multipoles as a function of the longitudinal distance z. The multipoles are the Fourier expansion of the radial field at a radius R=1 cm. Notice that the values of the quadrupole and sextupole multipoles have been multiplied by 100 to be visible in the graph.

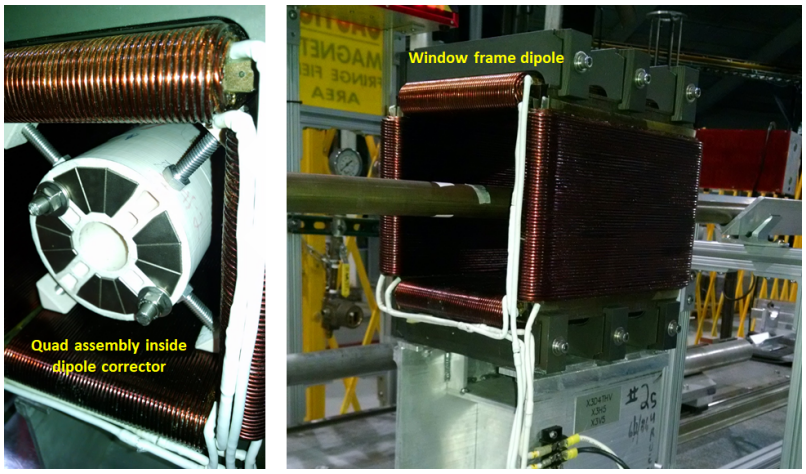

Figure 10: (Left) The window frame dipole magnet surrounding the permanent Halbach type magnet. (Right) An expanded view of the window frame magnet. The cylindrical pipe through the dipole magnet is the rotating coil which measures the integrated multipoles of the PM-Quad+WF-Dipole assembly.

Effect of Window Frame Yoke Only

Normal Terms: B_ref = 0.037748 T.m (at R_ref = 10 mm)

Harmonic Number	PMQ Only in T.m (1) Run 2/01	PMQ + WF (0A) in T.m (5) Run 10/02	Normalized to B_ref (in "units" of 1E-4) PMQ Only	PMQ + WF (0A)	Change
1	-2.605E-10	-4.721E-11	0.00	0.00	0.00
2	**3.775E-02**	3.796E-02	10000.00	10055.85	55.85
3	-5.700E-06	-6.980E-06	-1.51	-1.85	-0.34
4	-4.375E-06	2.328E-06	-1.16	0.62	1.78
5	-6.699E-06	-5.863E-06	-1.77	-1.55	0.22
6	-7.103E-04	-7.115E-04	-188.17	-188.49	-0.33
7	-1.986E-06	-2.011E-06	-0.53	-0.53	-0.01
8	-1.352E-06	-1.006E-06	-0.36	-0.27	0.09
9	3.260E-07	2.609E-07	0.09	0.07	-0.02
10	-9.706E-06	-9.733E-06	-2.57	-2.58	-0.01
11	1.039E-07	1.258E-07	0.03	0.03	0.01
12	8.371E-08	3.132E-08	0.02	0.01	-0.01
13	-2.289E-08	-1.424E-08	-0.01	0.00	0.00
14	4.206E-07	4.127E-07	0.11	0.11	0.00
15	-8.415E-10	-2.036E-09	0.00	0.00	0.00

Effect of Window Frame Yoke Only

Skew Terms: B_ref = 0.037748 T.m (at R_ref = 10 mm)

Harmonic Number	PMQ Only in T.m (1) Run 2/01	PMQ + WF (0A) in T.m (5) Run 10/02	Normalized to B_ref (in "units" of 1E-4) PMQ Only	PMQ + WF (0A)	Change
1	9.605E-11	2.870E-11	0.00	0.00	0.00
2	-1.379E-11	7.030E-12	0.00	0.00	0.00
3	-3.337E-05	-3.500E-05	-8.84	-9.27	-0.43
4	-9.634E-06	-1.032E-05	-2.55	-2.73	-0.18
5	-2.898E-05	-1.477E-05	-7.68	-3.91	3.76
6	-6.135E-06	-6.495E-06	-1.63	-1.72	-0.10
7	4.046E-06	4.053E-06	1.07	1.07	0.00
8	-3.741E-07	-3.879E-07	-0.10	-0.10	0.00
9	2.946E-07	3.704E-07	0.08	0.10	0.02
10	1.982E-08	-5.434E-10	0.01	0.00	-0.01
11	3.545E-08	4.723E-08	0.01	0.01	0.00
12	7.834E-10	4.462E-09	0.00	0.00	0.00
13	1.323E-08	7.317E-09	0.00	0.00	0.00
14	-1.898E-09	-1.110E-08	0.00	0.00	0.00
15	-1.086E-08	-3.017E-09	0.00	0.00	0.00

Figure 11: The effect of the WF yoke on the integrated normal (left) and skew (right) multipoles of the PM quadrupole.

Superposition for Horizontal Field Dipole

Normal Terms: B_ref used for Normalization= 0.037959 T.m (at Reference radius = 10 mm)

Harmonic Number	PMQ + WF (0A) in T.m 5	SkD Only (10A) in T.m 2	PMQ + SkD (10A) in T.m	Normalized to B_ref (in "units" of 1E-4)			
	Run 10/02	Run 5/01	Run 11/03	PMQ+WF(0A)	SkD Only (10A)	PMQ+SkD (10A)	Superposition Error
1	-4.721E-11	-4.322E-05	-6.223E-05	0.00	-11.39	-16.39	5.01
2	**3.796E-02**	3.662E-06	3.795E-02	10000.00	0.96	9998.02	2.95
3	-6.980E-06	-1.976E-07	-6.775E-06	-1.84	-0.05	-1.78	-0.11
4	2.328E-06	-3.922E-08	1.859E-06	0.61	-0.01	0.49	0.11
5	-5.863E-06	7.459E-09	-5.499E-06	-1.54	0.00	-1.45	-0.09
6	-7.115E-04	-7.467E-09	-7.119E-04	-187.45	0.00	-187.55	0.10
7	-2.011E-06	-2.511E-09	-2.085E-06	-0.53	0.00	-0.55	0.02
8	-1.006E-06	4.542E-09	-1.024E-06	-0.26	0.00	-0.27	0.01
9	2.609E-07	8.358E-10	2.591E-07	0.07	0.00	0.07	0.00
10	-9.733E-06	-1.582E-09	-9.732E-06	-2.56	0.00	-2.56	0.00
11	1.258E-07	-4.136E-10	1.251E-07	0.03	0.00	0.03	0.00
12	3.132E-08	7.563E-10	3.366E-08	0.01	0.00	0.01	0.00
13	-1.424E-08	-1.432E-11	-1.159E-08	0.00	0.00	0.00	0.00
14	4.127E-07	-3.192E-10	4.146E-07	0.11	0.00	0.11	0.00
15	-2.036E-09	-3.192E-11	-4.322E-09	0.00	0.00	0.00	0.00

Superposition for Horizontal Field Dipole

Skew Terms: B_ref used for Normalization= 0.037959 T.m (at Reference radius = 10 mm)

Harmonic Number	PMQ + WF (0A) in T.m 5	SkD Only (10A) in T.m 2	PMQ + SkD (10A) in T.m 6	Normalized to B_ref (in "units" of 1E-4)			
	Run 10/02	Run 5/01	Run 11/03	PMQ+WF(0A)	SkD Only (10A)	PMQ+SkD (10A)	Superposition Error
1	2.870E-11	5.680E-03	5.658E-03	0.00	1496.37	1490.51	5.86
2	7.030E-12	3.259E-06	-8.500E-07	0.00	0.86	-0.22	1.08
3	-3.500E-05	-7.188E-06	-5.042E-05	-9.22	-1.89	-13.28	2.17
4	-1.032E-05	-1.644E-08	-1.011E-05	-2.72	0.00	-2.66	-0.06
5	-1.477E-05	7.578E-08	-9.787E-06	-3.89	0.02	-2.58	-1.29
6	-6.495E-06	2.810E-09	-6.614E-06	-1.71	0.00	-1.74	0.03
7	4.053E-06	-6.512E-09	3.933E-06	1.07	0.00	1.04	0.03
8	-3.879E-07	-1.089E-09	-3.807E-07	-0.10	0.00	-0.10	0.00
9	3.704E-07	2.525E-09	3.954E-07	0.10	0.00	0.10	-0.01
10	-5.434E-10	9.721E-10	-6.038E-09	0.00	0.00	0.00	0.00
11	4.723E-08	-8.825E-10	3.787E-08	0.01	0.00	0.01	0.00
12	4.462E-09	6.928E-13	1.388E-08	0.00	0.00	0.00	0.00
13	7.317E-09	1.955E-10	6.069E-09	0.00	0.00	0.00	0.00
14	-1.110E-08	-7.127E-11	-9.620E-09	0.00	0.00	0.00	0.00
15	-3.017E-09	-5.052E-10	-2.756E-09	0.00	0.00	0.00	0.00

Figure 12: The effect of the Horizontal field generated by the WF dipole magnet which is excited at its nominal current on the integrated normal (left) and skew (right) multipoles of the PM quadrupole.

Superposition for Vertical Field Dipole

Normal Terms: B_ref used for Normalization= 0.037959 T.m (at Reference radius = 10 mm)

Harmonic Number	PMQ + WF (0A) in T.m 5	NorD Only (10A) in T.m 3	PMQ + NorD (10A) in T.m 7	Normalized to B_ref (in "units" of 1E-4)			
	Run 10/02	Run 6/01	Run 12/03	PMQ+WF(0A)	NorD Only (10A)	PMQ+NorD (10A)	Superposition Error
1	-4.721E-11	5.725E-03	5.685E-03	0.00	1508.27	1497.69	10.59
2	**3.796E-02**	-1.402E-06	3.791E-02	10000.00	-0.37	9987.18	12.45
3	-6.980E-06	7.155E-06	9.719E-06	-1.84	1.88	2.56	-2.51
4	2.328E-06	-2.784E-09	1.932E-06	0.61	0.00	0.51	0.10
5	-5.863E-06	6.489E-08	-6.789E-07	-1.54	0.02	-0.18	-1.35
6	-7.115E-04	-5.749E-09	-7.120E-04	-187.45	0.00	-187.56	0.12
7	-2.011E-06	-3.712E-09	-1.783E-06	-0.53	0.00	-0.47	-0.06
8	-1.006E-06	4.475E-09	-1.007E-06	-0.26	0.00	-0.27	0.00
9	2.609E-07	4.326E-10	2.720E-07	0.07	0.00	0.07	0.00
10	-9.733E-06	-1.647E-09	-9.710E-06	-2.56	0.00	-2.56	-0.01
11	1.258E-07	-6.315E-10	9.174E-08	0.03	0.00	0.02	0.01
12	3.132E-08	8.695E-10	4.591E-08	0.01	0.00	0.01	0.00
13	-1.424E-08	-3.212E-11	-1.465E-08	0.00	0.00	0.00	0.00
14	4.127E-07	-2.452E-10	4.154E-07	0.11	0.00	0.11	0.00
15	-2.036E-09	-4.463E-11	-7.737E-10	0.00	0.00	0.00	0.00

Superposition for Vertical Field Dipole

Skew Terms: B_ref used for Normalization= 0.037959 T.m (at Reference radius = 10 mm)

Harmonic Number	PMQ + WF (0A) in T.m 5	NorD Only (10A) in T.m 3	PMQ + NorD (10A) in T.m 7	Normalized to B_ref (in "units" of 1E-4)			
	Run 10/02	Run 6/01	Run 12/03	PMQ+WF(0A)	NorD Only (10A)	PMQ+NorD (10A)	Superposition Error
1	2.870E-11	-5.418E-06	-4.744E-05	0.00	-1.43	-12.50	11.07
2	7.030E-12	3.439E-06	-4.819E-06	0.00	0.91	-1.27	2.18
3	-3.500E-05	-1.950E-07	-3.480E-05	-9.22	-0.05	-9.17	-0.10
4	-1.032E-05	4.866E-08	-9.881E-06	-2.72	0.01	-2.60	-0.10
5	-1.477E-05	1.669E-08	-5.127E-06	-3.89	0.00	-1.35	-2.54
6	-6.495E-06	3.208E-09	-6.636E-06	-1.71	0.00	-1.75	0.04
7	4.053E-06	-6.325E-09	4.095E-06	1.07	0.00	1.08	-0.01
8	-3.879E-07	-7.565E-10	-3.980E-07	-0.10	0.00	-0.10	0.00
9	3.704E-07	2.941E-09	5.991E-07	0.10	0.00	0.16	-0.06
10	-5.434E-10	4.014E-10	1.228E-09	0.00	0.00	0.00	0.00
11	4.723E-08	-3.895E-10	4.418E-08	0.01	0.00	0.01	0.00
12	4.462E-09	-2.470E-10	6.559E-09	0.00	0.00	0.00	0.00
13	7.317E-09	4.730E-10	-1.156E-09	0.00	0.00	0.00	0.00
14	-1.110E-08	-8.983E-11	-8.974E-09	0.00	0.00	0.00	0.00
15	-3.017E-09	-2.019E-10	-2.024E-09	0.00	0.00	0.00	0.00

Figure 13: The effect of the Vertical field generated by the WF dipole magnet on the integrated normal (left) and skew (right) multipoles of the PM quadrupole.

Superposition for (Hor+Vert) Field Dipole

Normal Terms: B_ref used for Normalization= 0.037959 T.m (at Reference radius = 10 mm)

Harmonic Number	PMQ + WF (0A) in T.m (5) Run 10/02	RD+SD (10A) in T.m (4) Run 7/01	PMQ + RD+SD (10A) in T.m (6) Run 13/03	Normalized to B_ref (in "units" of 1E-4) PMQ+WF(0A)	RD+SD (10A)	PMQ+RD+SD (10A)	Superposition Error
1	-4.721E-11	5.720E-03	5.639E-03	0.00	1506.95	1485.64	21.31
2	**3.796E-02**	2.070E-06	3.787E-02	10000.00	0.55	9976.48	24.07
3	-6.980E-06	6.987E-06	9.909E-06	-1.84	1.84	2.61	-2.61
4	2.328E-06	-6.309E-08	1.258E-06	0.61	-0.02	0.33	0.27
5	-5.863E-06	6.691E-08	-7.527E-07	-1.54	0.02	-0.20	-1.33
6	-7.115E-04	-8.239E-09	-7.118E-04	-187.45	0.00	-187.51	0.06
7	-2.011E-06	-3.069E-09	-1.771E-06	-0.53	0.00	-0.47	-0.06
8	-1.006E-06	4.732E-09	-1.005E-06	-0.26	0.00	-0.26	0.00
9	2.609E-07	4.250E-10	2.364E-07	0.07	0.00	0.06	0.01
10	-9.733E-06	-1.612E-09	-9.674E-06	-2.56	0.00	-2.55	-0.02
11	1.258E-07	-3.361E-10	1.073E-07	0.03	0.00	0.03	0.00
12	3.132E-08	9.653E-10	3.670E-08	0.01	0.00	0.01	0.00
13	-1.424E-08	3.038E-10	-1.839E-08	0.00	0.00	0.00	0.00
14	4.127E-07	-1.415E-10	4.115E-07	0.11	0.00	0.11	0.00
15	-2.036E-09	-5.396E-11	-1.212E-09	0.00	0.00	0.00	0.00

Superposition for (Hor+Vert) Field Dipole

Skew Terms: B_ref used for Normalization= 0.037959 T.m (at Reference radius = 10 mm)

Harmonic Number	PMQ + WF (0A) in T.m (5) Run 10/02	RD+SD (10A) in T.m (4) Run 7/01	PMQ + RD+SD (10A) in T.m (6) Run 13/03	Normalized to B_ref (in "units" of 1E-4) PMQ+WF(0A)	RD+SD (10A)	PMQ+RD+SD (10A)	Superposition Error
1	2.870E-11	5.691E-03	5.557E-03	0.00	1499.20	1463.98	35.22
2	7.030E-12	6.664E-06	-7.180E-06	0.00	1.76	-1.89	3.65
3	-3.500E-05	-7.192E-06	-4.848E-05	-9.22	-1.89	-12.77	1.66
4	-1.032E-05	3.764E-08	-9.621E-06	-2.72	0.01	-2.53	-0.17
5	-1.477E-05	7.513E-08	4.029E-06	-3.89	0.02	1.06	-4.93
6	-6.495E-06	6.407E-09	-6.191E-06	-1.71	0.00	-1.63	-0.08
7	4.053E-06	-6.080E-09	3.958E-06	1.07	0.00	1.04	0.02
8	-3.879E-07	-9.959E-10	-4.049E-07	-0.10	0.00	-0.11	0.00
9	3.704E-07	2.681E-09	7.552E-07	0.10	0.00	0.20	-0.10
10	-5.434E-10	4.827E-10	-3.408E-09	0.00	0.00	0.00	0.00
11	4.723E-08	-8.859E-10	3.923E-08	0.01	0.00	0.01	0.00
12	4.462E-09	-4.632E-10	5.817E-09	0.00	0.00	0.00	0.00
13	7.317E-09	5.117E-10	-1.397E-08	0.00	0.00	0.00	0.01
14	-1.110E-08	-2.011E-10	-9.489E-09	0.00	0.00	0.00	0.00
15	-3.017E-09	-2.328E-10	-1.966E-09	0.00	0.00	0.00	0.00

Figure 14: The effect of the Horizontal and Vertical fields generated by the WF dipole magnet on the integrated normal (left) and skew (right) multipoles of the PM quadrupole.

Summary

- Field superposition was tested for a permanent magnet quadrupole in horizontal and vertical dipole fields of ~20 mT each.
- Presence of iron yoke of the window frame dipole enhances the quadrupole field by ~ 0.5%.
- Use of a 1 m long rotating coil causes some error in the analysis due to double counting of background fields.
- For higher harmonics, the deviation from a perfect superposition is less than 5 units (0.05% of the quadrupole field) at a reference radius of 10 mm.
- Error in the dipole and quadrupole terms can be much more, up to ~30 units (normalized to quadrupole field.)

Figure 15: The summary of the measurements as provided by [3]

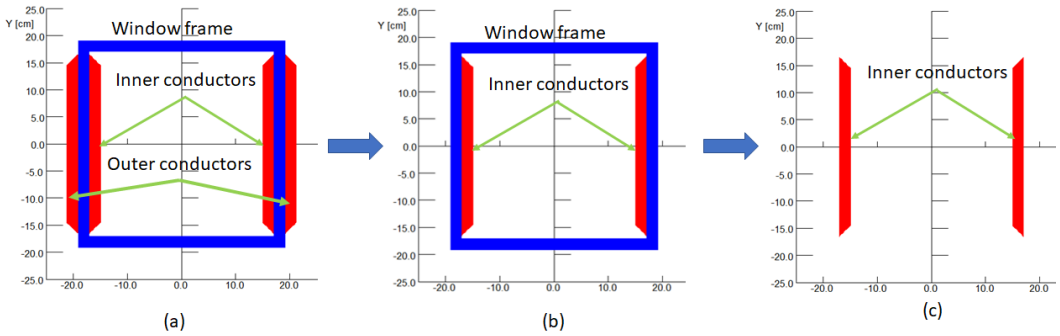

Figure 16: The cross section of a WF magnet (a) with two racetrack coils wound around the vertical sides of the frame to generate normal dipole field. In this appendix this magnet will be represented by the two inner conductors of the coil as shown in (c).

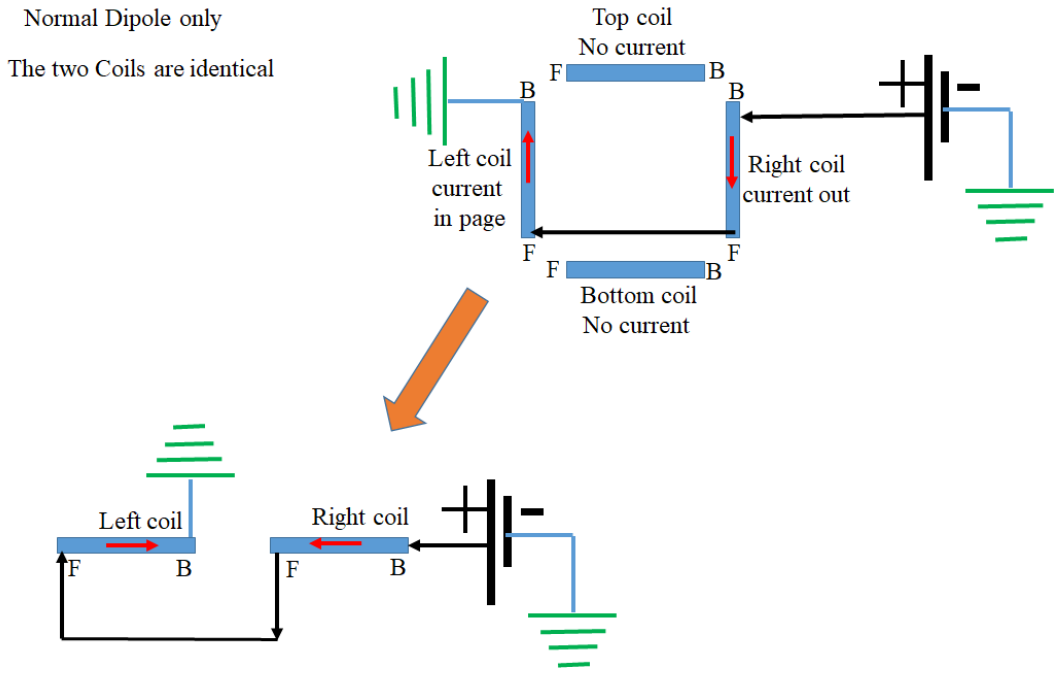

Figure 17: A schematic diagram showing the connection of the coils to generate a normal dipole field. Note that the top and bottom coils are not connected to a power supply.

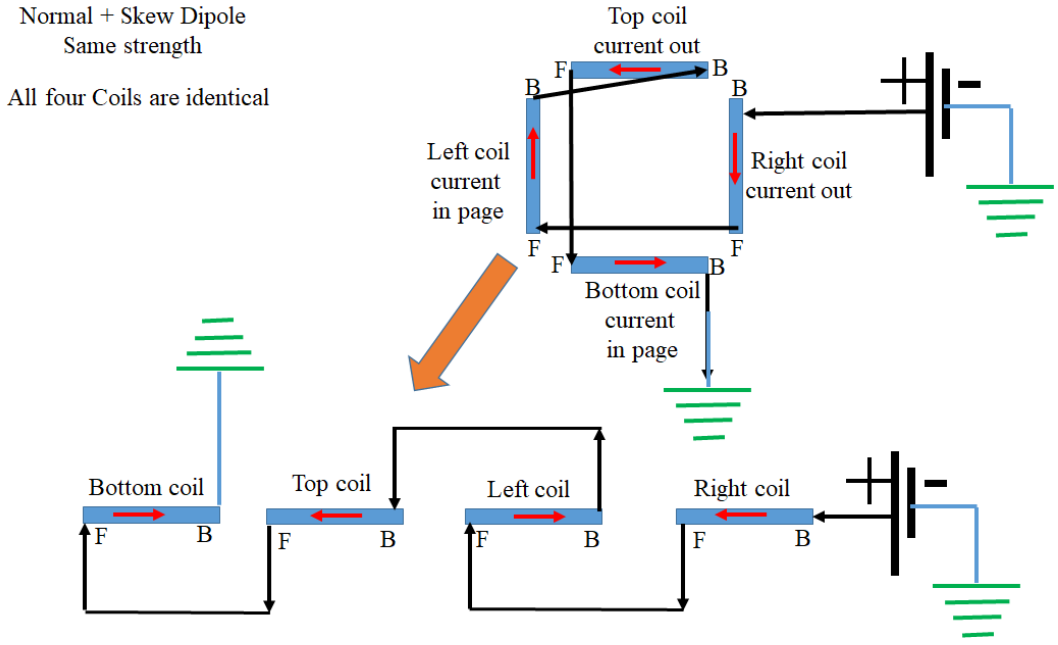

Figure 18: A schematic diagram showing the connection of the coils to generate simultaneous a normal and skew dipole field. If the coils which generate normal field are identical to those which generate vertical field the strength of the dipoles is the same.

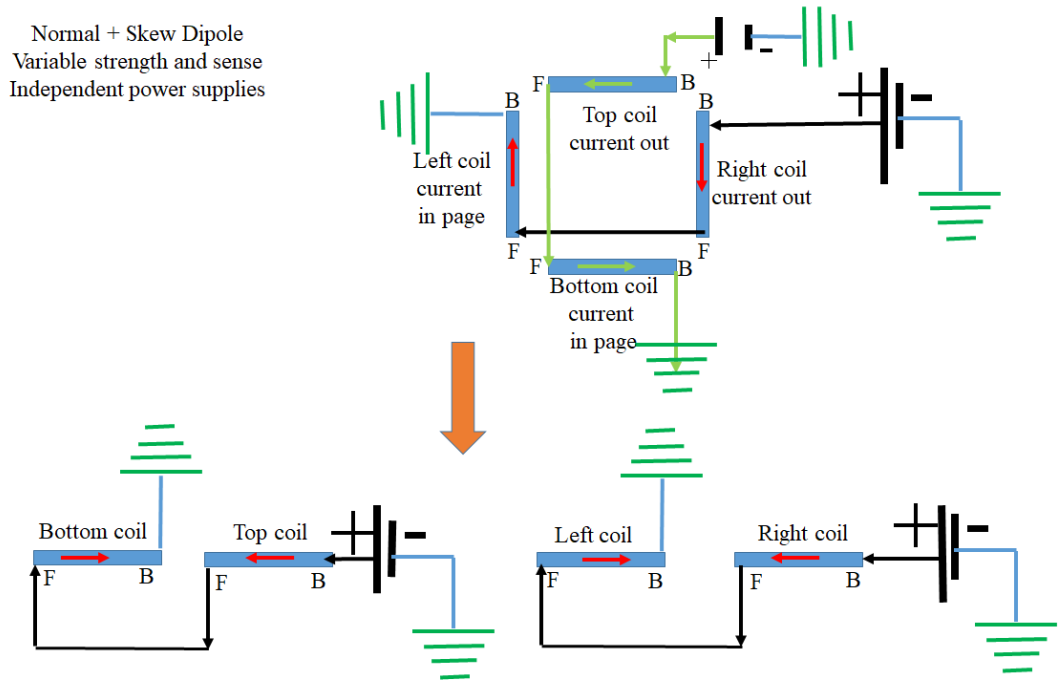

Figure 19: A schematic diagram showing the wire connection of the coils with two independent power supplies. Such a connection allows independent variation of the normal and skew dipole strength, as well as independent variation of the sense of the dipole fields by changing the polarity of the power supplies.

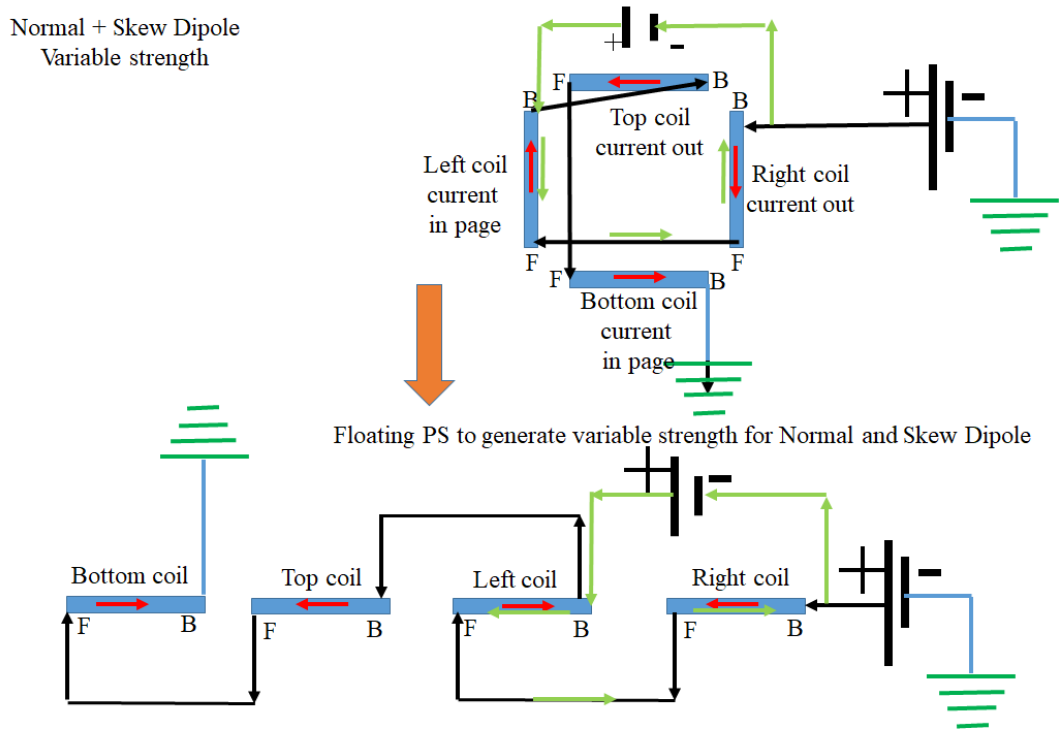

Figure 20: A schematic diagram for the wire connection of the coils of the WF magnet which generates both normal and skew dipoles but of variable strengths between the normal and the skew dipoles. The coils which generate the normal dipole field do not have to be identical to those which generate the skew dipole field.

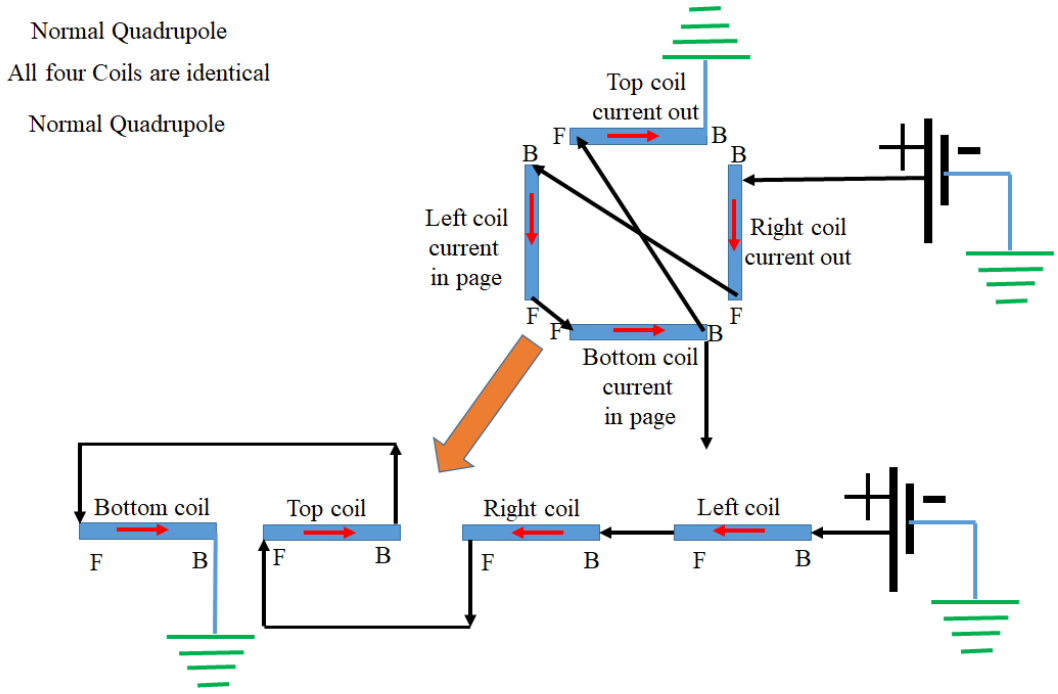

Figure 21: A schematic diagram for the connections of the four identical coils of the WF magnet to generate a normal quadrupole. If the vertical coils are not identical with the horizontal ones the strength of the quadupole will correspond to the the coils with the least ampere-turns.

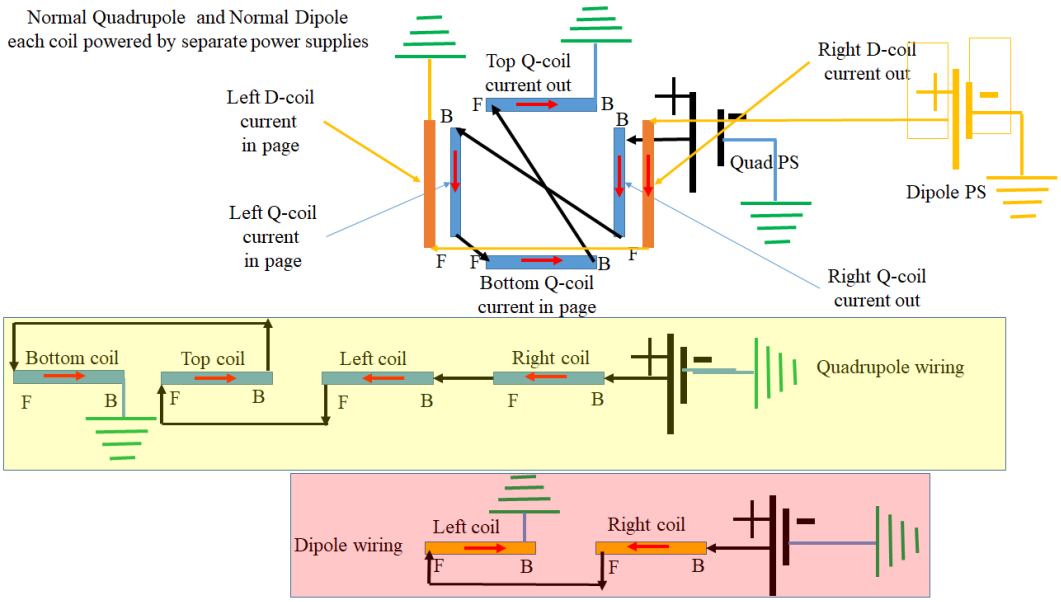

Figure 22: A schematic diagram for the connections of the Quadrupole coils by a power supply and the connection of the Dipole coils by a separate power supply. This method requires six racetrack coils and two power supplies.

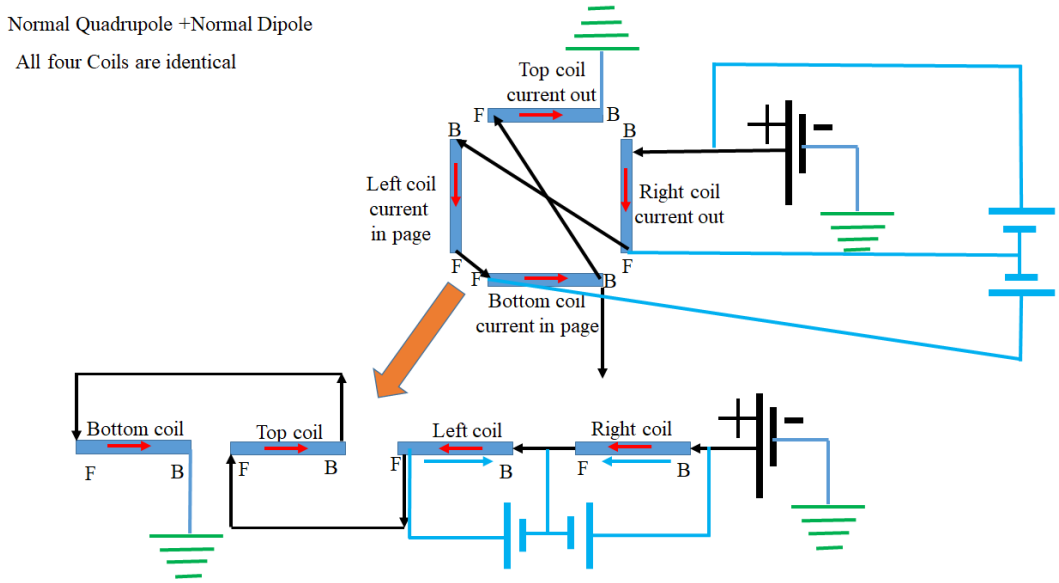

Figure 23: A schematic diagram of wiring four coils with three power supplies (two PS floating) to generate a combined quadrupole and dipole field. This method requires four coils and three power supplies.

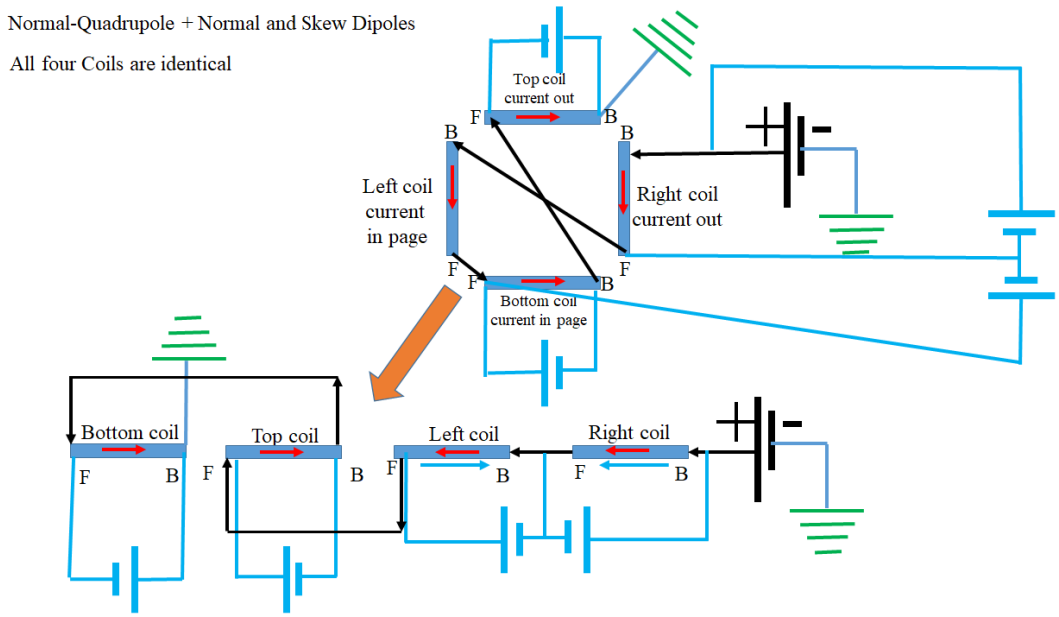

Figure 24: A schematic diagram of wiring four coils with five power supplies (4 PS are floating) to generate a Normal quadrupole combined with a Normal and Skew dipole field. This method requires four coils and six power supplies.