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Notes on calculating various parameters of ions circulating in Booster and destined for NSRL

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Notes on calculating various parameters of ions circulating in Booster and destined for NSRL

C.J. Gardner

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These notes are meant to serve as a self-contained tutorial for anyone wishing to calculate various parameters of ions circulating in Booster and destined for the NASA Space Radiation Laboratory (NSRL).

Sections 1 through **4** give the information needed to compute ion masses.

Sections 5 through **20** give the formulae needed to do the calculations.

The formulae are applied to iron, gold, tungsten, thorium, argon, bromine, krypton, xenon, and tantalum ions, which serve as illustrative examples. Results are given in **Sections 21** through **50**. Each section consists of a one-page data sheet for a given ion. The data include the ion kinetic energy at magnetic rigidities 15.8 and 17.5 Tm. At these rigidities the Fe20+ ion has kinetic energies of 1000 and 1162 MeV per nucleon respectively. The Fe24+ ion has kinetic energies of 1303 and 1503 MeV per nucleon respectively. The highest rigidity attainable in the Booster bending magnets is 17.5 Tm. This along with a high charge state gives the highest kinetic energy attainable for a given ion.

The revolution frequency f of any ion circulating in Booster is limited to F_L = 1,485,739.211 Hz by the velocity of light. The Booster RF cavities operate at frequency hf where h is a positive integer called the harmonic number. For a given h the cavities therefore need to reach a frequency of at most hF_L. If harmonic number 3 is used, one has $3F_L$ = 4.457,217.633 Hz. This is comfortably below the cavity operational limit of 5 MHz. At Booster injection a harmonic number greater than 3 is often needed in order to have a frequency hf that is greater than the low-frequency limit (some 340 kHz) of the cavities. After the injected beam has been accelerated to a sufficiently high frequency, the circulating beam may be debunched and then rebunched at harmonic number 3 if necessary.

1 Ion mass [1, 2]

Let Z and N be the number of protons and neutrons in the nucleus of a given atom. The mass number of the atom is then

$$A = N + Z. \tag{1}$$

This is also called the number of nucleons.

The mass energy equivalent of the ion formed by removing Q electrons from the neutral atom is

$$mc^2 = am_u c^2 - Qm_e c^2 + E_Q \tag{2}$$

where a is the relative atomic mass [1] of the neutral atom,

$$m_u c^2 = 931.494\,0954(57)\text{ MeV} \tag{3}$$

is the mass energy equivalent of the atomic mass constant [2], and

$$m_e c^2 = 0.510\,998\,9461(31)\text{ MeV} \tag{4}$$

is the electron mass energy equivalent [2]. The atomic binding energy E_Q is the energy required to remove the Q electrons from the neutral atom. In most cases this is quite small and can be neglected. A possible exception is the energy required to remove all 79 electrons from the neutral gold atom. This amounts to 0.517 MeV [3] which is slightly larger than the mass energy equivalent of an electron. In these notes we will take $E_Q = 0$.

2 Mass energy equivalents of atomic nuclei that have just one or two protons [2]

$$m_p c^2 = 938.272\,0813(58)\text{ MeV } (\textbf{proton}) \tag{5}$$

$$m_d c^2 = 1875.612\,928(12)\text{ MeV } (\textbf{deuteron}) \tag{6}$$

$$m_t c^2 = 2808.921\,112(17)\text{ MeV } (\textbf{triton}) \tag{7}$$

$$m_h c^2 = 2808.391\,586(17)\text{ MeV } (\textbf{helion}) \tag{8}$$

$$m_\alpha c^2 = 3727.379\,378(23)\text{ MeV } (\textbf{alpha}) \tag{9}$$

All of the nuclei are stable except the triton which decays into a helion, an electron, and an electron antineutrino. The half-life is 4,500 ± 8 days.

3 Table A of atomic parameters [1, 4]

atom	symbol	Z	A	relative atomic mass (a)	abundance
Helium	He	2	4	4.002 603 254 13(6)	0.999 998 66(3)
Lithium	Li	3	7	7.016 003 4366(45)	0.9241(4)
Beryllium	Be	4	9	9.012 183 065(82)	1.000000
Boron	B	5	11	11.009 305 36(45)	0.801(7)
Carbon	C	6	12	12.000 000 000 000	0.9893(8)
Nitrogen	N	7	14	14.003 074 004 43(20)	0.996 36(20)
Oxygen	O	8	16	15.994 914 619 57(17)	0.997 57(16)
Fluorine	F	9	19	18.998 403 162 73(92)	1.000000
Neon	Ne	10	20	19.992 440 1762(17)	0.9048(3)
Sodium	Na	11	23	22.989 769 2820(19)	1.000000
Magnesium	Mg	12	24	23.985 041 697(14)	0.7899(4)
Aluminum	Al	13	27	26.981 538 53(11)	1.000000
Silicon	Si	14	28	27.976 926 534 65(44)	0.922 23(19)
Phosphorus	P	15	31	30.973 761 998 42(70)	1.000000
Sulfur	S	16	32	31.972 071 1744(14)	0.9499(26)
Chlorine	Cl	17	35	34.968 852 682(37)	0.7576(10)
Argon	Ar	18	40	39.962 383 1237(24)	0.996 035(25)
Potassium	K	19	39	38.963 706 4864(49)	0.932 581(44)
Calcium	Ca	20	40	39.962 590 863(22)	0.969 41(156)
Scandium	Sc	21	45	44.955 908 28(77)	1.000000
Titanium	Ti	22	48	47.947 941 98(38)	0.7372(3)
Vanadium	V	23	51	50.943 957 04(94)	0.997 50(4)
Chromium	Cr	24	52	51.940 506 23(63)	0.837 89(18)
Manganese	Mn	25	55	54.938 043 91(48)	1.000000
Iron	Fe	26	56	55.934 936 33(49)	0.917 54(36)
Cobalt	Co	27	59	58.933 194 29(56)	1.000000
Nickel	Ni	28	58	57.935 342 41(52)	0.680 77(19)
Copper	Cu	29	63	62.929 597 72(56)	0.6915(15)
Zinc	Zn	30	64	63.929 142 01(71)	0.4917(75)
Gallium	Ga	31	69	68.925 5735(13)	0.601 08(9)
Germanium	Ge	32	74	73.921 177 761(13)	0.3650(20)
Arsenic	As	33	75	74.921 594 57(95)	1.000000
Selenium	Se	34	80	79.916 5218(13)	0.4961(41)
Bromine	Br	35	79	78.918 3376(14)	0.5069(7)

4 Table B of atomic parameters [1, 4]

atom	symbol	Z	A	relative atomic mass (a)	abundance
Krypton	Kr	36	84	83.911 497 7282(44)	0.569 87(15)
Rubidium	Rb	37	85	84.911 789 7379(54)	0.7217(2)
Strontium	Sr	38	88	87.905 6125(12)	0.8258(1)
Yttrium	Y	39	89	88.905 8403(24)	1.000000
Zirconium	Zr	40	90	89.904 6977(20)	0.5145(40)
Zirconium	Zr	40	96	95.908 2714(21)	0.0280(9)
Niobium	Nb	41	93	92.906 3730(20)	1.000000
Molybdenum	Mo	42	98	97.905 404 82(49)	0.2439(37)
Ruthenium	Ru	44	96	95.907 590 25(49)	0.0554(14)
Ruthenium	Ru	44	102	101.904 3441(12)	0.3155(14)
Rhodium	Rh	45	103	102.905 4980(26)	1.000000
Palladium	Pd	46	106	105.903 4804(12)	0.2733(3)
Silver	Ag	47	107	106.905 0916(26)	0.518 39(8)
Cadmium	Cd	48	114	113.903 365 09(43)	0.2873(42)
Indium	In	49	115	114.903 878 776(12)	0.9571(5)
Tin	Sn	50	120	119.902 201 63(97)	0.3258(9)
Antimony	Sb	51	121	120.903 8120(30)	0.5721(5)
Tellurium	Te	52	130	129.906 222 748(12)	0.3408(62)
Iodine	I	53	127	126.904 4719(39)	1.000000
Xenon	Xe	54	132	131.904 155 0856(56)	0.269 086(33)
Xenon	Xe	54	129	128.904 780 8611(60)	0.264 006(82)
Cesium	Cs	55	133	132.905 451 9610(80)	1.000000
Barium	Ba	56	138	137.905 247 00(31)	0.716 98(42)
Tantalum	Ta	73	181	180.947 9958(20)	0.999 8799(32)
Tungsten	W	74	184	183.950 930 92(94)	0.3064(2)
Rhenium	Re	75	187	186.955 7501(16)	0.6260(2)
Iridium	Ir	77	193	192.962 9216(21)	0.627(2)
Gold	Au	79	197	196.966 568 79(71)	1.000000
Lead	Pb	82	208	207.976 6525(13)	0.524(1)
Bismuth	Bi	83	209	208.980 3991(16)	1.000000
Radon	Rn	86	222	222.017 5782(25)	half-life 3.8 d
Thorium	Th	90	232	232.038 0558(21)	1.000000
Uranium	U	92	238	238.050 7884(20)	0.992 742(10)

5 Frequency, momentum, and energy

An ion in a synchrotron moving along a closed orbit of radius R with velocity $c\beta$ has revolution frequency

$$f = F_L \beta \tag{10}$$

where

$$F_L = \frac{c}{2\pi R} \tag{11}$$

is the frequency limit given by the velocity of light. In Booster we have

$$2\pi R = 201.780 \text{ meters (m)} \tag{12}$$

which gives

$$F_L = 1,485,739.211 \text{ Hz.} \tag{13}$$

The momentum of the ion is

$$p = mc\beta\gamma \tag{14}$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{15}$$

The energy is

$$E = mc^2\gamma \tag{16}$$

and the kinetic energy is

$$W = mc^2(\gamma - 1). \tag{17}$$

6 Magnetic rigidity

In the bending magnets of the synchrotron the radius of curvature of the ion trajectory is

$$\rho = k\,\frac{cp}{QB} \tag{18}$$

where B is the bend field and

$$k = 10^9/299792458. \tag{19}$$

Here cp and B must be given in units of GeV and Tesla (T) respectively. The units of ρ are then meters. Multiplying both sides of (18) by B gives

$$B\rho = k\,\frac{cp}{Q} \tag{20}$$

which is the magnetic rigidity of the ion.

7 Kinetic energy for a given rigidity

Using (14) in (20) we have

$$\beta\gamma = \frac{1}{k}\left(\frac{Q}{mc^2}\right)B\rho \tag{21}$$

where

$$k = 10^9/299792458 \tag{22}$$

and mc^2 and $B\rho$ are given in units of GeV and Tm respectively. The identity

$$\gamma = \sqrt{1 + (\beta\gamma)^2} \tag{23}$$

then gives γ, which in turn gives the kinetic energy

$$W = mc^2(\gamma - 1). \tag{24}$$

The kinetic energy per nucleon is W/A.

8 Maximum attainable kinetic energy

In the Booster bending magnets the nominal radius of curvature is

$$\rho = 13.8656 \text{ meters} \tag{25}$$

and the maximum attainable magnetic field is [5]

$$B_M = 1.2621163 \text{ T}. \tag{26}$$

The maximum attainable $B\rho$ is then

$$(B\rho)_M = \rho B_M = 17.5 \text{ Tm}. \tag{27}$$

Substituting $(B\rho)_M$ for $B\rho$ in (21) gives

$$(\beta\gamma)_M = \frac{1}{k}\left(\frac{Q}{mc^2}\right)(B\rho)_M \tag{28}$$

which is the maximum attainable $\beta\gamma$ for a given ion. The maximum attainable γ is then

$$\gamma_M = \sqrt{1 + (\beta\gamma)_M^2} \tag{29}$$

which gives maximum attainable kinetic energy

$$W_M = mc^2\left(\gamma_M - 1\right). \tag{30}$$

Having obtained $(\beta\gamma)_M$, we may also compute

$$\beta_M = \frac{(\beta\gamma)_M}{\gamma_M} \tag{31}$$

which is the maximum attainable β for a given ion. The corresponding revolution frequency is

$$f_M = F_L \beta_M \tag{32}$$

where F_L is given by (13). Since $\beta_M < 1$ we always have

$$f_M < 1,485,739.211 \text{ Hz}. \tag{33}$$

9 Rigidity for a given kinetic energy

Using (14) in (20) we have

$$B\rho = k\left(\frac{mc^2}{Q}\right)\beta\gamma. \tag{34}$$

Inverting the relation

$$W = mc^2\left(\gamma - 1\right) \tag{35}$$

we compute

$$\gamma = 1 + \frac{W}{mc^2}. \tag{36}$$

The identity

$$\beta\gamma = \sqrt{\gamma^2 - 1} \tag{37}$$

then gives $\beta\gamma$. Inserting this into (34) gives $B\rho$. Here

$$k = 10^9/299792458 \tag{38}$$

and mc^2 must be given in units of GeV. The units of $B\rho$ are then Tm.

Having obtained $\beta\gamma$, we can also compute

$$\beta = \frac{(\beta\gamma)}{\gamma} \tag{39}$$

which gives frequency

$$f = F_L \beta. \tag{40}$$

10 Rigidity for a given revolution frequency

For a given revolution frequency f we compute

$$\beta = \frac{f}{F_L} \tag{41}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{42}$$

and

$$B\rho = k\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{43}$$

where F_L is given by (13). Having obtained γ, we can also compute the kinetic energy

$$W = mc^2(\gamma - 1). \tag{44}$$

11 Revolution frequency for a given rigidity

In this case we compute

$$\beta\gamma = \frac{1}{k}\left(\frac{Q}{mc^2}\right)B\rho \tag{45}$$

$$\gamma = \sqrt{1+(\beta\gamma)^2} \tag{46}$$

$$\beta = \frac{(\beta\gamma)}{\gamma} \tag{47}$$

and finally

$$f = F_L\beta \tag{48}$$

where F_L is given by (13). Note again that since $\beta < 1$ we always have

$$f < 1,485,739.211 \text{ Hz}. \tag{49}$$

For RF harmonic number 3 this gives

$$hf < 4.457,217.633 \text{ Hz}. \tag{50}$$

These upper limits are well within the range of the Booster RF cavities which can operate at frequencies as low as 340 kHz and as high as 5 MHz.

12 Revolution frequency and rigidity at injection

The nominal revolution frequency of EBIS ions in Booster at injection is

$$f = 96.640\,\text{kHz}. \tag{51}$$

This means that in order to have a frequency hf that is greater than the low-frequency limit of the RF cavities, we must use harmonic number 4 instead of 3. This gives

$$hf = 386.560\,\text{kHz} \tag{52}$$

which is comfortably above the 340 kHz limit. After the injected beam has been accelerated to a sufficiently high frequency, the circulating beam may be debunched and then rebunched at harmonic number 3 if necessary. This is done on a relatively short magnetic field porch where the revolution frequency is 200 kHz.

The nominal β of EBIS ions circulating in Booster at injection is

$$\beta = \frac{f}{F_L} = 6.50450626079 \times 10^{-2} \tag{53}$$

which gives

$$\gamma = 1 + 2.12216640606 \times 10^{-3} \tag{54}$$

and

$$\beta\gamma = 6.51830990547 \times 10^{-2}. \tag{55}$$

The nominal rigidity of a given EBIS ion at injection is then

$$B\rho = k\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{56}$$

where

$$k = 10^9/299792458 \tag{57}$$

and the units of $B\rho$ and mc^2 are Tm and GeV.

13 The ETB bend

In the bending magnet [6, 7] of the EBIS-to-Booster (ETB) transfer line the nominal radius of curvature of the ion trajectory is

$$\rho = 1.3 \text{ meters} \tag{58}$$

and the maximum attainable magnetic field is

$$B_B = 0.964 \text{ T}. \tag{59}$$

The maximum attainable $B\rho$ is then

$$(B\rho)_B = \rho B_B = 1.2532 \text{ Tm}. \tag{60}$$

According to (55) and (56) we must then have

$$k\left(\frac{mc^2}{Q}\right)\beta\gamma \leq (B\rho)_B \tag{61}$$

where

$$\beta\gamma = 6.51830990547 \times 10^{-2}. \tag{62}$$

Thus we must have

$$k\left(\frac{mc^2}{Q}\right) \leq 19.2258 \text{ Tm} \tag{63}$$

and therefore

$$\frac{mc^2}{Q} \leq 5.7638 \text{ GeV}. \tag{64}$$

Of all the gin joints, um ions, listed in **Sections 21** through **50** the one that comes closest to this limit is the Au32+ ion which has ratio

$$\frac{mc^2}{Q} = 5.7330 \text{ GeV}. \tag{65}$$

A close second is the U39+ ion for which

$$\frac{mc^2}{Q} = 5.6852 \text{ GeV}. \tag{66}$$

The Th39+ ion has ratio

$$\frac{mc^2}{Q} = 5.5416 \text{ GeV}. \tag{67}$$

14 The Inflector

Ions from EBIS and Tandem are injected into Booster by means of an electrostatic inflector [8, 9]. The voltage V_I required for particles with mass m, velocity $c\beta$, and charge eQ to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma \tag{68}$$

where e is the elementary positive electronic charge. Here $G = 0.021$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory.

The inflector is conditioned to a voltage of 80 kV, but for normal operation we do not exceed 75 kV. We must therefore have

$$\frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma \leq 75 \text{ keV} \tag{69}$$

which gives

$$\left(\frac{mc^2}{Q}\right)\beta^2\gamma \leq 31.2 \text{ MeV}. \tag{70}$$

Using again the nominal revolution frequency

$$f = 96.640 \text{ kHz} \tag{71}$$

of EBIS ions in Booster at injection, we have

$$\beta = \frac{f}{F_L} = 6.50450626079 \times 10^{-2} \tag{72}$$

$$\gamma = 1 + 2.12216640606 \times 10^{-3} \tag{73}$$

and

$$\beta^2\gamma = 4.23983875899 \times 10^{-3}. \tag{74}$$

Using this result in (70) then gives the requirement

$$\frac{mc^2}{Q} \leq 7.36 \text{ GeV} \tag{75}$$

for all EBIS ions that are to be injected into Booster. This condition is easily met as shown in the previous section.

15 Scaling parameters at Booster injection

Suppose we have a working setup for a given EBIS ion in Booster and wish to introduce a new EBIS ion for which a setup has yet to be established. In this case one can take advantage of the fact that, nominally, all ions from EBIS arrive at Booster with the same velocity (and therefore the same β and γ). This makes it possible to obtain the magnetic field and inflector settings for the new setup by scaling the corresponding settings of the established setup.

Let B_0 and V_{I0} be the field and inflector setpoints for the established setup, and let m_0 and Q_0 be the mass and charge of the associated ion. Then according to (56) and (68) we have

$$B_0\rho = k\left(\frac{m_0c^2}{Q_0}\right)\beta\gamma \tag{76}$$

and

$$eV_{I0} = \frac{G}{R_I}\left(\frac{m_0c^2}{Q_0}\right)\beta^2\gamma. \tag{77}$$

The corresponding equations for the new setup are

$$B\rho = k\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{78}$$

and

$$eV_I = \frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma \tag{79}$$

where m and Q are the mass and charge of the new ion. Dividing equation (78) by (76), and (79) by (77), we obtain

$$\frac{B}{B_0} = \frac{m/Q}{m_0/Q_0} \tag{80}$$

and

$$\frac{V_I}{V_{I0}} = \frac{m/Q}{m_0/Q_0} = \frac{B}{B_0}. \tag{81}$$

Thus we see that the scaling factor needed to obtain B and V_I from B_0 and V_{I0} is simply the ratio of the mass-to-charge ratios of the two ions.

16 Ions with identical mass-to-charge ratios

If the two EBIS ions considered in the previous section have the same mass-to-charge ratio then we have

$$\frac{m}{Q} = \frac{m_0}{Q_0} \tag{82}$$

and equation (81) gives

$$\frac{V_I}{V_{I0}} = \frac{B}{B_0} = 1. \tag{83}$$

The two ions therefore require the same inflector and field settings and both will be transported through the inflector and circulate in Booster if they are present in the incoming beam. If only one of the two ions is desired then the other is said to be a contaminant.

The numbers in Tables A and B show that there are many examples of ions that have nearly the same mass to charge ratios. Simple examples include the fully stripped ions He2+, C6+, N7+, O8+, Ne10+, Mg12+, Si14+, S16+, and Ca20+, all of which have the same mass number to charge ratio $A/Q = 2$.

Examples of contaminants relevant for the delivery of ions to NSRL are the N5+ and N6+ ions which have mass to charge ratios very close to those of Fe20+ and Fe24+ respectively. These contaminant ions are produced from nitrogen gas present in EBIS.

Having two ions with the same mass to charge ratio is sometimes useful. A case in point is the scheme proposed by Ed Beebe for the delivery of thorium ions to NSRL. Here a source target consisting of a small sample of tungsten welding-electrode material is used. This material nominally contains 2% thorium in addition to the tungsten. In EBIS one then has the production of both tungsten and thorium ions. Some of these have nearly the same mass to charge ratio as shown in **Sections 26** through **31**. The copiously produced tungsten ions are easily seen on instrumentation and therefore serve to establish a working setup in Booster. The setup then should work for any thorium ion that has a mass to charge ratio sufficiently close to that of a tungsten ion. This has been demonstrated by Nick Kling and confirmed by Bragg curve measurements in the NSRL target room. The setup has been used since June 2017 to provide thorium ions for NSRL.

17 Approximations

If the terms Qm_ec^2 and E_Q on the right-hand side of (2) are neglected, we have the approximation

$$mc^2 = am_uc^2. \tag{84}$$

Furthermore, the "A" and "relative atomic mass" columns of Tables A and B show that to a good approximation we have

$$a = A. \tag{85}$$

We then have

$$mc^2 = Am_uc^2 \tag{86}$$

which gives the approximations

$$\frac{mc^2}{Q} = \frac{A}{Q}\, m_uc^2 \tag{87}$$

and

$$k\left(\frac{mc^2}{Q}\right) = \frac{A}{Q}\, km_uc^2. \tag{88}$$

Here

$$m_uc^2 = 0.931\,494\,0954(57) \text{ GeV} \tag{89}$$

and

$$km_uc^2 = 3.10713 \text{ Tm}. \tag{90}$$

Thus if an approximate number is all that is needed, one may use (87) or (88) in any of the formulae where the mass-to-charge ratio appears. In particular the formula

$$B\rho = k\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{91}$$

becomes

$$B\rho = \frac{A}{Q}\left(km_uc^2\right)\beta\gamma \tag{92}$$

where km_uc^2 is given by (90). The inverse of (92) is

$$\beta\gamma = \frac{Q}{A}\left(\frac{B\rho}{km_uc^2}\right). \tag{93}$$

18 Relative magnitudes of ion mass terms

Returning to the exact equation for the ion mass we have

$$mc^2 = am_uc^2 - Qm_ec^2 + E_Q \tag{94}$$

which we can write as

$$mc^2 = Am_uc^2 \left\{ 1 - \frac{\Delta}{A} - \frac{Q}{A}\left(\frac{m_e}{m_u}\right) + \frac{q}{A}\left(\frac{m_e}{m_u}\right) \right\} \tag{95}$$

where

$$\Delta = A - a, \quad q = \frac{E_Q}{m_ec^2} \tag{96}$$

and, as given in [2],

$$\frac{m_e}{m_u} = 5.485\,799\,090\,70(16) \times 10^{-4}. \tag{97}$$

Tables AA and BB in the next two sections show that Δ/A goes from -22.9 parts per 10,000 for **lithium** to a peak of 11.6 parts per 10,000 for **iron**, and then back to -2.1 parts per 10,000 for **uranium**. Thus we have

$$\frac{-2.29}{1000} \le \frac{\Delta}{A} \le \frac{1.16}{1000}. \tag{98}$$

For ions that have more than two protons we have $Q/A \le 1/2$, which gives

$$\frac{Q}{A}\left(\frac{m_e}{m_u}\right) \le \frac{1}{2} \times 5.485\,799\,090\,70(16) \times 10^{-4}. \tag{99}$$

(For fully stripped ions that have just one or two protons one should simply use the masses listed in Section 2.)

As already mentioned, we have taken $E_Q = 0$ in these notes which is generally a very good approximation. We note, however, that for fully stripped heavy ions such as Au79+ and U92+ one has $E_Q = 0.517$ and 0.762 MeV respectively [3]. Equation (96) then gives $q = 1.01$ and 1.49. In such cases one may wish to keep the last term in (95) if the next-to-last term has been retained.

19 Table AA: $(A-a)/A$ in parts per 10,000

atom	symbol	Z	A	a	$(A-a)/A$
Helium	He	2	4	4.002 603 254 13(6)	−6.50814
Lithium	Li	3	7	7.016 003 4366(45)	−22.86205
Beryllium	Be	4	9	9.012 183 065(82)	−13.53674
Boron	B	5	11	11.009 305 36(45)	−8.45942
Carbon	C	6	12	12.000 000 000 000	0.000 000
Nitrogen	N	7	14	14.003 074 004 43(20)	−2.19572
Oxygen	O	8	16	15.994 914 619 57(17)	3.17836
Fluorine	F	9	19	18.998 403 162 73(92)	0.840441
Neon	Ne	10	20	19.992 440 1762(17)	3.77991
Sodium	Na	11	23	22.989 769 2820(19)	4.44814
Magnesium	Mg	12	24	23.985 041 697(14)	6.23263
Aluminum	Al	13	27	26.981 538 53(11)	6.83758
Silicon	Si	14	28	27.976 926 534 65(44)	8.24052
Phosphorus	P	15	31	30.973 761 998 42(70)	8.46387
Sulfur	S	16	32	31.972 071 1744(14)	8.72776
Chlorine	Cl	17	35	34.968 852 682(37)	8.89923
Argon	Ar	18	40	39.962 383 1237(24)	9.40422
Potassium	K	19	39	38.963 706 4864(49)	9.30603
Calcium	Ca	20	40	39.962 590 863(22)	9.35228
Scandium	Sc	21	45	44.955 908 28(77)	9.79816
Titanium	Ti	22	48	47.947 941 98(38)	10.84542
Vanadium	V	23	51	50.943 957 04(94)	10.98882
Chromium	Cr	24	52	51.940 506 23(63)	11.44111
Manganese	Mn	25	55	54.938 043 91(48)	11.26474
Iron	Fe	26	56	55.934 936 33(49)	11.61851
Cobalt	Co	27	59	58.933 194 29(56)	11.32300
Nickel	Ni	28	58	57.935 342 41(52)	11.14786
Copper	Cu	29	63	62.929 597 72(56)	11.17497
Zinc	Zn	30	64	63.929 142 01(71)	11.07156
Gallium	Ga	31	69	68.925 5735(13)	10.78645
Germanium	Ge	32	74	73.921 177 761(13)	10.65165
Arsenic	As	33	75	74.921 594 57(95)	10.45406
Selenium	Se	34	80	79.916 5218(13)	10.43478
Bromine	Br	35	79	78.918 3376(14)	10.33701

20 Table BB: $(A-a)/A$ in parts per 10,000

atom	symbol	Z	A	a	$(A-a)/A$
Krypton	Kr	36	84	83.911 497 7282(44)	10.53598
Rubidium	Rb	37	85	84.911 789 7379(54)	10.37768
Strontium	Sr	38	88	87.905 6125(12)	10.72585
Yttrium	Y	39	89	88.905 8403(24)	10.57974
Zirconium	Zr	40	90	89.904 6977(20)	10.58914
Zirconium	Zr	40	96	95.908 2714(21)	9.55506
Niobium	Nb	41	93	92.906 3730(20)	10.06742
Molybdenum	Mo	42	98	97.905 404 82(49)	9.65257
Ruthenium	Ru	44	96	95.907 590 25(49)	9.62602
Ruthenium	Ru	44	102	101.904 3441(12)	9.37803
Rhodium	Rh	45	103	102.905 4980(26)	9.17495
Palladium	Pd	46	106	105.903 4804(12)	9.10562
Silver	Ag	47	107	106.905 0916(26)	8.86994
Cadmium	Cd	48	114	113.903 365 09(43)	8.47675
Indium	In	49	115	114.903 878 776(12)	8.35837
Tin	Sn	50	120	119.902 201 63(97)	8.14986
Antimony	Sb	51	121	120.903 8120(30)	7.94942
Tellurium	Te	52	130	129.906 222 748(12)	7.21363
Iodine	I	53	127	126.904 4719(39)	7.52190
Xenon	Xe	54	132	131.904 155 0856(56)	7.26098
Cesium	Cs	55	133	132.905 451 9610(80)	7.10888
Barium	Ba	56	138	137.905 247 00(31)	6.86616
Tantalum	Ta	73	181	180.947 9958(20)	2.87316
Tungsten	W	74	184	183.950 930 92(94)	2.66680
Rhenium	Re	75	187	186.955 7501(16)	2.36630
Iridium	Ir	77	193	192.962 9216(21)	1.92116
Gold	Au	79	197	196.966 568 79(71)	1.69702
Lead	Pb	82	208	207.976 6525(13)	1.12248
Bismuth	Bi	83	209	208.980 3991(16)	0.937842
Radon	Rn	86	222	222.017 5782(25)	−0.791811
Thorium	Th	90	232	232.038 0558(21)	−1.640336
Uranium	U	92	238	238.050 7884(20)	−2.133966

21 Data Sheet for EBIS Fe20+ in Booster

1. **Number of nucleons:** $A = 56$
2. **Relative atomic mass:** $a = 55.934\,936\,33(49)$
3. **Number of protons:** $Z = 26$
4. **Charge:** $Q = 20$
5. **Mass:** $mc^2 = 52.0928429390$ GeV
6. $mc^2/Q = 2.60464214695$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.566320607928$ Tm
3. $B = 408.435702695$ Gauss
4. $V_I = 26.530$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.18028182021$ Tm
2. $B = 0.851230253441$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.90567798339$ MHz ($h = 3$)
3. $\underline{W = 1000.34782634}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.99228452476$ MHz ($h = 3$)
3. $\underline{W = 1161.68104195}$ MeV per nucleon

22 Data Sheet for EBIS Fe24+ in Booster

1. **Number of nucleons:** $A = 56$
2. **Relative atomic mass:** $a = 55.934\,936\,33(49)$
3. **Number of protons:** $Z = 26$
4. **Charge:** $Q = 24$
5. **Mass:** $mc^2 = 52.0907989433$ GeV
6. $mc^2/Q = 2.17044995597$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.471915322410$ Tm
3. $B = 340.349730564$ Gauss
4. $V_I = 22.108$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 0.983529590699$ Tm
2. $B = 0.709330711040$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.05207700884$ MHz ($h = 3$)
3. $\underline{W = 1302.79909459}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.11867195464$ MHz ($h = 3$)
3. $\underline{W = 1503.06764856}$ MeV per nucleon

23 Data Sheet for EBIS Au32+ in Booster

1. **Number of nucleons:** $A = 197$
2. **Relative atomic mass:** $a = 196.966\,568\,79(71)$
3. **Number of protons:** $Z = 79$
4. **Charge:** $Q = 32$
5. **Mass:** $mc^2 = 183.456843853$ GeV
6. $mc^2/Q = 5.73302637040$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 1.24651710146$ Tm
3. $B = 898.999755840$ Gauss
4. $V_I = 58.396$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.59789499594$ Tm
2. $B = 1.87362609331$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.78531164171$ MHz ($h = 4$)
3. $\underline{W = 276.734590796}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.01209986501$ MHz ($h = 4$)
3. $\underline{W = 331.078270960}$ MeV per nucleon

24 Data Sheet for EBIS Au41+ in Booster

1. **Number of nucleons:** $A = 197$
2. **Relative atomic mass:** $a = 196.966\,568\,79(71)$
3. **Number of protons:** $Z = 79$
4. **Charge:** $Q = 41$
5. **Mass:** $mc^2 = 183.452244862$ GeV
6. $mc^2/Q = 4.47444499664$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.972867007318$ Tm
3. $B = 701.640756489$ Gauss
4. $V_I = 45.576$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.02757453313$ Tm
2. $B = 1.46230565798$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.32020924679$ MHz ($h = 4$)
3. $\underline{W = 424.875011056}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.52177283789$ MHz ($h = 4$)
3. $\underline{W = 503.830854894}$ MeV per nucleon

25 Data Sheet for EBIS Au43+ in Booster

1. **Number of nucleons:** $A = 197$
2. **Relative atomic mass:** $a = 196.966\,568\,79(71)$
3. **Number of protons:** $Z = 79$
4. **Charge:** $Q = 43$
5. **Mass:** $mc^2 = 183.451222864$ GeV
6. $mc^2/Q = 4.26630750847$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz $(h = 4)$
2. $B\rho = 0.927612211387$ Tm
3. $B = 669.002575718$ Gauss
4. $V_I = 43.456$ kV

On porch with $f = 200$ **kHz:**

1. $B\rho = 1.93325797080$ Tm
2. $B = 1.39428367384$ kG

At rigidity $B\rho = 15.8$ **Tm:**

1. $B = 11.3951073159$ kG
2. $hf = 4.41585410365$ MHz $(h = 4)$
3. $\underline{W = 460.226398006}$ MeV per nucleon

At rigidity $B\rho = 17.5$ **Tm:**

1. $B = 12.6211631664$ kG
2. $hf = 4.61084874347$ MHz $(h = 4)$
3. $\underline{W = 544.763026751}$ MeV per nucleon

26 Data Sheet for EBIS W31+ in Booster

1. **Number of nucleons:** $A = 184$
2. **Relative atomic mass:** $a = 183.950\,930\,92(94)$
3. **Number of protons:** $Z = 74$
4. **Charge:** $Q = 31$
5. **Mass:** $mc^2 = 171.333365028$ GeV
6. $mc^2/Q = 5.52688274284$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 1.20169582548$ Tm
3. $B = 866.674233703$ Gauss
4. $V_I = 56.296$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.50448194254$ Tm
2. $B = 1.80625572823$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.86733923799$ MHz ($h = 4$)
3. $\underline{W = 295.183439404}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.09149839834$ MHz ($h = 4$)
3. $\underline{W = 352.715618135}$ MeV per nucleon

27 Data Sheet for EBIS Th39+ in Booster

1. **Number of nucleons:** $A = 232$
2. **Relative atomic mass:** $a = 232.038\,0558(21)$
3. **Number of protons:** $Z = 90$
4. **Charge:** $Q = 39$
5. **Mass:** $mc^2 = 216.122149927$ GeV
6. $mc^2/Q = 5.54159358787$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 1.20489436649$ Tm
3. $B = 868.981051296$ Gauss
4. $V_I = 56.446$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.51114809549$ Tm
2. $B = 1.81106341990$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.86141036773$ MHz ($h = 4$)
3. $\underline{W = 293.932560184}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.08577429010$ MHz ($h = 4$)
3. $\underline{W = 351.252678301}$ MeV per nucleon

28 Data Sheet for EBIS W38+ in Booster

1. **Number of nucleons:** $A = 184$
2. **Relative atomic mass:** $a = 183.950\,930\,92(94)$
3. **Number of protons:** $Z = 74$
4. **Charge:** $Q = 38$
5. **Mass:** $mc^2 = 171.329788035$ GeV
6. $mc^2/Q = 4.50867863251$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.980310338256$ Tm
3. $B = 707.008956162$ Gauss
4. $V_I = 45.925$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.04308735054$ Tm
2. $B = 1.47349364654$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.30464770716$ MHz ($h = 4$)
3. $\underline{W = 419.403187362}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.50721980852$ MHz ($h = 4$)
3. $\underline{W = 497.485234990}$ MeV per nucleon

29 Data Sheet for EBIS Th48+ in Booster

1. **Number of nucleons:** $A = 232$
2. **Relative atomic mass:** $a = 232.038\,0558(21)$
3. **Number of protons:** $Z = 90$
4. **Charge:** $Q = 48$
5. **Mass:** $mc^2 = 216.117550936$ GeV
6. $mc^2/Q = 4.50244897784$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.978955840548$ Tm
3. $B = 706.032079786$ Gauss
4. $V_I = 45.861$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.04026441068$ Tm
2. $B = 1.47145771599$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.30747579602$ MHz ($h = 4$)
3. $\underline{W = 420.564935347}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.50986586425$ MHz ($h = 4$)
3. $\underline{W = 498.837273105}$ MeV per nucleon

30 Data Sheet for EBIS W42+ in Booster

1. **Number of nucleons:** $A = 184$
2. **Relative atomic mass:** $a = 183.950\,930\,92(94)$
3. **Number of protons:** $Z = 74$
4. **Charge:** $Q = 42$
5. **Mass:** $mc^2 = 171.327744040$ GeV
6. $mc^2/Q = 4.07923200094$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.886936867453$ Tm
3. $B = 639.667138424$ Gauss
4. $V_I = 41.550$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.84848554984$ Tm
2. $B = 1.33314501344$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.37740329593$ MHz ($h = 3$)
3. $\underline{W = 495.759905049}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.51872946861$ MHz ($h = 3$)
3. $\underline{W = 585.810179024}$ MeV per nucleon

31 Data Sheet for EBIS Th53+ in Booster

1. **Number of nucleons:** $A = 232$
2. **Relative atomic mass:** $a = 232.038\,0558(21)$
3. **Number of protons:** $Z = 90$
4. **Charge:** $Q = 53$
5. **Mass:** $mc^2 = 216.114995942$ GeV
6. $mc^2/Q = 4.07764143286$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.886591034347$ Tm
3. $B = 639.417720364$ Gauss
4. $V_I = 41.534$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.84776479110$ Tm
2. $B = 1.33262519552$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.37796411098$ MHz ($h = 3$)
3. $\underline{W = 496.293206522}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.51924642528$ MHz ($h = 3$)
3. $\underline{W = 586.431489872}$ MeV per nucleon

32 Data Sheet for EBIS Ar11+ in Booster

1. **Number of nucleons:** $A = 40$
2. **Relative atomic mass:** $a = 39.962\,383\,1237(24)$
3. **Number of protons:** $Z = 18$
4. **Charge:** $Q = 11$
5. **Mass:** $mc^2 = 37.2191029294$ GeV
6. $mc^2/Q = 3.38355481177$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.735677574825$ Tm
3. $B = 530.577526270$ Gauss
4. $V_I = 34.464$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.53324257487$ Tm
2. $B = 1.10578884064$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.62692097819$ MHz ($h = 3$)
3. $\underline{W = 670.319310544}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.74577488021$ MHz ($h = 3$)
3. $\underline{W = 786.298344378}$ MeV per nucleon

33 Data Sheet for EBIS Ar13+ in Booster

1. **Number of nucleons:** $A = 40$
2. **Relative atomic mass:** $a = 39.962\,383\,1237(24)$
3. **Number of protons:** $Z = 18$
4. **Charge:** $Q = 13$
5. **Mass:** $mc^2 = 37.2180809315$ GeV
6. $mc^2/Q = 2.86292930243$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.622479316363$ Tm
3. $B = 448.937886830$ Gauss
4. $V_I = 29.161$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.29732347768$ Tm
2. $B = 0.935641788082$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.81458814692$ MHz ($h = 3$)
3. $\underline{W = 868.325018183}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.91257214381$ MHz ($h = 3$)
3. $\underline{W = 1011.97000220}$ MeV per nucleon

34 Data Sheet for EBIS Ar14+ in Booster

1. **Number of nucleons:** $A = 40$
2. **Relative atomic mass:** $a = 39.962\,383\,1237(24)$
3. **Number of protons:** $Z = 18$
4. **Charge:** $Q = 14$
5. **Mass:** $mc^2 = 37.2175699326$ GeV
6. $mc^2/Q = 2.65839785233$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.578008571967$ Tm
3. $B = 416.865171336$ Gauss
4. $V_I = 27.078$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.20464097522$ Tm
2. $B = 0.868798303150$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.88690654644$ MHz ($h = 3$)
3. $\underline{W = 970.663436799}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.97592485169$ MHz ($h = 3$)
3. $\underline{W = 1128.06832273}$ MeV per nucleon

35 Data Sheet for EBIS Ar15+ in Booster

1. **Number of nucleons:** $A = 40$
2. **Relative atomic mass:** $a = 39.962\,383\,1237(24)$
3. **Number of protons:** $Z = 18$
4. **Charge:** $Q = 15$
5. **Mass:** $mc^2 = 37.2170589336$ GeV
6. $mc^2/Q = 2.48113726224$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.539467260157$ Tm
3. $B = 389.068817907$ Gauss
4. $V_I = 25.272$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.12431613974$ Tm
2. $B = 0.810867282875$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.94834470079$ MHz ($h = 3$)
3. $\underline{W = 1074.77417733}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.02933935584$ MHz ($h = 3$)
3. $\underline{W = 1245.87979807}$ MeV per nucleon

36 Data Sheet for EBIS Br17+ in Booster

1. **Number of nucleons:** $A = 79$
2. **Relative atomic mass:** $a = 78.918\,3376(14)$
3. **Number of protons:** $Z = 35$
4. **Charge:** $Q = 17$
5. **Mass:** $mc^2 = 73.5032785111$ GeV
6. $mc^2/Q = 4.32372226536$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.940095753536$ Tm
3. $B = 678.005822709$ Gauss
4. $V_I = 44.041$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.95927520847$ Tm
2. $B = 1.41304754822$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.38929946547$ MHz ($h = 4$)
3. $\underline{W = 449.667403598}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.58618197692$ MHz ($h = 4$)
3. $\underline{W = 532.536809684}$ MeV per nucleon

37 Data Sheet for EBIS Br25+ in Booster

1. **Number of nucleons:** $A = 79$
2. **Relative atomic mass:** $a = 78.918\,3376(14)$
3. **Number of protons:** $Z = 35$
4. **Charge:** $Q = 25$
5. **Mass:** $mc^2 = 73.4991905195$ GeV
6. $mc^2/Q = 2.93996762078$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.639229558747$ Tm
3. $B = 461.018317813$ Gauss
4. $V_I = 29.946$ kV

On porch with $f = 200$ **kHz:**

1. $B\rho = 1.33223304356$ Tm
2. $B = 0.960818892482$ kG

At rigidity $B\rho = 15.8$ **Tm:**

1. $B = 11.3951073159$ kG
2. $hf = 3.78705477453$ MHz ($h = 3$)
3. $\underline{W = 833.851391204}$ MeV per nucleon

At rigidity $B\rho = 17.5$ **Tm:**

1. $B = 12.6211631664$ kG
2. $hf = 3.88831663744$ MHz ($h = 3$)
3. $\underline{W = 972.784294877}$ MeV per nucleon

38 Data Sheet for EBIS Kr18+ in Booster

1. **Number of nucleons:** $A = 84$
2. **Relative atomic mass:** $a = 83.911\,497\,7282(44)$
3. **Number of protons:** $Z = 36$
4. **Charge:** $Q = 18$
5. **Mass:** $mc^2 = 78.1538666890$ GeV
6. $mc^2/Q = 4.34188148272$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.944044065218$ Tm
3. $B = 680.853381907$ Gauss
4. $V_I = 44.226$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.96750397576$ Tm
2. $B = 1.41898221192$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.38092712486$ MHz ($h = 4$)
3. $\underline{W = 446.513173997}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.57839462317$ MHz ($h = 4$)
3. $\underline{W = 528.885877343}$ MeV per nucleon

39 Data Sheet for EBIS Kr26+ in Booster

1. **Number of nucleons:** $A = 84$
2. **Relative atomic mass:** $a = 83.911\,497\,7282(44)$
3. **Number of protons:** $Z = 36$
4. **Charge:** $Q = 26$
5. **Mass:** $mc^2 = 78.1497786974$ GeV
6. $mc^2/Q = 3.00576071913$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.653534782018$ Tm
3. $B = 471.335378215$ Gauss
4. $V_I = 30.616$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.36204688880$ Tm
2. $B = 0.982320915648$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.76344451942$ MHz ($h = 3$)
3. $\underline{W = 806.047077038}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.86745783035$ MHz ($h = 3$)
3. $\underline{W = 941.150813879}$ MeV per nucleon

40 Data Sheet for EBIS Kr28+ in Booster

1. **Number of nucleons:** $A = 84$
2. **Relative atomic mass:** $a = 83.911\,497\,7282(44)$
3. **Number of protons:** $Z = 36$
4. **Charge:** $Q = 28$
5. **Mass:** $mc^2 = 78.1487566995$ GeV
6. $mc^2/Q = 2.79102702498$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.606845790076$ Tm
3. $B = 437.662841908$ Gauss
4. $V_I = 28.429$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.26474128554$ Tm
2. $B = 0.912143207318$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.84015480711$ MHz ($h = 3$)
3. $\underline{W = 902.274370893}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.93502819814$ MHz ($h = 3$)
3. $\underline{W = 1050.51631623}$ MeV per nucleon

41 Data Sheet for EBIS Xe27+ in Booster

1. **Number of nucleons:** $A = 132$
2. **Relative atomic mass:** $a = 131.904\,155\,0856(56)$
3. **Number of protons:** $Z = 54$
4. **Charge:** $Q = 27$
5. **Mass:** $mc^2 = 122.854144649$ GeV
6. $mc^2/Q = 4.55015350553$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.989328112668$ Tm
3. $B = 713.512659148$ Gauss
4. $V_I = 46.347$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.06188150186$ Tm
2. $B = 1.48704816370$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.28586192871$ MHz ($h = 4$)
3. $\underline{W = 412.769313072}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.48962912953$ MHz ($h = 4$)
3. $\underline{W = 489.785003395}$ MeV per nucleon

42 Data Sheet for EBIS 129Xe27+ in Booster

1. **Number of nucleons:** $A = 129$
2. **Relative atomic mass:** $a = 128.904\,780\,8611(60)$
3. **Number of protons:** $Z = 54$
4. **Charge:** $Q = 27$
5. **Mass:** $mc^2 = 120.060245269$ GeV
6. $mc^2/Q = 4.44667575072$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.966829211972$ Tm
3. $B = 697.286242190$ Gauss
4. $V_I = 45.293$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.01499102481$ Tm
2. $B = 1.45323031445$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.33286863644$ MHz ($h = 4$)
3. $\underline{W = 429.112513742}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.53359939424$ MHz ($h = 4$)
3. $\underline{W = 508.736027609}$ MeV per nucleon

43 Data Sheet for EBIS Xe35+ in Booster

1. **Number of nucleons:** $A = 132$
2. **Relative atomic mass:** $a = 131.904\,155\,0856(56)$
3. **Number of protons:** $Z = 54$
4. **Charge:** $Q = 35$
5. **Mass:** $mc^2 = 122.850056658$ GeV
6. $mc^2/Q = 3.51000161880$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.763170577160$ Tm
3. $B = 550.405735893$ Gauss
4. $V_I = 35.752$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.59054137415$ Tm
2. $B = 1.14711326892$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.58115207782$ MHz ($h = 3$)
3. $\underline{W = 632.512110087}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.70457560403$ MHz ($h = 3$)
3. $\underline{W = 743.020165729}$ MeV per nucleon

44 Data Sheet for EBIS 129Xe35+ in Booster

1. **Number of nucleons:** $A = 129$
2. **Relative atomic mass:** $a = 128.904\,780\,8611(60)$
3. **Number of protons:** $Z = 54$
4. **Charge:** $Q = 35$
5. **Mass:** $mc^2 = 120.056157278$ GeV
6. $mc^2/Q = 3.43017592222$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.745814282337$ Tm
3. $B = 537.888214240$ Gauss
4. $V_I = 34.939$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.55436872043$ Tm
2. $B = 1.12102521379$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.61003881511$ MHz ($h = 3$)
3. $\underline{W = 656.080599459}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.73060184840$ MHz ($h = 3$)
3. $\underline{W = 770.009843283}$ MeV per nucleon

45 Data Sheet for EBIS Xe36+ in Booster

1. **Number of nucleons:** $A = 132$
2. **Relative atomic mass:** $a = 131.904\,155\,0856(56)$
3. **Number of protons:** $Z = 54$
4. **Charge:** $Q = 36$
5. **Mass:** $mc^2 = 122.849545659$ GeV
6. $mc^2/Q = 3.41248737941$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.741968308206$ Tm
3. $B = 535.114461838$ Gauss
4. $V_I = 34.759$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.54635323718$ Tm
2. $B = 1.11524437253$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.61644344630$ MHz ($h = 3$)
3. $\underline{W = 661.488268511}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.73636133238$ MHz ($h = 3$)
3. $\underline{W = 776.198882600}$ MeV per nucleon

46 Data Sheet for EBIS Xe43+ in Booster

1. **Number of nucleons:** $A = 132$
2. **Relative atomic mass:** $a = 131.904\,155\,0856(56)$
3. **Number of protons:** $Z = 54$
4. **Charge:** $Q = 43$
5. **Mass:** $mc^2 = 122.845968666$ GeV
6. $mc^2/Q = 2.85688299224$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.621164682771$ Tm
3. $B = 447.989760826$ Gauss
4. $V_I = 29.100$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.29458361954$ Tm
2. $B = 0.933665776844$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.81674325885$ MHz ($h = 3$)
3. $\underline{W = 871.300675906}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.91446753820$ MHz ($h = 3$)
3. $\underline{W = 1015.35582030}$ MeV per nucleon

47 Data Sheet for EBIS Ta38+ in Booster

1. **Number of nucleons:** $A = 181$
2. **Relative atomic mass:** $a = 180.947\,9958(20)$
3. **Number of protons:** $Z = 73$
4. **Charge:** $Q = 38$
5. **Mass:** $mc^2 = 168.532571702$ GeV
6. $mc^2/Q = 4.43506767637$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz $(h = 4)$
2. $B\rho = 0.964305298379$ Tm
3. $B = 695.465972175$ Gauss
4. $V_I = 45.175$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 2.00973087837$ Tm
2. $B = 1.44943664780$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 4.33817003839$ MHz $(h = 4)$
3. $\underline{W = 431.199831675}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 4.53854871844$ MHz $(h = 4)$
3. $\underline{W = 511.160045588}$ MeV per nucleon

48 Data Sheet for EBIS Ta47+ in Booster

1. **Number of nucleons:** $A = 181$
2. **Relative atomic mass:** $a = 180.947\,9958(20)$
3. **Number of protons:** $Z = 73$
4. **Charge:** $Q = 47$
5. **Mass:** $mc^2 = 168.527972712$ GeV
6. $mc^2/Q = 3.58570154706$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.779629816846$ Tm
3. $B = 562.276293017$ Gauss
4. $V_I = 36.523$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.62484445460$ Tm
2. $B = 1.17185297037$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.55379640369$ MHz ($h = 3$)
3. $\underline{W = 611.559658932}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.67985455088$ MHz ($h = 3$)
3. $\underline{W = 719.009864809}$ MeV per nucleon

49 Data Sheet for EBIS Ta55+ in Booster

1. **Number of nucleons:** $A = 181$
2. **Relative atomic mass:** $a = 180.947\,9958(20)$
3. **Number of protons:** $Z = 73$
4. **Charge:** $Q = 55$
5. **Mass:** $mc^2 = 168.523884720$ GeV
6. $mc^2/Q = 3.06407063128$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.666212955461$ Tm
3. $B = 480.478995111$ Gauss
4. $V_I = 31.210$ kV

On porch with $f = 200$ kHz:

1. $B\rho = 1.38846976202$ Tm
2. $B = 1.00137733818$ kG

At rigidity $B\rho = 15.8$ Tm:

1. $B = 11.3951073159$ kG
2. $hf = 3.74245908675$ MHz ($h = 3$)
3. $\underline{W = 783.157203140}$ MeV per nucleon

At rigidity $B\rho = 17.5$ Tm:

1. $B = 12.6211631664$ kG
2. $hf = 3.84887203454$ MHz ($h = 3$)
3. $\underline{W = 915.105379771}$ MeV per nucleon

50 Data Sheet for EBIS Ta59+ in Booster

1. **Number of nucleons:** $A = 181$
2. **Relative atomic mass:** $a = 180.947\,9958(20)$
3. **Number of protons:** $Z = 73$
4. **Charge:** $Q = 59$
5. **Mass:** $mc^2 = 168.521840724$ GeV
6. $mc^2/Q = 2.85630238516$ GeV

At injection:

1. $f = 96.640$ kHz, $hf = 386.560$ kHz ($h = 4$)
2. $B\rho = 0.621038442875$ Tm
3. $B = 447.898715436$ Gauss
4. $V_I = 29.094$ kV

On porch with $f = 200$ **kHz:**

1. $B\rho = 1.29432052007$ Tm
2. $B = 0.933476027050$ kG

At rigidity $B\rho = 15.8$ **Tm:**

1. $B = 11.3951073159$ kG
2. $hf = 3.81695015935$ MHz ($h = 3$)
3. $\underline{W = 871.951941763}$ MeV per nucleon

At rigidity $B\rho = 17.5$ **Tm:**

1. $B = 12.6211631664$ kG
2. $hf = 3.91464948056$ MHz ($h = 3$)
3. $\underline{W = 1016.10686647}$ MeV per nucleon

References

[1] J.S. Coursey, D.J. Schwab, J.J. Tsai, and R.A. Dragoset, "Atomic Weights and Isotopic Compositions", NIST Physical Measurement Laboratory, https://www.nist.gov/pml/atomic-weights-and-isotopic-compositions-relative-atomic-masses.

[2] P.J. Mohr and B.N. Taylor, "Values of Fundamental Physical Constants," NIST Physical Measurement Laboratory, https://www.nist.gov/pml/fundamental-physical-constants.

[3] C.J. Gardner, "Notes on the setup of Ruthenium and Zirconium ions in Booster and AGS for RHIC Run 18," C-A/AP/Note 608, July 2018, Sections 10 and 28.

[4] Theodore Gray, "The Elements, A Visual Exploration of Every Known Atom in the Universe," Black Dog & Leventhal Publishers, New York, 2009.

[5] R. Thern, "Booster Dipole Production Measurements," Booster Technical Note No. 190, 13 March 1991. The current required to give the field quoted in equation (26) is obtained from Thern's data by extrapolation. This amounts to some 5588 amps, which is somewhat larger than the present limit of 5500 amps.

[6] Takeshi Kanesue, et al, "Design Study of the Dipole Magnet for the RHIC EBIS High Energy Transport Line," Proceedings of PAC07, Albuquerque, New Mexico, USA, THPAN033, pp. 3301–3303 (2007).

[7] Takeshi Kanesue, et al, "Dipole Magnet for use of RHIC EBIS HEBT Line," Proceedings of EPAC08, Genoa, Italy, WEPC153, pp. 2365–2367 (2008).

[8] C.J. Gardner, "Booster Inflector Specifications," Booster Technical Note No. 159, 28 February 1990. The gap between the cathode and septum of the inflector was increased from 17 to 21 mm in order to accommodate EBIS beams.

[9] C.J. Gardner, "Injection of large transverse emittance EBIS beams in Booster," C-A/AP/Note 436, October 2011, Section 7.