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Simple formula for surface roughness wakes

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For energy recovery linacs (ERLs) the energy spread induced by wakefields grows throughout acceleration and decelleration. Too much fractional momentum spread at low energy will complicate beam transport. This note uses some analytical results and a simple model of surface roughness correlation functions to obtain a wakefield formula that is very fast and easy to compute.

The resistive wall wake will be used for comparison. For resistive wall we used the low frequency approximation for the longitudinal wake potential

$$W(s) = \frac{d}{ds} H(s) \frac{cL}{2\pi b} \sqrt{\frac{Z_0 \rho_e}{\pi s}}, \tag{1}$$

where $Z_0 = 377\Omega$, s is the lag distance, $H(s) = 1$ for $s > 0$ and is zero otherwise, c is the speed of light, L is the length of the resistive section, b is the pipe radius, and ρ_e is the electrical resisitivity. When applying equation (1) and in formulas below we use integration by parts to obtain actual voltages. The numerics are very straightforward and will not be discussed.

For the wake potential due to surface roughness we used Stupakov's formula [1]. Define

$$W(s) = \frac{d}{ds} H(s) Re(\Phi(s)).$$

In MKS units

$$\Phi(s) = \int_0^\infty dk_z \int_{-\infty}^{\infty} dk_x |k_z|^{3/2} \frac{< |\hat{s}(k_x, k_z)|^2 >}{\epsilon_0 b^2 \sqrt{\pi}} \frac{1-i}{\sqrt{s}} \exp\left(i \frac{k_x^2 + k_z^2}{2k_z} s \right) \tag{2}$$

where the angular brackets denote statistical averages and

$$\hat{s}(k_x, k_z) = \int_0^L dz \int_0^{2\pi b} dx \frac{h(z,x)}{(2\pi)^2} \exp(ik_z z + ik_x x), \tag{3}$$

with surface roughness $h(x, z)$ where z is measured along the beam direction. For $h = h_0 \cos kz$ one has

$$< |\hat{s}(k_x, k_z)|^2 >= \frac{h_0^2 Lb}{8\pi} \delta(k_x)\delta(k_z - k).$$

and

$$W_0(s) = \frac{d}{ds} H(s) L h_0^2 k^{3/2} \frac{\cos(ks/2) + \sin(ks/2)}{8 s^{1/2} \epsilon_0 \pi^{3/2} b}. \tag{4}$$

Figures 1 and 2 show the input and results of an ABCI [2] simulation and equation (4). For these parameters the agreement is excellent. Other parameters have been checked and the amplitude of the wake is always good within a factor of 2. With this level of agreement it seems likely the theory is reasonably accurate. We go on to consider the wake due to surface roughness.

We need a statistical model of wall roughness to get $< \hat{s}^2 >$. For simplicity we take a stationary random process and a correlation function given by

$$< h(x_1, z_1) h(x_2, z_2) > \quad = C(x_1 - x_2, z_1 - z_2) \tag{5}$$

$$= h_0^2 \exp\left(-\frac{(x_1 - x_2)^2}{2\sigma_x^2} - \frac{(z_1 - z_2)^2}{2\sigma_z^2} \right), \tag{6}$$

where h_0 is the rms distortion, σ_z is the correlation length along the axis of the pipe and σ_x is along the circumference. Using the Wiener-Khinchin theorem

$$< |\hat{s}(k_x, k_z)|^2 > \quad = \frac{bL}{(2\pi)^3} \int_{-\infty}^{\infty} dx dz C(x, z) \exp(ik_z z + ik_x x) \tag{7}$$

$$= \frac{bL h_0^2 \sigma_x \sigma_z}{(2\pi)^2} \exp(-k_x^2 \sigma_x^2/2 - k_z^2 \sigma_z^2/2). \tag{8}$$

Inserting (8) in (2) and doing the k_x integration yields.

$$\Phi(s) = \frac{\sqrt{2}(1 - i) h_0^2 \sigma_x \sigma_z}{(2\pi)^2 \epsilon_0 b \sqrt{s}} \int_0^{\infty} dk_z \frac{k_z^2 \exp(-k_z^2 \sigma_z^2/2 + ik_z s/2)}{\sqrt{\sigma_x^2 k_x - is}}. \tag{9}$$

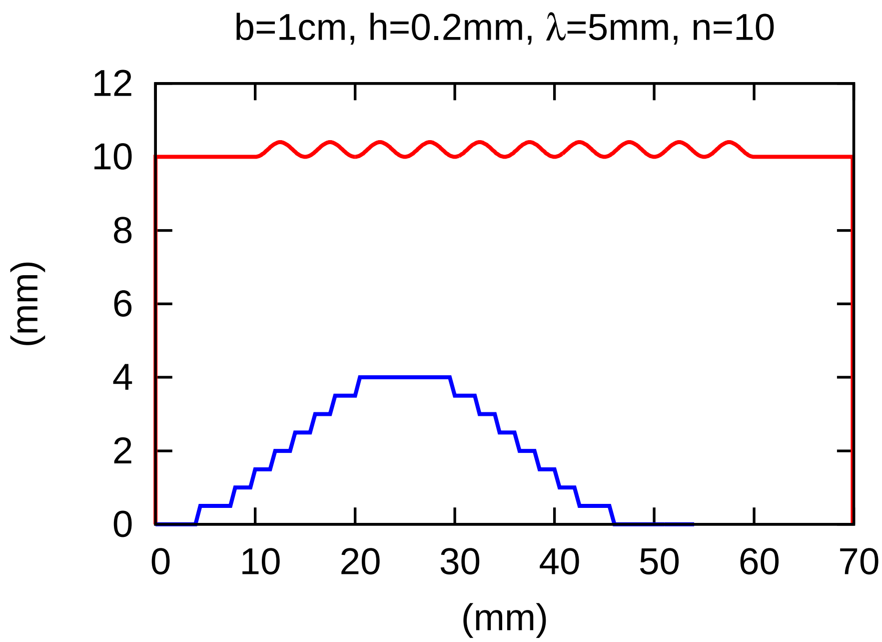

Figure 1: geometry to compare Stupakov's formula with ABCI. The red curve is the ideal geometry. The blue staircase is a 10 fold zoom of an actual convolution used in the simulation.

In (9) the square root has a positive real part and a negative imaginary part for $s > 0$. The integral is done numerically. Figure 3 shows the net voltage for a pipe of radius 1 cm and length equal to 4 passes up and 4 passes down in CBETA. Generalizing equation (2) to flat chambers has been considered by Stupapakov and Bane [4] who employed earlier work [5]. The main point here is that the surface impedance due to wall roughness is a spatially localized thing. We can take an effective surface impedance as a function of frequency and use impedance boundary conditions. Figure 8 in [3] shows the low frequency, longitudinal resistive wall wake for elliptical pipes. For all values the impedance is within 10% of the wake for a round pipe with the smaller apeture. It is likely we will not know $< \hat{s}^2 >$ to better than 10%.

References

[1] G. Stupakov, SLAC-PUB 8743 (2002)

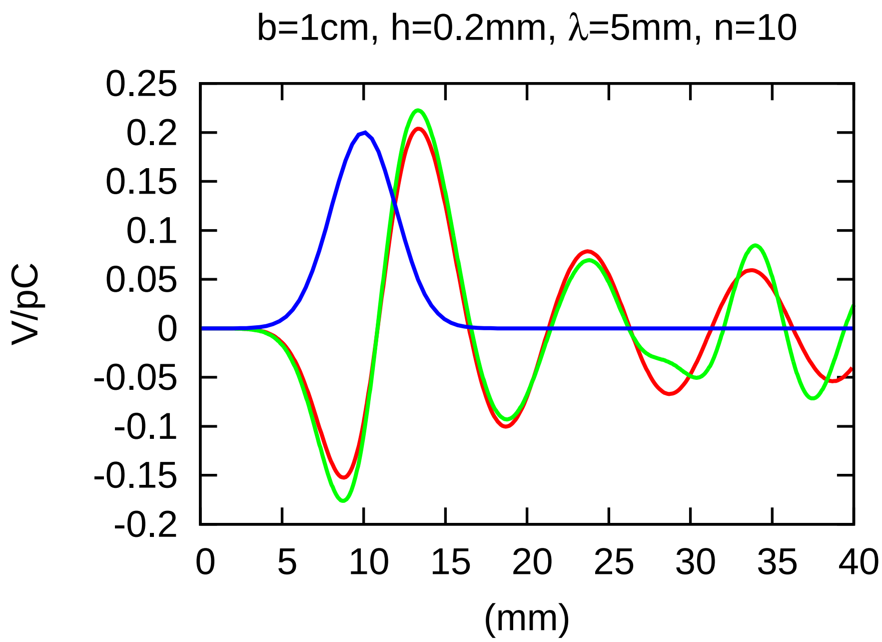

Figure 2: wakefields from Stupakov's formula and ABCI.

[2] Y.H. Chin, ABCI code available at http://abci.kek.jp

[3] K. Yokoya, *Particle Accelerators* Vol 41, p 221-248, (1993)

[4] K. Bane, G. Stupakov, *Phys. Rev. ST Accel. Beams* **18**,034401 (2015).

[5] H. Henke, O. Napoly, EPAC90, p1046

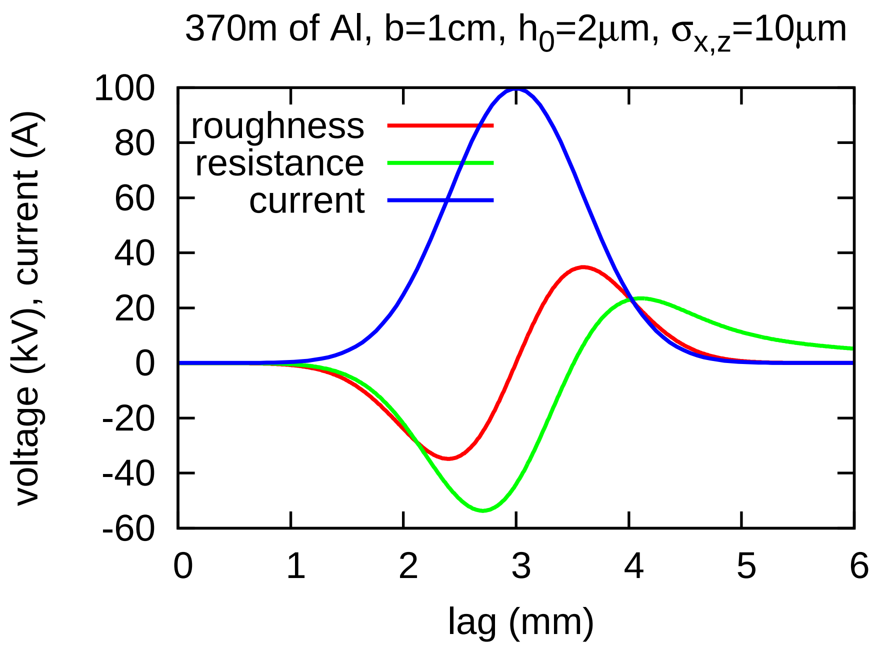

Figure 3: bunch current and induced voltage