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Corrections to the elastic p p analyzing power parameterization at high energies

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Corrections to the elastic $p^{\uparrow}p^{\uparrow}$ analyzing power parametrization at high energies

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The HJET polarimeter was designed to measure the absolute polarization of the proton beams at RHIC. It can also be used for the precise measurement of the elastic pp single and double spin analyzing powers. Recently Boris Kopeliovich pointed out that analyzing power parametrization, which is conventionally used for these measurements, was derived with specific simplifications and for the experimental accuracy achieved at HJET, the corrections should be applied. In this note we evaluate the corrections to analyzing powers due to (i) the differences between electromagnetic and hadronic form factors, and (ii) the $\sim m_p^2/s$ term in elastic pp electromagnetic amplitude. The corresponding variations of the measured hadronic spin-flip amplitudes are about experimental uncertainties of the HJET measurements. It should also be noted that the evaluated corrections might be essential for the elastic pp forward real-to-imaginary amplitude ratios ρ listed in PDG.

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I. INTRODUCTION

The Polarized Atomic Hydrogen Gas Jet Target (HJET) [1] is used to measure absolute polarization of the proton beams at the Relativistic Heavy Ion Collider (RHIC). For that, vertically polarized RHIC beams are scattered on HJET vertically polarized target (the jet) and the recoil proton spin-correlated asymmetries [2]

$$\frac{d^2\sigma}{dtd\varphi} = \frac{1}{2\pi}\frac{d\sigma}{dt}\times\big[1 + A_\mathrm{N}\sin\varphi\,(P_j+P_b) + \left(A_\mathrm{NN}\sin^2\varphi + A_\mathrm{SS}\cos^2\varphi\right)P_bP_j\big] \quad (1)$$

are studied. Here, P_j and P_b are jet and beam polarization, respectively. Positive signs of the $P_{j,b}$ correspond to the spin up direction. Generally, the analyzing powers $A_\mathrm{N}(s,t)$, $A_\mathrm{NN}(s,t)$, and $A_\mathrm{SS}(s,t)$ are functions of the invariant variables s, center of mass energy squared, and t, 4-momentum transfer squared. The azimuthal angle φ is defined in Fig. 1.

For HJET detectors, $\sin\varphi = \pm1$ and, thus, three spin correlated asymmetries can be experimentally determined:

$$a_\mathrm{N}^\mathrm{j,b} = \langle A_\mathrm{N}\rangle|P_{j,b}|, \quad a_\mathrm{NN} = \langle A_\mathrm{NN}\rangle|P_jP_b| \quad (2)$$

where analyzing powers are averaged over HJET acceptance t-range

$$0.001 \lesssim -t \lesssim 0.020\,\mathrm{GeV}^2. \quad (3)$$

consistent with the Coulomb-nuclear interference (CNI) region.

For elastic pp scattering, analyzing power $\langle A_\mathrm{N}\rangle$ is the same for the jet and beam asymmetry. Since jet polarization,

$$|P_j| = 0.957 \pm 0.001, \quad (4)$$

is well known, the beam polarization, $|P_b| = (a_\mathrm{N}^b/a_\mathrm{N}^j)\,|P_j|$, can be determined with actually no knowledge of the analyzing power $A_\mathrm{N}(t)$.

Main upgrade of HJET done in 2015 along with the development of new methods in data analysis, allowed us to reduce the systematic uncertainties of the beam polarization measurements to a $\sigma_P^\mathrm{syst}/P \lesssim 0.5\%$ [3] level. Such a small systematic uncertainty of measurements combined with large statistics $\sim 2\times10^9$ elastic pp events per RHIC Run accumulated in 2015 ($E_\mathrm{lab} = 100\,\mathrm{GeV}$) and 2017 ($E_\mathrm{lab} = 255\,\mathrm{GeV}$) allowed us to experimentally determine the analyzing powers

$$A_\mathrm{N}\ \ (t) = a_\mathrm{N}^\mathrm{j}(t)/|P_j| \quad (5)$$

$$A_\mathrm{NN}(t) = \frac{a_\mathrm{N}^\mathrm{j}(t)\,a_\mathrm{NN}(t)}{a_\mathrm{N}^\mathrm{b}(t)}\times\frac{1}{P_j^2} \quad (6)$$

with a high precision and, as a result, to isolate single and double spin-flip hadronic amplitudes in high energy elastic pp scattering.

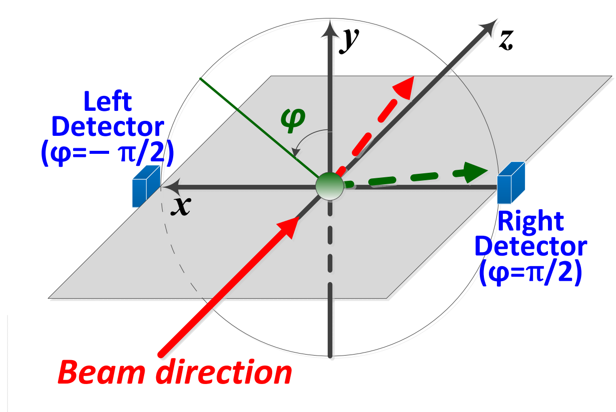

FIG. 1. A schematic view of the $p^{\uparrow}p^{\uparrow}$ spin correlated asymmetries measurement at HJET. The recoil protons are counted in left/right symmetric detectors. Beam moves along z-axis. The transverse polarization direction is along the y-axis.

II. PARAMETRIZATION OF THE CNI ANALYZING POWERS AT HIGH ENERGIES

All experimental studies of elastic pp analyzing powers at high energies [4–6] were based on the theoretical approach given in Ref. [7]. The elastic $p^\uparrow p^\uparrow$ scattering is described by five helicity amplitudes:

$$\begin{aligned}
\phi_1(s,t) &= \langle ++|M|++\rangle,\\
\phi_2(s,t) &= \langle ++|M|--\rangle,\\
\phi_3(s,t) &= \langle +-|M|+-\rangle,\\
\phi_4(s,t) &= \langle +-|M|-+\rangle,\\
\phi_5(s,t) &= \langle ++|M|+-\rangle.
\end{aligned} \tag{7}$$

For the scattering in the CNI region, the hadronic and electromagnetic components of the elastic pp amplitude should be explicitly indicated

$$\phi_i = \phi_i^{\mathrm{h}} + \phi_i^{\mathrm{em}} \exp(i\delta_C) \tag{8}$$

The Coulomb phase is approximately independent of helicity [8, 9]

$$\delta_C = \alpha \ln \frac{-2}{t\,(B+8/\Lambda^2)} - \alpha\gamma \quad \sim \; 0.02, \tag{9}$$

where $\gamma = 0.5772$ is Euler constant and $\Lambda^2 = 0.71\,\mathrm{GeV}^2$. The differential cross section slope $B(s)$ depends on energy as $B_0 + B_1 \ln s$ and is about $11.5\,\mathrm{GeV}^{-2}$ for RHIC energies. To the lowest order in α, the fine structure constant, the electromagnetic amplitudes were calculated in Ref. [8].

For very low t, the hadronic amplitude is dominated by the

$$\phi_+(s,t) = \left[\phi_1(s,t) + \phi_3(s,t)\right]/2 \tag{10}$$

term. According to the optical theorem,

$$\mathrm{Im}\,\phi_+^{\mathrm{h}}(s,0) = \frac{\sigma_{\mathrm{tot}}(s)\,s}{8\pi}\sqrt{1-4m_p^2/s} \tag{11}$$

where m_p is proton mass and $\sigma_{\mathrm{tot}}(s)$ is total pp cross section. Therefore, $\phi_+^{\mathrm{h}}(s,t)$ can be presented as

$$\phi_+^{\mathrm{h}}(s,t) = (\rho+i)\,\frac{\alpha s}{t_c}\left(1-4m_p^2/s\right)^{1/2} e^{Bt/2} \tag{12}$$

where

$$t_c = -8\pi\alpha/\sigma_{\mathrm{tot}} \approx -1.84\times10^{-3}\,\mathrm{GeV}^2 \tag{13}$$

for HJET energies and

$$\rho(s) = \mathrm{Re}\,\phi_+^{\mathrm{h}}(s,0)\,/\,\mathrm{Im}\,\phi_+^{\mathrm{h}}(s,0). \tag{14}$$

Using the following expressions [7]:

$$\frac{d\sigma}{dt} = \frac{2\pi}{s(s-4m_p^2)}\left(|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+4|\phi_5|^2\right) \tag{15}$$

and

$$A_{\mathrm{N}}\frac{d\sigma}{dt} = \frac{-4\pi}{s(s-4m_p^2)}\,\mathrm{Im}\left[\phi_5^*\left(\phi_1+\phi_2+\phi_3+\phi_4\right)\right] \tag{16}$$

the single spin analyzing power $A_{\mathrm{N}}(t)$ was calculated [7] as

$$A_{\mathrm{N}}(t) = \frac{\sqrt{-t}}{m_p}\,\frac{(t_c/t)\,f_{\mathrm{N}}^0 + f_{\mathrm{N}}^1}{f_{\mathrm{cs}}(t)} \tag{17}$$

where

$$f_{\mathrm{N}}^0(r_5) = \varkappa\,(1-\rho\delta_C) - 2(I_5 - \delta_C R_5) \tag{18}$$

$$f_{\mathrm{N}}^1(r_5) = -2(R_5 - \rho I_5) \tag{19}$$

$$f_{\mathrm{cs}}(t) \;= \left(\frac{t_c}{t}\right)^2 - 2(\rho+\delta_C)\frac{t_c}{t} + 1 + \rho^2 \tag{20}$$

The following notations were used: $\varkappa = \mu_p - 1 = 1.792$ and

$$r_5 = \frac{m\phi_5^{\mathrm{h}}}{\sqrt{-t}\,\mathrm{Im}\,\phi_+^{\mathrm{h}}} = R_5 + iI_5 \tag{21}$$

The dependence of $A_{\mathrm{N}}(t)$ on the double spin-flip amplitude

$$r_2 = \frac{\phi_2^{\mathrm{h}}}{2\,\mathrm{Im}\,\phi_+^{\mathrm{h}}} = R_2 + iI_2 \tag{22}$$

was neglected in Eqs. (18,19).

It should be pointed out that $A_N(t)$ dependence on r_5 amplitude is accumulated in a t-linear function

$$f_{\mathrm{N}}(t,r_5) = f_{\mathrm{N}}^0 + f_{\mathrm{N}}^1 t/t_c \approx \varkappa - 2I_5 - 2R_5\,t/t_c \tag{23}$$

Similarly, from the equation [**?**]

$$A_{\mathrm{NN}}\frac{d\sigma}{dt} = \frac{4\pi}{s(s-4m^2)}\,\left[2|\phi_5|^2 + \mathrm{Re}\left(\phi_1\phi_2^* - \phi_3\phi_4^*\right)\right] \tag{24}$$

one gets for the double spin analyzing power

$$A_{\mathrm{NN}}(t) = \frac{(t_c/t)\,f_{\mathrm{NN}}^0 + f_{\mathrm{NN}}^1}{f_{\mathrm{cs}}(t)} \tag{25}$$

where

$$f_{\mathrm{NN}}^0 = -2(R_2 + \delta_C I_2) \tag{26}$$

$$f_{\mathrm{NN}}^1 = 2I_2 + 2\rho R_2 - (\rho\varkappa - 4R_5)\frac{\varkappa t_c}{2m_p^2} \tag{27}$$

III. PRECISION MEASUREMENT OF THE ELASTIC pp ANALYZING POWERS AT HJET

The preliminary analysis of the HJET data acquired in RHIC Runs 2015 and 2017 has been done using the Ref. [7] analyzing powers. The values of the $\sigma_{\mathrm{tot}}(s)$ and $\rho(s)$ were taken from the Ref. [10] fit. The slope $B(s)$ was derived from Ref. [11]. The results could be summarized

as:
Run 2015 (100 GeV): $\sqrt{s} = 13.76\,\text{GeV}$, $\rho = -0.079$, $\sigma_{\text{tot}} = 38.39\,\text{mb}$, $B = 11.2 \pm 0.2\,\text{GeV}^2$.

$$R_5 = (-15.5 \pm 0.9_{\text{stat}} \pm 1.0_{\text{syst}}) \times 10^{-3} \quad (28)$$
$$I_5 = (-0.7 \pm 2.9_{\text{stat}} \pm 3.5_{\text{syst}}) \times 10^{-3} \quad (29)$$
$$R_2 = (-3.65 \pm 0.28_{\text{stat}}) \times 10^{-3} \quad (30)$$
$$I_2 = (-0.10 \pm 0.12_{\text{stat}}) \times 10^{-3} \quad (31)$$

Run 2017 (255 GeV): $\sqrt{s} = 21.92\,\text{GeV}$, $\rho = -0.009$, $\sigma_{\text{tot}} = 39.19\,\text{mb}$, $B = 11.6 \pm 0.2\,\text{GeV}^2$.

$$R_5 = (-7.3 \pm 0.5_{\text{stat}} \pm 0.8_{\text{syst}}) \times 10^{-3} \quad (32)$$
$$I_5 = (21.5 \pm 2.5_{\text{stat}} \pm 2.5_{\text{syst}}) \times 10^{-3} \quad (33)$$
$$R_2 = (-2.15 \pm 0.20_{\text{stat}}) \times 10^{-3} \quad (34)$$
$$I_2 = (-0.35 \pm 0.07_{\text{stat}}) \times 10^{-3} \quad (35)$$

For r_2, systematic errors are small in these measurements.

Recently, it was pointed out by Boris Kopeliovich [12] that $A_N(t)$ in Ref. [7] was derived with some simplifications, namely
(i) it was implicitly assumed that electromagnetic form factor is equal to hadronic form factor $\exp(Bt/2)$;
(ii) the elastic *pp* electric form factor was approximated, $G_E^{pp} = G_E^2(t)$, by an electric form factor $G_E(t)$ determined in electron-proton scattering experiments. No absorptive corrections on inelastic collisions were taken into account. Such correction are currently undetermined but a theoretical study is in progress.

IV. CORRECTIONS TO THE ANALYZING POWERS

To calculate corrections to Eq. (17), it is convenient to use the scaled amplitudes:

$$\varphi_i(s,t) = \phi_i(s,t)/\operatorname{Im}\phi_+^{\text{h}}(s,t) \quad (36)$$

Since a possible dependence of ρ, r_2, and r_5 on t may be neglected in the CNI region, the scaled hadronic amplitudes can be approximated by

$$\varphi_1^{\text{h}} = \varphi_3^{\text{h}} = \rho(s) + i$$
$$\varphi_2^{\text{h}} = 2r_2(s)$$
$$\varphi_4^{\text{h}} = r_4(s) \times (-t/m_p^2) \approx 0 \quad (37)$$
$$\varphi_5^{\text{h}} = r_5(s) \times \sqrt{-t}/m_p \quad (38)$$

For the electromagnetic amplitudes, we should account the corrections $\sim m_p^2/s$ which can be significant for $E_{\text{Lab}} = 100\,\text{GeV}$. Neglecting the terms $\sim (m_p^2/s)^2$ and $\sim t/s$, one can derive from Ref. [8]:

$$\varphi_1^{\text{em}} = \varphi_3^{\text{em}} = \varphi_0^{\text{em}},$$
$$\varphi_2^{\text{em}} = -\varphi_4^{\text{em}} = \varphi_0^{\text{em}} F_\varkappa^2,$$
$$\varphi_5^{\text{em}} = \varphi_0^{\text{em}} F_\varkappa. \quad (39)$$

The following shorthand were used:

$$\varphi_0^{\text{em}}(s,t) = \frac{t_c(s)}{t} \times G_0^2(t)\, G_E^2(t) \exp(-Bt/2) \quad (40)$$
$$G_E(t) = 1 + r_p^2 t/6 \quad (41)$$
$$G_0(t) = \frac{1 - \mu_p t/4m_p^2}{1 - t/4m_p^2} \quad (42)$$
$$F_\varkappa(s,t) = \frac{-\sqrt{-t}\,\varkappa'}{2m_p}, \quad \varkappa' = \frac{\varkappa - 2m_p^2/s}{1 - \mu_p t/4m_p^2} \quad (43)$$

The correction factor $(1 - 2m_p^2/s)$ common for all five electromagnetic amplitudes (7) was canceled by a similar factor in $\operatorname{Im}\varphi_+(t)$ (Eq. 11). The remaining s-dependent corrections are accumulated in the value of $\varkappa'$. Proton's electric form factor $G_E(t)$ was approximated by proton charge radius $r_p = \langle r_p^2 \rangle^{1/2}$.

The electromagnetic form factor related corrections can be accounted by the substitution $t_c \to t_c + bt$ where

$$b/t_c = \frac{d}{dt}\left[G_0^2(t) G_E^2(t) e^{-Bt/2}\right]_{t=0}. \quad (44)$$

Since electric form factor G_E in the dipole form [9, 13]

$$G_D(t) = \left(1 - t/\Lambda^2\right)^{-2}, \quad \Lambda^2 = 0.71\,\text{GeV}^2 \quad (45)$$

was commonly used in elastic *pp* data analysis, it is convenient to explicitly isolate the corresponding term b_D in (44):

$$b = b_D + b_{\text{nf}} \quad (46)$$

where

$$b_D/t_c = \frac{d}{dt}\left(G_D^2(t) e^{-Bt/2}\right)\Big|_{t=0} = \left(\frac{4}{\Lambda^2} - \frac{B}{2}\right) \quad (47)$$

For the HJET energies,

$$100\,\text{GeV:} \quad b_D = (-0.06 \pm 0.19) \times 10^{-3} \quad (48)$$
$$255\,\text{GeV:} \quad b_D = (+0.31 \pm 0.19) \times 10^{-3} \quad (49)$$

The specified errors correspond to the systematic uncertainties in the values of $B(s)$ [11].

For the non-flip amplitude:

$$b_{\text{nf}}/t_c = r_p^2/3 - 4/\Lambda^2 - \varkappa/2m_p^2 \quad (50)$$

Currently, PDG [14] gives two values of proton charge radius:

$$r_{ep} = 0.8751 \pm 0.0061\,\text{fm} \quad (51)$$
$$r_{\mu p} = 0.84086 \pm 0.00026 \pm 0.00029\,\text{fm} \quad (52)$$

obtained in three kinds of measurements: with atomic Hydrogen, with electron scattering off Hydrogen, and with muonic Hydrogen. The discrepancy between the methods is not resolved yet. Assuming $r_p = 0.858 \pm 0.017$ one gets

$$b_{\text{nf}} = (0.64 \pm 0.46) \times 10^{-3} \quad (53)$$

Spin-flip contributions are negligible for the hadronic and interference part of the $d\sigma/dt$. However, they effectively change the parameter $b_{\rm nf}$ for the electromagnetic part $|\phi_+^{\rm em}|^2$ in Eq. (20):

$$b_{\rm nf} \to b_{\rm cs} = b_{\rm nf} - \varkappa^2 t_c/4m_p^2 \approx 2.32\times10^{-3} \tag{54}$$

Applying the corrections to $f_{\rm cs}(t,\rho)$ one gets

$$\begin{aligned} f_{\rm cs}(t,\rho) \to\ & f_{\rm cs}(t,\rho-b_D-b_{\rm cs}) \\ & -\ \rho\varkappa^2 t_c/4m_p^2 - (b_D+b_{\rm nf})\,\delta_C \\ \approx\ & f_{\rm cs}(t,\rho-b_D-b_{\rm cs}) \end{aligned} \tag{55}$$

Experimental determination of the real to imaginary ratio ρ at high energies is based on analysis of the differential cross section $d\sigma/dt\,(t)$, which is proportional to $f_{\rm cs}(t,\rho)$. Proton-proton electromagnetic form factor was approximated by $G_D^2(t)$ in almost all experimental studies of ρ. Therefore, a biased value of ρ was measured in these experiments

$$\rho^{\rm exp} = \rho - b_{\rm cs} = \rho - (2.3\pm0.5)\times10^{-3} \tag{56}$$

The bias is small compared to the uncertainty of measurements in any of the experiment listed in PDG, but it is substantial for the global fit [15]. Since in analyzing power measurements, the values of ρ from the global fit are used, we should replace

$$\rho \to \rho + b_{\rm cs} \tag{57}$$

Thus, the leading order corrections to the Ref. [7] analyzing power $A_N(t)$ (17) can be approximated as

$$f_{\rm cs}(t,\rho) \to f_{\rm cs}(t,\rho-b_D) \tag{58}$$
$$f_{\rm N}^0 \to f_{\rm N}^0 - 2m_p^2/s \tag{59}$$
$$f_{\rm N}^1 \to f_{\rm N}^1 + \varkappa\,(b_D+b_{\rm nf}+b_\varkappa) \tag{60}$$

where

$$b_\varkappa = \mu_p t_c/4m_p^2 \approx -1.46\times10^{-3} \tag{61}$$

accounts the spin-flip contribution (see Eq. 43) to the electromagnetic form factor.

For $A_{\rm NN}(t)$, the corrections are small compared to uncertainties of the measurement at HJET. Also we can neglect the correction to the Coulomb phase $\delta_C(r^2,B)$.

V. NUMERICAL ESTIMATES OF THE CORRECTIONS

The effect of the substitution (58) can be parameterized by the effective correction $b_D\Delta f_{\rm N}(t)$ to the linear function $f_{\rm N}(t)$:

$$\frac{1}{f_{\rm cs}(t,\rho-b_D)} = \frac{1+b_D\Delta f_{\rm N}(t)}{f_{\rm cs}(t,\rho)} \tag{62}$$

The dependence of the $\Delta f_{\rm N}(t)$ on ρ and b_D can be neglected. The calculated value of this correction is shown in Fig. 2. For a relatively large b_D, the $\Delta f_{\rm N}(t)$ results in a non-linearity of $f_N(t)$ which may be detected in the data analysis. For HJET, the statistical uncertainties of such an evaluation of b_D were found to be $\sigma_{b_D}\sim 0.02$. For smaller b_D, the correction may be approximated by a linear function

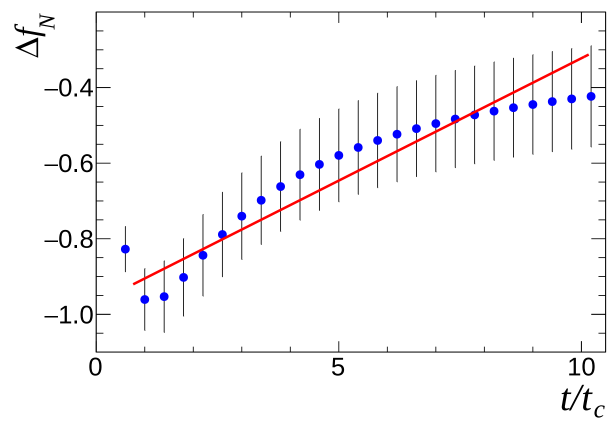

FIG. 2. Calculation of the correction function $\Delta f_{\rm N}(t)$ (blue points). The displayed error bars $\sigma_{0.1}(t)$ correspond to the HJET measurement statistical uncertainties if $b_D=0.1$. The error dependence on b_D can be approximated by $\sigma(t,b_D)=\sigma_{0.1}(t)\times0.1/b_D$. The red line is a linear fit.

$$\Delta f_{\rm N}(t) = c_0 + c_1\,t/t_c \tag{63}$$

which is equivalent to the following substitutions:

$$f_{\rm N}^0 \to f_{\rm N}^0 + c_0\varkappa b_D \tag{64}$$
$$f_{\rm N}^1 \to f_{\rm N}^1 + c_1\varkappa b_D \tag{65}$$

Obviously, the values of c_0 and c_1 depend on the t-range and experimental uncertainties (statistical errors and errors due to the background suppression). In the HJET data analysis, it was evaluated, that $c_0\sim-1.0$ and $c_0\sim0.1$.

Combining (59,60,64,65) we can find the corrections to the measured hadronic form factors:

$$\Delta I_5 = (\varkappa/2)\times c_0 b_D - m_p^2/s \tag{66}$$
$$\Delta R_5 = (\varkappa/2)\times[(1+c_1)b_D + b_{\rm nf} + b_\varkappa] + \rho\Delta I_5 \tag{67}$$

For HJET measurements, the calculation gives:

$$100\,\text{GeV:}\quad \Delta R_5 = (-0.4\pm0.2\pm0.4)\times10^{-3} \tag{68}$$
$$\Delta I_5 = (-4.6\pm0.2)\times10^{-3} \tag{69}$$
$$255\,\text{GeV:}\quad \Delta R_5 = (-0.4\pm0.2\pm0.4)\times10^{-3} \tag{70}$$
$$\Delta I_5 = (-2.1\pm0.2)\times10^{-3} \tag{71}$$

The first error corresponds to uncertainties in the slope B, the second one is due to uncertainties in proton charge radius. Both errors are strongly correlated for Re / Im and 100/255 GeV. The found corrections are comparable with HJET experimental uncertainties and, thus, should be properly accounted.

For the other experiments [4–6], the corrections to r_5 are small compared to the experimental uncertainties. However, they might be noticable if the Re/Im error correlations are taken into account.

VI. POSSIBLE EFFECT OF THE ABSORPTIVE CORRECTION

A measured r_5 dependence on the absorptive corrections could be easily estimated if the corresponding modification of the elastic *pp* electromagnetic form factor $\mathcal{F}^{\text{em}}$ can be approximated by a linear function in the CNI region:

$$\mathcal{F}^{\text{em}} \to \mathcal{F}^{\text{em}} \times [1 + a(s)t/t_c]\,, \tag{72}$$

Generally, $a(s)$ is spin-flip dependent. It can be effectively accounted by the substitutions

$$\rho \to \rho + a_{\text{nf}} \tag{73}$$
$$b_\varkappa \to b_\varkappa + a_{\text{sf}} \tag{74}$$

where a_{nf} and a_{sf} are absorptive corrections to non-flip and spin-flip amplitudes, respectively. The dominant absorptive corrections to r_5 and r_2 can be written as

$$\Delta_{\text{a}} R_5 = a_{\text{sf}}\varkappa/2 + a_{\text{nf}} I_5 \tag{75}$$
$$\Delta_{\text{a}} I_2 = a_{\text{nf}} \frac{\varkappa^2 t_c}{2m_p^2} \tag{76}$$

The correction to ρ (Eq. 73) might be essential in the A_{N} measurement if $a_{\text{nf}} > 0.01$. However, in this case, the unpolarized *pp* measurements of the forward real to imaginary ratio ρ are also strongly affected and must be revisited.

VII. SUMMARY

In this note, the corrections to the analyzing powers given in Ref. [7] were studied. For already published experimental results, the corrections to the measured single and double spin-flip amplitude parameters r_5 and r_2 could be evaluated with a sufficient accuracy.

The improved expressions for $A_{\text{N}}(t)$ and $A_{\text{NN}}(t)$ could be written in exactly the same, if $\Delta_{\text{N}}^a = \Delta_{\text{NN}}^a = 0$, form as in Ref. [7]:

$$\frac{m_p}{\sqrt{-t}}\,A_{\text{N}}(t) = \frac{\left[\varkappa'(1-\rho'\delta_C+I_2) - 2(I_5-\delta_C R_5)\right]\,t_c'/t - 2\left[(1+I_2)R_5 - (\rho'+R_2)I_5\right] + \Delta_{\text{N}}^a\varkappa}{(t_c/t)^2 - 2(\tilde\rho+\delta_C)\,t_c/t + 1 + \tilde\rho^2} \tag{77}$$

$$A_{\text{NN}}(t) = \frac{-2(R_2+\delta_C I_2)\,t_c'/t + 2(I_2+\rho' R_2) - (\rho'\varkappa' - 4R_5)\,\varkappa' t_c/2m_p^2 + \Delta_{\text{NN}}^a \varkappa t/m_p^2}{(t_c/t)^2 - 2(\tilde\rho+\delta_C)t_c/t + 1 + \tilde\rho^2} \tag{78}$$

but with a following modification of some parameters:

$$t_c' = t_c \times \left[1 + \left(r_p^2/3 - B/2 - \varkappa/2m_p^2\right)t\right], \tag{79}$$
$$\rho' = \rho + \left(r_p^2/3 - 4/\Lambda^2 - \varkappa/2m_p^2 - \varkappa^2/4m_p^2\right)t_c, \tag{80}$$
$$\tilde\rho\ = \rho - \left(4/\Lambda^2 - B/2\right)t_c, \tag{81}$$
$$\varkappa' = \left(\varkappa - 2m_p^2/s\right) / \left(1 - \mu_p t/4m_p^2\right). \tag{82}$$

For completeness, we also added double spin-flip hadronic amplitude r_2, which in fact is small, to the single spin analyzing power $A_{\text{N}}(t)$.

The absorptive corrections are currently undetermined, but once calculated may be introduced by the following substitutions:

$$r_p^2/3 \to r_p^2/3 + a_{\text{nf}}(s) \tag{83}$$
$$\Delta_{\text{N}}^a \ = a_{\text{sf}}(s) - a_{\text{nf}}(s) \tag{84}$$
$$\begin{aligned}\Delta_{\text{NN}}^a \ &= \rho\varkappa \times [a_{\text{df}}(s) - a_{\text{nf}}(s)] \\ &\quad - 4R_5 \times [a_{\text{sf}}(s) - a_{\text{nf}}(s)]\end{aligned} \tag{85}$$

where a_{df} is the absorptive correction to the double spin-flip electromagnetic form factor.

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[1] A. Zelenski *et al.*, Nucl. Instrum. Meth. A **536**, 248 (2005).
[2] J. Ashkin *et al.*, AIP Conf. Proc. **42**, 142 (1978); E. Leader, Spin in Particle Physics, p. 119, Cambridge University Press, 2001.
[3] A. Poblaguev *et al.*, PoS PSTP **2017**, 022 (2018).
[4] H. Okada *et al.*, Phys. Lett. B **638**, 405 (2006).
[5] I. G. Alekseev *et al.*, Phys. Rev. D **79**, 094014 (2009).
[6] L. Adamczyk *et al.*, Phys. Lett. B **719**, 62 (2013).

[7] N. H. Buttimore, B. Z. Kopeliovich, E. Leader, J. Soffer and T. L. Trueman, Phys. Rev. D **59**, 114010 (1999).
[8] N. H. Buttimore, E. Gotsman and E. Leader, Phys. Rev. D **18**, 694 (1978) Erratum: [Phys. Rev. D **35**, no. 1, 407 (1987)].
[9] R. Cahn, Z. Phys. C **15**, 253 (1982).
[10] D. A. Fagundes, M. J. Menon and P. V. R. G. Silva, Nucl. Phys. A **966**, 185 (2017).
[11] V. Bartenev *et al.*, Phys. Rev. Lett. **31**, 1088 (1973); Phys. Rev. Lett. **31**, 1367 (1973).
[12] M. Krelina, Talk at CFNS workshop on Forward Physics And Instrumentation From Colliders To Cosmic Rays, 17-19 Oct 2018, Stony Brook. https://indico.bnl.gov/event/4737/contributions/24347/attachments/20363/27218/ForwardPhysWorkshop2018-Krelina-v1.pdf
[13] J. R. Dunning *et al.*, Phys. Rev. Lett. **13**, 631 (1964); L. H. Chan *et al.*, Phys. Rev. **141**, 1298 (1966).
[14] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[15] COMPETE Collaboration, http://nuclth02.phys.ulg.ac.be/compete/predictor/.