Notes on adjusting gold ion momentum in RHIC to optimize electron cooling at injection

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The proposed electron cooling of low-energy gold ions in RHIC and the detection of ion-electron recombination are reviewed in [1]. Optimal cooling requires that the gold ions have the same velocity as the electrons in the cooling section of the RHIC ring. This can be accomplished by making small adjustments of either the ion or the electron momentum. Such adjustments are necessary because although we can set the momentum with good accuracy, this may not be good enough to land precisely on the sweet spot where, as detected by the recombination monitor, the cooling is maximized.

We examine here two ways in which adjustments of the ion momentum could be made. One way is to keep the ion revolution frequency fixed while adjusting the machine bending field and the closed orbit radius. Another is to keep the radius fixed while adjusting the frequency and field. Taking the fractional shift dp/p in ion momentum to be 1/10,000, we calculate the relevant parameter changes for both cases and compare the results. It is shown that only the fixed-radius case is practicable.

The differential relations required for the calculations are given in **Section 1**.

The calculated effects of changing the ion momentum while keeping the frequency fixed are given in **Section 2**. The effects of changing the momentum while keeping the radius fixed are given in **Section 3**.

The constraints imposed by the electron RF frequency are discussed in **Section 4**.

A summary of the results along with conclusions is given in **Section 5**.

1 Fundamental differential relations

From the defining relation

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{1}$$

we have

$$\gamma = \sqrt{(\beta\gamma)^2 + 1}, \quad \beta = \frac{\beta\gamma}{\sqrt{(\beta\gamma)^2 + 1}} \tag{2}$$

$$\frac{d\gamma}{d(\beta\gamma)} = \beta, \quad \frac{d\beta}{d(\beta\gamma)} = \frac{1}{\gamma^3} \tag{3}$$

and therefore

$$\frac{d\gamma}{\gamma} = \beta^2 \frac{d(\beta\gamma)}{\beta\gamma}, \quad \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{d(\beta\gamma)}{\beta\gamma}. \tag{4}$$

In terms of the ion momentum

$$p = mc\beta\gamma \tag{5}$$

we have

$$\frac{d\gamma}{\gamma} = \beta^2 \frac{dp}{p} = \left(\frac{\gamma^2 - 1}{\gamma^2}\right)\frac{dp}{p} \tag{6}$$

and

$$\frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}. \tag{7}$$

The ion revolution frequency is

$$f = \frac{c\beta}{2\pi R} \tag{8}$$

which gives the differential relation

$$\frac{df}{f} = \frac{d\beta}{\beta} - \frac{dR}{R} = \frac{1}{\gamma^2}\frac{dp}{p} - \frac{dR}{R}. \tag{9}$$

The dispersive properties of the machine lattice give the differential relation

$$\frac{dR}{R} = \frac{1}{\gamma_t^2}\left(\frac{dp}{p} - \frac{db}{b}\right) \tag{10}$$

where b is the machine bending field and γ_t is the transition gamma of the lattice. Substituting this dR/R into (9) then gives

$$\frac{df}{f} = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}\right)\frac{dp}{p} + \frac{1}{\gamma_t^2}\frac{db}{b} \tag{11}$$

which is the fundamental differential relation connecting df, dp, and db.

2

2 Fixed frequency

If we want to keep the frequency fixed while varying the ion momentum, we must take $df = 0$ in (9) and (11). This gives

$$\frac{dR}{R} = \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p} \tag{12}$$

and

$$\frac{db}{b} = \left(\frac{\gamma^2 - \gamma_t^2}{\gamma^2}\right)\frac{dp}{p}. \tag{13}$$

We also have

$$\Delta X = \left(\frac{dp}{p} - \frac{db}{b}\right)D \tag{14}$$

which is the shift in the closed orbit at a point in the lattice where the periodic dispersion is D. Putting the value of db/b from (13) into (14) gives

$$\Delta X = \left(\frac{\gamma_t^2}{\gamma^2}\frac{dp}{p}\right)D. \tag{15}$$

In RHIC the nominal radius is

$$R_R = \frac{1}{2\pi}(3833.845181) = 610.175411605 \text{ meters} \tag{16}$$

and we take

$$\gamma_t = 22.89. \tag{17}$$

In AGS at extraction the nominal radius is

$$R_A = \frac{4}{19}R_R = 128.457981391 \text{ meters} \tag{18}$$

and we take

$$\gamma_t = 8.5. \tag{19}$$

For Au79+ ions circulating in RHIC at injection with energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon we have, **respectively:**

$$\begin{aligned}
\gamma \quad = \quad & 4.13474444648, \tag{20}\\
& 4.88651616403,\\
& 6.17526767981,\\
& 7.83990505437,\\
& 10.5204666076.
\end{aligned}$$

3

This gives, **in parts per thousand:**

$$\frac{1}{\gamma^2} = 58.4928328260, \tag{21}$$
$$41.8794838577,$$
$$26.2233652798,$$
$$16.2696568693,$$
$$9.03503825076.$$

Taking

$$\frac{dp}{p} = \frac{1}{10,000} \tag{22}$$

in (12) then gives, **in parts per million:**

$$\frac{dR}{R} = \frac{d\beta}{\beta} = 5.84928328260, \tag{23}$$
$$4.18794838577,$$
$$2.62233652798,$$
$$1.62696568693,$$
$$0.903503825076.$$

In RHIC this gives, **in mm:**

$$dR_R = 3.5691,\ 2.5554,\ 1.6001,\ 0.9927,\ 0.5513. \tag{24}$$

According to (15) we also have, **in parts per 1000:**

$$\frac{\Delta X_R}{D} = 3.0647,\ 2.1943,\ 1.3740,\ 0.8525,\ 0.4734. \tag{25}$$

In AGS we have

$$dR_A = \frac{4}{19}\, dR_R. \tag{26}$$

Using (17), (19), and (20) we have in RHIC

$$\frac{\gamma^2 - \gamma_t^2}{\gamma^2} = -29.6474425941, \tag{27}$$
$$-20.9428435141,$$
$$-12.7397873074,$$
$$-7.52452088293,$$
$$-3.73392726507$$

4

and in AGS

$$\frac{\gamma^2 - \gamma_t^2}{\gamma^2} = -3.22610717168, \tag{28}$$
$$= -2.02579270872,$$
$$= -0.894638141466,$$
$$= -0.175482708805,$$
$$= +0.347218486383. \tag{29}$$

Multiplying these numbers by dp/p gives db/b as per equation (13).

3   Fixed radius

If we want to keep the radius fixed while varying the ion momentum, we must take $dR = 0$ in (9) and (10). This gives

$$\frac{df}{f} = \frac{1}{\gamma^2} \frac{dp}{p} \tag{30}$$

and

$$\frac{db}{b} = \frac{dp}{p}. \tag{31}$$

Taking again

$$\gamma = 4.13474444648, \tag{32}$$
$$4.88651616403,$$
$$6.17526767981,$$
$$7.83990505437,$$
$$10.5204666076$$

and

$$\frac{dp}{p} = \frac{1}{10,000} \tag{33}$$

we have, **in parts per million:**

$$\frac{df}{f} = 5.84928328260, \tag{34}$$
$$4.18794838577,$$
$$2.62233652798,$$
$$1.62696568693,$$
$$0.903503825076.$$

5

In RHIC one has, **in MHz:**

$$hf = \begin{aligned}&9.10498412755,\\&9.18496377307,\\&9.25970290448,\\&9.30690779975,\\&9.34106786594\end{aligned} \tag{35}$$

where

$$h = 120. \tag{36}$$

This gives, **in Hz:**

$$d(hf) = 53.258,\ 38.466,\ 24.282,\ 15.142,\ 8.4397 \tag{37}$$

at gold ion energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon, respectively.

In AGS one has revolution frequency

$$f_A = \frac{19f}{4} \tag{38}$$

and

$$h_A = 12. \tag{39}$$

This gives

$$h_A f_A = \left(\frac{19 h_A}{4h}\right) hf = \left(\frac{19}{40}\right) hf \tag{40}$$

and

$$d(h_A f_A) = \left(\frac{19}{40}\right) d(hf). \tag{41}$$

Thus we have, **in Hz:**

$$d(h_A f_A) = 25.297,\ 18.271,\ 11.534,\ 7.192,\ 4.009. \tag{42}$$

The fractional shift in the bending field in RHIC and in AGS is

$$\frac{db}{b} = \frac{dp}{p} = 1/10,000. \tag{43}$$

6

4   Electron RF frequency

The RF frequency f_E used for acceleration of the cooling electrons needs to be synchronized with the RHIC revolution frequency f. This is accomplished by imposing the constraint

$$nf = f_E \tag{44}$$

where n is a positive integer. For different frequencies f' and f'_E we would have

$$n'f' = f'_E \tag{45}$$

where n' is again a positive integer. Writing

$$n' = n + \Delta n, \quad f' = f + \Delta f, \quad f'_E = f_E + \Delta f_E \tag{46}$$

we then have

$$\frac{\Delta f_E}{f_E} = \frac{1}{nf} \left\{ (n + \Delta n)(f + \Delta f) - nf \right\} \tag{47}$$

$$\frac{\Delta f_E}{f_E} = \frac{1}{nf}(f \, \Delta n + n \, \Delta f + \Delta n \, \Delta f) \tag{48}$$

and

$$\frac{\Delta f_E}{f_E} = \frac{\Delta f}{f} \left(1 + \frac{\Delta n}{n} \right) + \frac{\Delta n}{n}. \tag{49}$$

Note that no approximations have been made in these equations; they are exact and the differences Δf_E, Δf, Δn needn't be considered small. If we take $\Delta n = 0$ then (49) simply reduces to

$$\frac{\Delta f_E}{f_E} = \frac{\Delta f}{f}. \tag{50}$$

The fractional shifts $\Delta f / f$ required to produce a fractional shift of 1/10,000 in gold ion momentum (at fixed radius) are listed in (34). Using these in (50) we have, **in parts per million,**

$$\frac{\Delta f_E}{f_E} = 5.849, \ 4.188, \ 2.622, \ 1.627, \ 0.9035 \tag{51}$$

for gold ion energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon, respectively. Taking the nominal value

$$f_E = 704 \text{ MHz} \tag{52}$$

7

then gives, **in KHz,**

$$\Delta f_E = 4.118, \ 2.948, \ 1.846, \ 1.145, \ 0.636. \tag{53}$$

These are the shifts in f_E that would be required to give $dp/p = 1/10,000$ (at fixed radius) while keeping n fixed.

If, on the other hand, we want to keep f_E fixed in (44) while varying f, then the integer n must change and only discrete changes in f are possible. These are obtained by setting $\Delta f_E = 0$ in (49), which gives

$$\frac{\Delta f}{f} = \frac{-\Delta n}{n + \Delta n}. \tag{54}$$

The value of n here is of order 9000. Taking $\Delta n = -1$ and $n = 9001$ then gives

$$\frac{\Delta f}{f} = \frac{1}{9000}. \tag{55}$$

This is essentially the smallest fractional shift in ion revolution frequency we could make. We would then have, according to (30),

$$\frac{dp}{p} = \gamma^2 \frac{df}{f} = \frac{\gamma^2}{9000} \tag{56}$$

which gives, **in parts per 1000,**

$$\frac{dp}{p} = 1.900, \ 2.653, \ 4.237, \ 6.829, \ 12.30 \tag{57}$$

at gold ion energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon, respectively. These are essentially the smallest fractional shifts in ion momentum we could make with the radius and the electron RF frequency fixed, and **are most likely too large to be useful.**

5 Summary and conclusions

For Au79+ ions circulating in RHIC at injection with energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon, the effects of a fractional shift in momentum of $dp/p = 1/10,000$ have been calculated for the case in which the frequency is fixed and for the case in which the radius is fixed. The results are as follows:

8

5.1 For a momentum shift with frequency fixed:

1. The shifts in the closed orbit radius in RHIC are, **in mm,**

$$dR_R = 3.5691,\ 2.5554,\ 1.6001,\ 0.9927,\ 0.5513. \qquad (58)$$

From left to right, these numbers correspond to energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon, respectively.

2. The corresponding fractional shifts in the bending field are, **in parts per 10,000,**

$$\frac{db}{b} = -29.6,\ -20.9,\ -12.7,\ -7.52,\ -3.73. \qquad (59)$$

3. We also have, **in parts per 1000,**

$$\frac{\Delta X_R}{D} = 3.0647,\ 2.1943,\ 1.3740,\ 0.8525,\ 0.4734 \qquad (60)$$

where ΔX_R is the shift in the closed orbit at a point in the lattice where the periodic dispersion is D.

4. In AGS the shifts in closed orbit radius are

$$dR_A = \frac{4}{19} dR_R. \qquad (61)$$

5. The corresponding fractional shifts in the AGS bending field are, **in parts per 10,000,**

$$\frac{db}{b} = -3.23,\ -2.03,\ -0.895,\ -0.175,\ +0.347. \qquad (62)$$

In (59) we see that at the lower energies in RHIC, the fractional shifts in bending field are much larger than the fractional momentum shift. This follows from the fact that at fixed frequency we have

$$\frac{db}{b} = \left(\frac{\gamma^2 - \gamma_t^2}{\gamma^2}\right) \frac{dp}{p}. \qquad (63)$$

This expression shows that db/b is large when gamma is well below transition gamma, which is what we have at the lower energies. The large values of db/b then produce large shifts in the closed orbit position as shown by equations (14), (15) and (60). These shifts can make circulating beam scrape against apertures in RHIC for fractional momentum shifts that exceed a few parts in 10,000. **This limits the usefulness of keeping the frequency fixed.**

5.2 For a momentum shift with radius fixed:

1. The shifts in RHIC RF frequency hf with $h = 120$ are, **in Hz,**

$$d(hf) = 53.258,\ 38.466,\ 24.282,\ 15.142,\ 8.4397. \qquad (64)$$

   From left to right, these numbers correspond to energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon, respectively.

2. The shifts in AGS RF frequency $h_A f_A$ with $h_A = 12$ are, **in Hz,**

$$d(h_A f_A) = 25.30,\ 18.27,\ 11.53,\ 7.192,\ 4.009. \qquad (65)$$

3. The fractional shift in the bending field in RHIC and AGS is

$$\frac{db}{b} = \frac{dp}{p} = 1/10,000. \qquad (66)$$

Comparing (59) and (66) we see that in RHIC at the four lower energies the fractional shift db/b in the bending field is much smaller for the case in which the radius is fixed. In practice, a change in momentum can be made simply by incrementing the programmed bending field while keeping the programmed radius fixed. Because the radius is fixed, there is no danger of circulating beam scraping against apertures. **This is therefore a simple and practical way to make small adjustments in the ion momentum.**

5.3 Electron RF frequency

The RF frequency f_E used for acceleration of the cooling electrons needs to be synchronized with the RHIC revolution frequency f. This is accomplished by imposing the constraint

$$nf = f_E \qquad (67)$$

where n is a positive integer.

If we want to make fractional momentum shift $dp/p = 1/10,000$ with the radius fixed and with n fixed, then we must have, **in parts per million,**

$$\frac{\Delta f_E}{f_E} = 5.849,\ 4.188,\ 2.622,\ 1.627,\ 0.9035 \qquad (68)$$

for gold ion energies 3.85, 4.55, 5.75, 7.30, and 9.79596126523 GeV per nucleon, respectively. Taking the nominal value

$$f_E = 704 \text{ MHz} \tag{69}$$

then gives, **in KHz,**

$$\Delta f_E = 4.118, \ 2.948, \ 1.846, \ 1.145, \ 0.636. \tag{70}$$

As of this writing, these shifts in f_E are thought to be achievable without too much trouble.

In **Section 4** it is shown that if we keep f_E fixed in (67) and vary n, then the resulting discrete values of dp/p most likely will be too large to be useful.

References

[1] F. Carlier, et al, "Radiative Recombination Detection to Monitor Electron Cooing Conditions during Low Energy RHIC Operations," TUPMR007, Proceedings of IPAC2016, Busan, Korea, pp. 1239–1242.