

Dispersion and electron cooling

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Dispersion and Electron Cooling

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It is known [1] that dispersion can be used to redistribute the electron cooling force. This note presents a simple model explaining why. Some addition details relevant to implementing a more detailed analysis are included.

I. THE COOLING FORCE

For non-magnetized cooling the force on an ion is a generalization of the Bethe-Bloch equation. The general form for the force is given by

$$\mathbf{F}(\mathbf{r}, \mathbf{u}) = 4\pi Z^2 r_e^2 m_e c^2 \ln \Lambda \int d^3 u_1 f_e(\mathbf{r}, \mathbf{u}_1) \frac{\mathbf{u}_1 - \mathbf{u}}{|\mathbf{u}_1 - \mathbf{u}|^3} \quad (1)$$

where Z is the ion atomic number, r_e is the classical electron radius, m_e is the electron mass, c is the speed of light, $\ln \Lambda$ is the Coulomb logarithm, and $\mathbf{u} = \mathbf{v}/c$ in the frame comoving with the ion beam. The electron phase space distribution is $f_e(\mathbf{r}, \mathbf{u})$ and $\int f_e(\mathbf{r}, \mathbf{u}_1) d^3 u_1 = n_e(\mathbf{r})$ is the local number density of electrons in the comoving frame. Note that $u_z = \Delta p/p \equiv \delta$, $u_x = \beta\gamma x'$, and $u_y = \beta\gamma y'$ where δ , x' and y' are the usual lab frame coordinates. Note that the integral in eq (1) has the form of an electrostatic field. We limit discussion to a gaussian distribution with $f_e \propto f_{ex} f_{ey} f_{ez}$ where

$$f_{ex} = \exp \left\{ \frac{-1}{2\epsilon_x} \left[\frac{1 + \alpha_x^2}{\beta_x} (x - D\delta)^2 + 2\alpha_x (x - D\delta) x' + \beta_x x'^2 \right] \right\} \quad (2)$$

$$f_{ey} = \exp \left\{ \frac{-1}{2\epsilon_y} \left[\frac{1 + \alpha_y^2}{\beta_y} y^2 + 2\alpha_y y y' + \beta_y y'^2 \right] \right\} \quad (3)$$

$$f_{ez} = \exp \left\{ -\frac{\delta^2}{2\sigma_p^2} - \frac{s_0^2}{2\sigma_s^2} \right\}. \quad (4)$$

The equations are expressed in laboratory coordinates but we take the longitudinal position with respect to the ion bunch s_0 as fixed in time since they move together. The functions $\beta_x(s)$ etc. vary as the electron bunch traverses the cooling section. The density in the comoving frame, expressed in lab frame coordinates is $n_e(\mathbf{r})$,

$$n_e(\mathbf{r}) = \int dx' dy' d\delta f_e(x, y, s_0, s)/\gamma = n_{e0} \exp \left[-\frac{x^2}{2(\epsilon_x \beta_x + D^2 \sigma_p^2)} - \frac{y^2}{2\epsilon_y \beta_y} - \frac{s_0^2}{2\sigma_s^2} \right] \quad (5)$$

where n_{e0} is the maximum density in the comoving frame. The velocity distribution in the rest frame requires a bit of work to get in standard form. One finds

$$f_v(u_x, u_y, u_z) = \frac{\exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\frac{(u_x - \bar{u}_x)^2}{\sigma_1^2} + \frac{(u_z - \bar{u}_z)^2}{\sigma_3^2} - 2\rho \frac{(u_x - \bar{u}_x)(u_z - \bar{u}_z)}{\sigma_3 \sigma_1} \right] - \frac{(u_y - \bar{u}_y)^2}{2\sigma_2^2} \right\}}{(2\pi)^{3/2} \sigma_1 \sigma_2 \sigma_3 \sqrt{1-\rho^2}}. \quad (6)$$

The various parameters in eq (6) are given by:

$$\bar{u}_x = -\frac{\gamma x \alpha_x \epsilon_x}{\epsilon_x \beta_x + D^2 \sigma_p^2}$$

$$\bar{u}_y = -\frac{\gamma y \alpha_y}{\beta_y}$$

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$$\begin{aligned}
\bar{u}_z &= \frac{Dx\sigma_p^2}{\epsilon_x\beta_x + D^2\sigma_p^2} \\
\sigma_1^2 &= \frac{\epsilon_x\gamma^2}{\beta_x} \left(1 + \frac{\alpha_x^2 D^2 \sigma_p^2}{\epsilon_x\beta_x + D^2\sigma_p^2} \right) \\
\sigma_2^2 &= \frac{\epsilon_y\gamma^2}{\beta_y} \\
\sigma_3^2 &= \frac{\sigma_p^2 \epsilon_x \beta_x}{\epsilon_x\beta_x + D^2\sigma_p^2} \\
\rho &= \frac{\alpha_x D \sigma_p}{\sqrt{\epsilon_x\beta_x + D^2\sigma_p^2(1 + \alpha_x^2)}}
\end{aligned}$$

To be explicit we note that $f_e(\mathbf{r}, \mathbf{u}) = C_n f_{ex} f_{ey} f_{ez} = \gamma n_e(\mathbf{r}) f_v(\mathbf{u}, \mathbf{r})$ with C_n a normalization constant. The point of redefining the distribution was so that the local distribution for \mathbf{u} was multiplied by the local number density. We now go on to present a simple model of how dispersion can be used to change the cooling rates.

Equations (5) and (6) are good for computer work but difficult analytically. To simplify the presentation we assume

$$f_e = n_e(\mathbf{r}) \exp \left\{ - (u_x^2 + u_y^2 + [u_z - kx]^2) / 2\sigma_u^2 \right\} / (2\pi)^{3/2} \sigma_u^3. \quad (7)$$

where the rms velocity spread is the same in all directions and the average longitudinal longitudinal velocity of the electrons varies with horizontal position due to dispersion. In addition to this we will only work to leading order in the beam frame velocities. In the comoving frame the change in \mathbf{u} for an ion at location \mathbf{r} is

$$\Delta \mathbf{u}(\mathbf{r}, \mathbf{u}) = \frac{2\sqrt{2\pi} Z^2 r_e^2 m_e \ln \Lambda \ell n_e(\mathbf{r})}{3\beta\gamma\sigma_u^3 m_I} (\hat{z} kx - \mathbf{u}) \approx C(\hat{z} kx - \mathbf{u}) \quad (8)$$

where ℓ is the length of the cooling section in the lab frame and we have assumed the cooling section is short so that the change in beam parameters is negligible over the cooling section. The new coefficient C is just the maximum cooling gain and

$$k = \frac{D\sigma_p^2}{\epsilon_x\beta_x + D^2\sigma_p^2}, \quad (9)$$

is defined above in \bar{u}_z .

Now consider an ion with initial energy error δ and initial horizontal emittance

$$\epsilon_{x0} = (x_0 - D_i \delta_0)^2 / 2\beta_{ix} + \beta_{ix} x_0'^2 / 2$$

after traversing the cooling section

$$\epsilon_{x1} = (x_1 - D_i \delta_1)^2 / 2\beta_{ix} + \beta_{ix} x_1'^2 / 2.$$

For a short cooling section $x_0 = x_1 = x$, $\delta_1 - \delta_0 = C(kx - \delta_0)$, and $x_1' = (1 - C)x_0'$. Substituting these in the formulas above and keeping first order terms the expressions for the change of a single particle are

$$\Delta \epsilon_x = \frac{-CD_i}{\beta_{ix}} (x - D_i \delta) (kx - \delta) - C\beta_{ix} x'^2 \quad (10)$$

$$\Delta \delta = -C(\delta - kx). \quad (11)$$

Set $x = x_\beta + \delta D_i$ with x_β the betatron amplitude and insert this in equations (10) and (11). Then average over betatron phase giving

$$\Delta \epsilon_x = -C(1 + kD_i) \epsilon_x \quad (12)$$

$$\Delta \delta = -C(1 - kD_i) \delta. \quad (13)$$

For positive kD_i the horizontal cooling is increased at the expense of the longitudinal. This agrees with sections 1.3 and 1.4 in [1]. The value added here is the explicit expression in equation (9) and a somewhat shorter derivation.

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[1] Y.S. Derbenev arXiv:1703:09735v2[physics.acc-ph]