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Combined Function Magnets in the eRHIC Electron Storage Ring

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Abstract

The maximum electron beam intensity, and therefore the luminosity, of the electron-ion collider eRHIC is limited by the available RF power, which has been chosen as 10 MW. To maximize the beam intensity in the presence of this limitation the bending radius needs to be chosen as large as possible. While this is generally accomplished by a FODO lattice in the arcs due to its high dipole packing factor, the quadrupoles in a separated-function lattice do not contribute to bending. The use of combined-function magnets instead of regular quadrupoles would allow the quadrupoles to contribute to the bending as well, which increases the actual bending radius of the machine. However, this measure also affects the damping distribution in the machine, and may even result in anti-damping, making the lattice not suitable as a storage ring. In the following we demonstrate how to convert a separated-function lattice to a combined-function lattice while simultaneously preserving the damping distribution.

1 Theory

The damping distribution in an electron storage ring is characterized by the parameter

$$\mathcal{D} = \frac{\oint \left[\frac{D}{R}\left(2k + \frac{1}{R^2}\right)\right] \mathrm{d}s}{\oint \frac{\mathrm{d}s}{R^2}}, \tag{1}$$

where D, R, and k denote the local dispersion, bending radius, and quadrupole strength.

In a storage ring with separate-function magnets, the quadrupoles do not contribute to this integral because $1/R = 0$ inside the quadrupoles.

In a lattice with combined-function magnets, both $1/R \neq 0$ and $k \neq 0$ at the quadrupoles, which in general modifies the damping distribution. However, the lattice can be configured in such a way as to not alter the damping distribution.

To first order, when replacing the quadrupoles in a separated-function machine with combined-function magnets, only the contribution of the quadrupoles themselves to the integral in Equation (1) changes,

$$\Delta\mathcal{D} = \frac{\oint \frac{D}{R} \cdot 2k \,\mathrm{d}s}{\oint \frac{\mathrm{d}s}{R^2}}. \tag{2}$$

In the following we assume that these combined function magnets are realized as transversely offset focusing and defocusing quadrupoles of equal length l with equal but opposite strengths $\hat{k}$ and $\check{k}$. The dispersion at these quadrupoles is denoted as $\hat{D}$ and $\check{D}$, and the local bending radius as $\hat{R}$ and $\check{R}$, respectively. We can thus rewrite the numerator of Equation (2) in good approximation as

$$l\frac{\hat{D}}{\hat{R}} \cdot 2\hat{k} + l\frac{\check{D}}{\check{R}} \cdot 2\check{k} = 0, \tag{3}$$

where we impose the condition that the damping distribution be unchanged by the introduction of combined-function magnets.

Replacing the inverse bending radii by

$$\frac{1}{\hat{R}} = \hat{k}\Delta\hat{x} \tag{4}$$

and

$$\frac{1}{\check{R}} = \check{k}\Delta\check{x}, \tag{5}$$

where $\Delta\hat{x}$ and $\Delta\check{x}$ are the transverse offsets of the respective quadrupoles, and assuming

$$\check{k} = -\hat{k}, \tag{6}$$

we can express Equation (3) as

$$\hat{D} \cdot 2 \cdot \hat{k}^2 \cdot \Delta\hat{x} + \check{D} \cdot 2 \cdot \hat{k}^2 \cdot \Delta\check{x} = 0 \tag{7}$$

$$\Rightarrow \hat{D} \cdot \Delta\hat{x} = -\check{D} \cdot \Delta\check{x}. \tag{8}$$

Thus, offsetting the quadrupoles in opposite directions, with the offsets scaling as the inverse dispersion at their location, leaves the damping distribution unchanged.

2 Numerical Example

The arc quadrupoles in the eRHIC electron storage ring have a length of $l_\mathrm{q} = 0.6\,\mathrm{m}$ and a strength of approximately $k = \hat{k} = -\check{k} = 4 \cdot 10^{-2}\,\mathrm{m}^{-2}$. The dispersion values at the quadrupoles are $\hat{D} = 0.95\,\mathrm{m}$ and $\check{D} = 0.55\,\mathrm{m}$, respectively.

The arc dipoles have a length of $l_\mathrm{d} = 6\,\mathrm{m}$ and a bending radius of $\rho_\mathrm{d} = 250\,\mathrm{m}$. To achieve the same bending radius in the vertically focusing quadrupoles these would therefore need to be offset by

$$\Delta\check{x} = \frac{1}{k \cdot \rho_\mathrm{d}} \tag{9}$$

$$= \frac{1}{0.04 \cdot 250}\,\mathrm{m} \tag{10}$$

$$= 0.1\,\mathrm{m}, \tag{11}$$

and the horizontally focusing magnets by

$$\Delta\hat{x} = \frac{\check{D}}{\hat{D}}\Delta\check{x} \tag{12}$$

$$= 0.06\,\mathrm{m} \tag{13}$$

for a local bending radius of 420 m.

Such large offsets cannot be realized using the eRHIC quadrupoles as presently designed, since the required offset exceeds the pole tip radius of those magnets by a factor 2.5. This scheme would therefore require dedicated combined-function quadrupoles.