

## RHIC polarization for Runs 9-17

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# RHIC polarization for Runs 9-17

The RHIC Polarimetry Group

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## 1 Introduction

This note describes the RHIC polarization measurements for use by the collider experiments for the Run9<sup>1</sup> and later polarized proton running periods. The measurement procedure is outlined [1] and the resulting polarization parameters are defined. The systematic uncertainties for each step of the procedure are discussed and estimated; when possible the uncertainties are evaluated using the present data. Finally the use of the provided results to determine mean polarization and uncertainty for a data set is described. The results used for this are compiled on the web pages linked at <https://wiki.bnl.gov/rhicspin/Results>; there, for each year the results are at the link 'Results'.

This is an extension of an earlier note for Runs 9-12 [2], with additional clarifications and the new results for Runs 13-17. A full list of the data sets presented here is in Table 1.

RHIC Run	$E_p$ (GeV)	species
9	100	$pp$
11	250	$pp$
12	100	$pp$
12	255	$pp$
13	255	$pp$
15	100	$pp$
15	104	$pAu$
15	104	$pAl$
17	255	$pp$

Table 1: Data sets covered in this note.

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<sup>1</sup>Earlier polarization values for Run9 100 GeV are consistent with the present analysis. The Run9 250 GeV polarization values were not reevaluated, because severe rate effects rendered the data unsuitable for analysis in the present framework.

## 2 Measurement procedure

### 2.1 Proton carbon polarimeters

The proton carbon (pC) polarimeters provide the basis of the polarization measurements. They supply the following information:

- The intensity averaged polarization of the beam,  $P = \int d^2x P(\vec{x})I(\vec{x}) / \int d^2x I(\vec{x})$ , where  $\vec{x} = (x, y)$  are the transverse beam coordinates, and  $P(\vec{x})$  and  $I(\vec{x})$  are the transverse polarization and intensity distributions, respectively.
- The transverse polarization profile parameter  $R = \sigma_I^2 / \sigma_P^2$ , the square of the ratio of the widths of the beam intensity and polarization distributions; the two pC polarimeters in each RHIC ring allow separate measurements of  $R$  in the horizontal ( $R_x$ ) and vertical ( $R_y$ ) directions;
- The pC detectors around the beam allow a measurement of both the magnitude  $P$  of the polarization and the azimuthal tilt of the spin vector at the pC polarimeters,  $\phi_{\text{pC}}$ ;
- With two or more measurements per RHIC fill the pC polarimeters measure the time dependence of  $P$  and  $R$  throughout fills, necessary for physics data collected during portions of fills;
- The two pC polarimeters in each RHIC ring allow cross checks with two independent measurements of the same beam.

It is important to note that the pC polarimeters are only sensitive to the component of the proton spin vector transverse to the beam direction, and provide no information about a possible longitudinal component.

The polarization is obtained from the measured asymmetry  $\epsilon$  via the relation  $P = \epsilon / A_N$ . The analyzing power  $A_N$  is determined for the pC polarimeters by normalizing to the hydrogen jet (H-jet) polarimeter absolute polarization values. Uncertainties from the H-jet thus contribute to a scale uncertainty on the pC measurements through the uncertainty in determining  $A_N$ .

The polarizations for single- and double-spin asymmetry (SSA and DSA) measurements with colliding beams are determined from the transverse averaged polarization  $P$  and profile parameter  $R$ . The corrections from  $P$  to  $P_{SSA}$  and  $P_{DSA}$  are scale factors which are algebraic functions of  $R$  [3]. For equal horizontal and vertical profiles  $R$ , to lowest order in  $R$ :

$$P_{SSA} \approx (1 + \frac{1}{2}R)P; \quad (1)$$

If both beams  $B$  and  $Y$  have equal profiles  $R$ , to lowest order in  $R$ :

$$P_{DSA}^2 \approx (1 + R)P_B P_Y \approx P_{SSA,B} \cdot P_{SSA,Y}. \quad (2)$$

The pC measurements for a fill are a set of polarization and profile values  $P_i$ ,  $R_{xi}$  and  $R_{yi}$ , their statistical uncertainties, and times in the fill  $t_i$ . They are fit to the forms:

$$P(t) = P_0 - P' \cdot t, \quad (3)$$

$$R_{x,y}(t) = R_{0x,y} + R'_{x,y} \cdot t. \quad (4)$$

$P_0, R_{0x,y}$  are the polarization and profiles at  $t = 0$ , usually taken as the start of a physics fill;  $P'$  is the absolute rate of polarization loss and  $R'_{x,y}$  are the rates of profile growth. These fits are performed for each pC polarimeter in use. When both polarimeters in a ring are used, they separately provide the horizontal and vertical profile parameters  $R_x(t)$  and  $R_y(t)$ ; their fit parameters and uncertainties for  $P(t)$  are combined in a weighted average to produce a linear parameterization for each beam. For the many short fills with only one pC measurement, the average values of  $P'$  and  $R'$  over the whole running period are used.

The parameters  $\{P_0, P', R_{0x}, R'_x, R_{0y}, R'_y\}$ , using the exact algebraic relations [3], are then used to determine a parameterization of the colliding beam polarizations linear in  $t$ :

$$P_{SSA}(t) = P_{0,SSA} - P'_{SSA} \cdot t. \quad (5)$$

Here  $P_{0,SSA}$  and  $P'_{SSA}$  have analogous meanings to the parameters in Eq. (3); their statistical uncertainties are determined by the statistical uncertainties on  $\{P_0, P', R_{0x}, R'_x, R_{0y}, R'_y\}$ . To lowest order in  $R$ , the DSA polarization is the product of these  $P_{SSA}$  parameterizations for the two beams as indicated in Eq. (2).

## 2.2 H-jet polarimeter

A polarized atomic hydrogen jet is used to measure the absolute polarization of the beam. In terms of measured asymmetries  $\epsilon$  with respect to the jet and the beam spin states, the transverse averaged polarization of the beam is determined:

$$P_{\text{beam}} = -\frac{\epsilon_{\text{beam}}}{\epsilon_{\text{jet}}} P_{\text{jet}}. \quad (6)$$

The polarization of the hydrogen jet  $P_{\text{jet}}$  is measured with a Breit-Rabi polarimeter. It is largely constant, and a mean value is used for each running period.

The H-jet polarimeter measures the beam intensity weighted average of  $P_{\text{beam}}$  over a fill:

$$P_{\text{H-jet}} \equiv \overline{P_{\text{beam}}} = \frac{\int dt I(t) P(t)}{\int dt I(t)}, \quad (7)$$

where  $P_{\text{H-jet}}$  is the result from the H-jet for each beam in each fill and  $I(t)$  is the proton beam intensity as a function of time.

### 2.2.1 Spin tilt correction

It is important to note the measurement capabilities of the pC and H-jet polarimeters:

- The pC polarimeters have six detectors arranged azimuthally around the proton beam. This allows a measurement of the spin vector in the plane transverse to the beam; thus, the polarization may be expressed as a magnitude  $P$  and tilt angle at the pC polarimeters  $\phi_{\text{pC}}$ , where  $\phi_{\text{pC}} = 0$  for a vertical spin vector.
- The H-jet has only two detector stations in the horizontal plane of the beam. Thus, asymmetries between these two detector stations measure only the vertical component of the transverse spin vector at the H-jet,  $P \cos \phi_{\text{H-jet}}$ .

Spin tracking analysis shows that the spin vector tilt does not change significantly in the 71 m between the pC and H-jet polarimeters [4], *i.e.*  $\phi_{\text{H-jet}} = \phi_{\text{pC}}$ . Thus the tilt angle measurements from the pC polarimeter can be used to correct the jet measurement to the polarization magnitude:  $P = P_{\text{H-jet}} / \cos \phi_{\text{pC}}$ .

The spin tilt measured by the pC polarimeter was found to be constant for each ring throughout a running period [5]. The mean values are tabulated in Table 2. For beam energies other than 255 GeV the spin tilt correction is less than 1% and was not applied. The spin tilt analysis was not available at the time the Run12 255 GeV polarizations were tabulated and they have not been corrected for this effect. To date, the spin tilt correction has been applied to the Run13 and Run17 255 GeV polarization results; the extreme case was the Run13 Blue beam with a correction of 4%.

$\phi_{\text{pC}}(^{\circ})$	Blu	Yel
Run9-100	6	5
Run11-250	3	1
Run12-100	3	3
Run12-255	11	7
Run13-255	16	9
Run15-100 pp	3	2
Run15-104 pAu	0	-
Run15-104 pAl	1	-
Run17-255	12	8

Table 2: Spin tilts measured by the pC polarimeter.

### 2.3 pC/H-jet normalization

To compare directly with the jet measurement, the beam intensity weighted average polarization from the pC is computed for each fill in terms of the parameters  $P_0$  and  $P'$  in

Eq. (3):

$$\overline{P}_{\text{pC}} = \left( 1 - \left( \frac{P'}{P_0} \right) \cdot \frac{\int dt t I(t)}{\int dt I(t)} \right) P_0. \quad (8)$$

Note that the ratio  $P'/P_0$  is independent of the pC polarization scale. The RHIC archive values of beam intensities are used to numerically evaluate the terms involving  $I(t)$ .

Over a set of fills the relative pC/H-jet normalization  $s$  is determined from a statistically weighted mean of the ratio:

$$s = \left\langle \frac{\overline{P}_{\text{pC}}}{P_{\text{H-jet}}} \right\rangle_{\text{fills}}, \quad (9)$$

with  $P_{\text{H-jet}}$  in some periods corrected for spin tilt as discussed in Section 2.2.1. The scale factor  $s$  is then applied to all pC polarization values to adjust them to the scale set by the H-jet.

For most running periods, a separate normalization was determined for each pC polarimeter, using all fills with both pC and H-jet measurements. For the Blue downstream pC polarimeter in Run11, a set of fills when a thick carbon target was used showed a significant deviation in scale; a separate normalization was determined for these data. In Run17, the carbon targets exhibited unprecedented lifetime, allowing dozens to hundreds of measurements with each target. This allowed a statistically significant normalization to be determined for each carbon target in Run17.

## 2.4 Special treatment Run13: $P_0$ from H-jet

In Run13 the pC polarimeters experienced a high loss of carbon ribbon targets, requiring two replacements of the target sets during the run. This resulted in a few periods when there were no viable targets and thus no pC measurements for one of the beams. Many fills in these periods were long enough to provide a statistically significant H-jet measurement. As described in Section 2.2, the H-jet measures the beam intensity averaged polarization throughout a fill:

$$P_{\text{H-jet}} = \frac{\int dt I(t) P(t)}{\int dt I(t)} = P_0 + P' \cdot \frac{\int dt t I(t)}{\int dt I(t)}, \quad (10)$$

where  $I(t)$  is the beam intensity throughout a fill. This allows a determination of the initial polarization for experiments, using run average values for unmeasured parameters:

$$P_{0,SSA} = \left( P_{\text{H-jet}} - \overline{P'} \cdot \frac{\int dt t I(t)}{\int dt I(t)} \right) \cdot \left( 1 + \frac{1}{2} \overline{R_0} \right). \quad (11)$$

Here  $\overline{P'}$  is the Run13 average of polarization decay, and  $\overline{R_0}$  the Run13 average of initial profile parameter. RHIC archive values of beam intensities are used to numerically evaluate the term involving  $I(t)$ . The values of  $P_{0,SSA}$  so determined are included in the tabulated results, highlighted in red.

### 3 Uncertainties

#### 3.1 H-jet scale and background

The scale of the H-jet polarization is provided by the Breit-Rabi polarimeter measurement of the atomic jet polarization. The measured H-jet asymmetries may be affected by contamination of the jet with molecular hydrogen  $H_2$ , which is not measured by the Breit-Rabi polarimeter and is unpolarized. The  $H_2$  contamination was measured in a test bench configuration to be approximately 2%, and the Breit-Rabi measurement is corrected for this. Since this measurement was performed only once several years ago, and never *in situ*, the uncertainty on polarization scale from this effect is conservatively taken to be 3%. This molecular background value (2%) and uncertainty (3%) apply to the polarimetry analysis for Runs 9-15. During Run17, the atomic dissociator was turned off during two RHIC fills, leaving only molecular  $H_2$  in the jet. Measurements in this configuration determined that in normal H-jet operation the atomic jet had a negligible contamination of 0.06%  $H_2$ , with negligible uncertainty.

Equation (6) is defined for elastic proton-proton scattering where the analyzing power is the same for jet and beam asymmetries. Background that is not separated from elastic events will dilute the measured asymmetries, but it will not affect the determination of the beam polarization if the background is not spin orientation (up or down) dependent. In Runs 9-13, the orientation dependent effect on the beam polarization measurement has been estimated to be less than 1%. In Run 15, the improved statistics allowed a full background correction of the jet and beam asymmetries. For Run17, collimators were removed to study diffuse  $H_2$  background in the scattering chamber. The removal of the collimators made a direct measurement of the background impossible. The background was estimated indirectly by varying selection cuts, with a relative uncertainty of approximately 1% on the final polarization values.

The uncertainties due to atomic hydrogen jet polarization and backgrounds for different running periods are listed in Table 3.

$\sigma(P)/P$ (%)	scale	Blu bkg.	Yel bkg.
Runs 9-13	3.	1.	1.
Run 15	3.	0.	0.
Run 17	0.	1.	1.3

Table 3: Relative systematic uncertainties on H-jet measurements due to jet polarization and backgrounds.



### 3.2 pC scale

The pC/H-jet ratios  $\overline{PpC}/\overline{Pjet}$  [6] averaged in Eq. (9) are proportional to the pC analyzing power  $A_N$ . This should be constant within uncertainties. If fit to a constant using only statistical uncertainties, a value of  $\chi^2/NDOF > 1$  indicates fill-to-fill systematic uncertainties on the ratios; these effects may be due to instabilities in either the pC or H-jet or both. The size of this effect may be estimated by including a constant systematic for each fill in the  $\chi^2$  calculation and requiring  $\chi^2/NDOF = 1$ . The values so obtained are listed in Table 4. Many of them are zero, indicating the systematic uncertainty is negligible in comparison to the statistical uncertainties with typical values of  $\approx 9\%$ . Asterisks in table entries indicate when there were known instabilities in a polarimeter; these account for many of the nonzero values.

$\sigma(P)/P$ (%)	B up	B dn	Y up	Y dn
Run9-100	0.	0.	0.	0.
Run11-250	2.6	2.5*	0.	0.*
Run12-100	0.	0.*	6.4*	0.*
Run12-255	0.	3.3*	5.6*	3.3*
Run13-255	0.	0.	0.	6.6
Run15-100 pp	0.	0.	0.6	4.3
Run15-104 pAu	5.	0.	-	-
Run15-104 pAl	0.	0.	-	-
Run17-255	0.	1.4	4.5	0.

Table 4: Relative fill-to-fill systematic uncertainties on the pC/H-jet ratio. Asterisks indicate there were known instabilities in a pC polarimeter.

After including possible systematic uncertainties, the mean in Eq. (9) is re-evaluated, with a possibly increased uncertainty. The overall relative uncertainties on this mean are listed for the individual polarimeters in the left columns of Table 5. For most fills the polarization of one beam is the average of the up- and downstream polarimeters; the relative uncertainty on  $A_N$  for this average is listed in the rightmost two columns of Table 5. These are also the overall relative uncertainties on  $A_N$  for one ring, and contribute a scale uncertainty to the pC measurements. They incorporate the statistical uncertainties of the H-jet and pC from an entire running period, and all fill-to-fill systematic uncertainties from both.

### 3.3 pC fill-to-fill systematics

The pC analyzing power  $A_N$  has a steep dependence on the energy of the scattered carbon nuclei; the measurement is thus sensitive to the energy scale of the measured nuclei.

$\sigma(A_N)/A_N$ (%)	B up	B dn	Y up	Y dn	Blu	Yel
Run9-100	1.6	1.1	1.2	1.1	1.0	0.8
Run11-250	1.4	1.8	1.3	1.7	1.1	1.1
Run12-100	1.3	2.0	2.0	1.5	1.2	1.3
Run12-255	1.3	2.2	2.0	1.6	1.3	1.3
Run13-255	1.0	0.9	1.0	1.1	0.7	0.8
Run15-100 pp	0.6	0.6	0.6	0.8	0.4	0.5
Run15-104 pAu	1.2	0.9	-	-	0.7	-
Run15-104 pAl	1.7	1.6	-	-	1.1	-
Run17-255	0.6	0.7	0.7	0.6	0.5	0.5

Table 5: Overall relative uncertainties (stat.  $\oplus$  syst.) on the pC analyzing power  $A_N$ .

Leading sources of systematic shifts in this energy scale include the dead layer of the Si detectors, and varying energy loss of nuclei in the carbon target en route to the detectors.

For most fills each RHIC beam has (intensity averaged) polarization measurements from both the up- and down-stream pC polarimeters. Measuring the same beam, they should yield the same polarization, within uncertainties [7]. Possible fill-to-fill systematic uncertainties may be estimated by requiring the ratio to be constant, adjusting  $\chi^2/NDOF = 1$  as described in Section 3.2. The contribution of these uncertainties to the polarization scale are listed in Table 6. They are small or negligible compared to the statistical uncertainties on the ratios from each fill with typical values of 5-10%. Note also that these uncertainties are already incorporated in the uncertainties on  $A_N$  in Table 5 through the pC/H-jet ratio used to determine  $A_N$ .

$\sigma(P)/P$ (%)	Blu	Yel
Run9-100	0.	1.0
Run11-250	3.2	0.9
Run12-100	0.	2.8
Run12-255	0.	3.1
Run13-255	2.8	1.4

$\sigma(P)/P$ (%)	Blu	Yel
Run15-100 pp	0.	0.
Run15-104 pAu	1.3	-
Run15-104 pAl	0.	-
Run17-255	0.	2.6

Table 6: Relative fill-to-fill systematic uncertainties on the pC polarization as estimated from the upstream/downstream ratio.

### 3.4 Profile correction procedure systematics

The profile parameter  $R$  is determined from a fit of the polarization versus intensity (rate) distribution:  $P(I) = P_{max} \cdot (I/I_{max})^R$  [8]. The fit parameters  $P_{max}$  and  $R$  determine  $P_{avg}$ , the average polarization across the beam:  $P_{avg} = P_{max}/\sqrt{1+R}$ . This may be compared to the directly measured average from a sweep measurement  $P$ ; differences are due to systematic effects of the profile correction procedure. This is used to estimate the uncertainty of the correction for colliding beams. Based on this study the fill-to-fill relative uncertainty on the profile correction is 2.2%.

## 4 Use of results

### 4.1 Tabulated parameters

The results of the polarization measurements are compiled on web pages [9]. Polarization values and statistical uncertainties for SSA with each beam are listed. For each the initial value and slope of the parameterization  $P(t) = P_0 - P' \cdot t$  are provided<sup>2</sup>; a Unix time stamp value for  $t = 0$  in this parameterization is also included. When there was only one polarization measurement in a fill, the mean values of  $P'$  and  $R'$  for that ring and running period are used. For some sets, a beam current weighted mean polarization is also listed:  $P_{Avrg} = \int dt I(t) P(t) / \int dt I(t)$ .

### 4.2 Mean polarization

For each fill  $i$  in a data set a time dependent luminosity  $L_i(t)$  is required; it should include effects such as deadtimes, varying trigger prescales etc. The appropriate  $P_i(t)$  from the web page is also needed. When available the initial and slope values should be used:  $P_i(t) = P_{0,i} - P'_i \cdot t$ . Fills with only a mean polarization were typically short and may be approximated as a constant:  $P_i(t) = P_{Avrg,i}$ . It is convenient to define the mean luminosity weighted polarization for fill  $i$ :

$$\mathcal{P}_i = \frac{1}{\mathcal{L}_i} \int dt L_i(t) P_i(t) = P_{0,i} - \frac{\int dt t L_i(t)}{\mathcal{L}_i} P'_i, \quad (12)$$

where  $\mathcal{L}_i = \int dt L_i(t)$  is the total luminosity for fill  $i$ . Then the polarization for the data set is determined from the luminosity weighted average over fills  $i$ :

$$P_{set} = \frac{\sum_i \mathcal{L}_i \cdot \mathcal{P}_i}{\sum_i \mathcal{L}_i}. \quad (13)$$

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<sup>2</sup>The slope values listed in the tables ('Slope' or 'dP/dT') are opposite in sign from  $P'$  in this note.

### 4.3 Polarization uncertainty

There are several contributions to the overall uncertainty on  $P$ . Each component may vary according to ring and running period. It is convenient to separate them into an overall scale uncertainty for a given running period, and a fill-to-fill uncertainty for subsets of a running period.

#### 4.3.1 Overall scale uncertainty

The contributions to the overall scale uncertainty are:

- H-jet scale: For SSA, the scale uncertainty is in the second column of Table 3. For DSA, the scale is fully correlated between the two beams and the uncertainty is twice this value.
- H-jet background: For SSA, the background uncertainty is listed in the rightmost two columns of Table 3. For DSA, the background is fully correlated between the two beams and the uncertainty is the sum of the Blu and Yel values.
- pC scale: The appropriate value for  $\sigma(\text{pC scale})/P$  for each beam is listed in the rightmost two columns of Table 5. For DSA the uncertainties for the two beams are taken as uncorrelated and are added in quadrature,  $\sigma(\text{pC scale})/P = \sigma(\text{Blu-pC scale})/P \oplus \sigma(\text{Yel-pC scale})/P$ .

The contributions are added in quadrature, giving the relative scale uncertainties for each running period listed in Table 7.

$\sigma(\text{scale})/P$ (%)	SSA-Blu	SSA-Yel	DSA
Run9-100	3.3	3.3	6.5
Run11-250	3.3	3.3	6.5
Run12-100	3.4	3.4	6.6
Run12-255	3.4	3.4	6.6
Run13-255	3.2	3.3	6.4
Run15-100 pp	3.0	3.0	6.0
Run15-104 pAu	3.1	-	-
Run15-104 pAl	3.2	-	-
Run17-255	1.1	1.4	2.4

Table 7: Overall scale relative uncertainties on polarization.

### 4.3.2 Fill-to-fill uncertainties

The contributions to the fill-to-fill uncertainties are:

- Fill-to-fill statistical and systematic uncertainties [10]: Equations (12,13) may be used to determine the fill-to-fill uncertainty on  $P$  through usual propagation of errors, taking the statistical uncertainties on  $P_{0,i}$  and  $P'_i$  from the web page. The systematic fill-to-fill uncertainties from Table 6 should be added to the statistical uncertainties in quadrature. For DSA the Blu and Yel SSA values should be added in quadrature. For example, for the polarization of a single fill  $\mathcal{P}_i$  in Eq. (12):

$$\sigma(\mathcal{P}_i) = \sigma(P_{0,i}) \oplus \frac{\int dt t L_i(t)}{\mathcal{L}_i} \cdot \sigma(P'_i) \oplus \mathcal{P}_i \cdot (\sigma(P)/P)_{\text{Table 6}} \quad (14)$$

and for the polarization of a set of fills  $P_{set}$  in Eq. (13):

$$\sigma(P_{set}) = \frac{\oplus_i \mathcal{L}_i \cdot \sigma(\mathcal{P}_i)}{\sum_i \mathcal{L}_i}. \quad (15)$$

However, this leads to double counting of uncertainties, since they already contribute to  $\sigma(\text{scale})$  through the uncertainties on  $A_N$  in Table 5. (Recall that the uncertainties on  $A_N$  incorporate all statistical and systematic uncertainties from both the H-jet and pC for an entire running period, as described in Section 3.2.) The  $A_N$  were evaluated using nearly entire run periods, so the overcounting is significant when the data set used for a measurement is an appreciable fraction of the run period. An approximate correction for the overcounting should be applied; since the errors are fill-to-fill the correction depends on the numbers of fills used. Suppose that  $N$  fills in the entire run period were used to determine  $A_N$ , and  $M$  fills are in the data set for the measurement, with  $M \leq N$ . The correction for overcounting is a scale factor  $\sqrt{1 - \frac{M}{N}}$  applied to Eq. 15, and the corrected fill-to-fill uncertainty for the polarization of a set of fills  $P_{set}$  is:

$$\sigma(\text{fill-to-fill}) = \sqrt{1 - \frac{M}{N}} \cdot \frac{\oplus_i \mathcal{L}_i \cdot \sigma(\mathcal{P}_i)}{\sum_i \mathcal{L}_i}. \quad (16)$$

The values of  $N$  for each running period are listed in Table 8. In each period there were several fills not used for the determination of  $A_N$ , usually because the fills were short and the statistics were too limited for an H-jet measurement. Thus it is possible that  $M > N$ ; in these cases it is reasonable to take  $\sigma(\text{fill-to-fill}) = 0$ .

- Profile correction: The relative uncertainty of the profiles correction for one beam in one fill is 2.2%. For a set of  $M$  fills it contributes a relative uncertainty on the polarization for an SSA measurement of  $\sigma(\text{profile})/P = 2.2\%/\sqrt{M}$ , and for a DSA measurement  $\sigma(\text{profile})/P = 3.1\%/\sqrt{M}$ .

$N$ (# fills)	Blu	Yel
Run9-100	117	116
Run11-250	65	65
Run12-100	56	55
Run12-255	49	49
Run13-255	138	139

$N$ (# fills)	Blu	Yel
Run15-100 pp	142	142
Run15-104 pAu	80	-
Run15-104 pAl	30	-
Run17-255	190	191

Table 8: Number of fill used to determine  $A_N$ .

For data sets consisting of a large fraction of a running period the fill-to-fill uncertainties are negligible.

### 4.3.3 Total uncertainty

These components of the uncertainty are then added in quadrature to give the overall uncertainty on  $P_{set}$ . Explicitly, in terms of  $\sigma(\text{scale})/P$  from Section 4.3.1, and  $\sigma(\text{fill-to-fill})$  and  $\sigma(\text{profile})/P$  from Section 4.3.2, the total uncertainty on the mean polarization for a data set is

$$\sigma(P_{set}) = P_{set} \cdot \frac{\sigma(\text{scale})}{P} \oplus \sigma(\text{fill-to-fill}) \oplus P_{set} \cdot \frac{\sigma(\text{profile})}{P}. \quad (17)$$

For data sets consisting of a large fraction of a running period the fill-to-fill uncertainties are negligible and  $\sigma(P_{set})/P_{set} = \sigma(\text{scale})/P$  from Table 7.

### 4.3.4 Scale uncertainty of different running periods

In different running periods, the pC configuration was altered, and the pC/H-jet ratios are not directly comparable. Also, there were no direct measurements of the jet H<sub>2</sub> contamination in different running periods; the 3% uncertainty for Runs 9-15 was assigned to span likely variations of the contamination between different periods. Given the lack of information, it is prudent to choose a maximally conservative estimate of the scale uncertainty when combining data from different running periods. This depends on whether identical or different measurements are being combined. Consider the example of a process measured in different kinematic regions  $A$  and  $B$ . Then:

- If region  $A$  was measured in both Run11-250 and Run12-255, select the larger of the uncertainties from Table 7, 3.4% in this case.
- If region  $A$  was measured in Run11-250, and region  $B$  in Run12-255, assign the relevant uncertainties from Table 7 to each, in this case 3.3% for  $A$  and 3.4% for  $B$ .

## References

- [1] Much more information on the polarimetry system is available on the polarimetry wiki and links therein: <https://wiki.bnl.gov/rhicspin/Polarimetry>. The tabulated polarization results are compiled on the web pages linked at <https://wiki.bnl.gov/rhicspin/Results>; there, for each year the results are at the link 'Fill by fill results'.
- [2] [http://public.bnl.gov/docs/cad/Documents/RHIC polarization for Runs 9-12.pdf](http://public.bnl.gov/docs/cad/Documents/RHIC%20polarization%20for%20Runs%209-12.pdf).
- [3] W. Fischer and A. Bazilevsky, Phys. Rev. ST Accel. Beams **15**, 041001 (2012).
- [4] <http://www.cadops.bnl.gov/AP/spinmeeting.htm>, see Feb. 25, 2015 meeting minutes and presentation 'Is It Possible?'
- [5] Spin tilts are at: <https://www.phy.bnl.gov/cnipol/summary/>. From there select a running period, and then select "Spin Angle (Radial Component) by p-Carbon". Spin tilts at both injection and store energies are shown.
- [6] H-jet/pC ratios are at: <https://www.phy.bnl.gov/cnipol/summary/>. From there select a running period, and then select "H-Jet/p-Carbon Polarization Ratio".
- [7] Upstream/downstream ratios for pC are at: <https://www.phy.bnl.gov/cnipol/summary/>. From there select a running period, and then select "Upstream/Downstream Scaled Polarization Ratio".
- [8] An example of a distribution of polarization vs. intensity and fit from one pC measurement is at:  
[https://www.phy.bnl.gov/cnipol/runs/20814.305/images//Rprof\\_ab/c\\_gepsvsintPMT.png](https://www.phy.bnl.gov/cnipol/runs/20814.305/images//Rprof_ab/c_gepsvsintPMT.png).
- [9] The results are available at <https://wiki.bnl.gov/rhicspin/Results>; there, for each year click on 'Results'.
- [10] An example calculation for the fill-to-fill uncertainty for a STAR analysis is linked at: <https://wiki.bnl.gov/rhicspin/upload/1/1c/ExampleFillToFill.pdf>.