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Bucket and bunch parameters for clean injection of low energy gold ions into RHIC

C. Gardner

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Collider Accelerator Department

Brookhaven National Laboratory

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Bucket and Bunch Parameters for Clean Injection of Low Energy Gold Ions into RHIC

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Collisions of gold nuclei in RHIC at five low energies, 3.85, 4.55, 5.75, 7.30, and 9.80 GeV per nucleon, are planned [1] in the search for a critical point in the nuclear matter phase diagram. As discussed in [2, 3, 4], having longer bunches of gold ions is advantageous for this program. The longer bunches are made possible by six new "9 MHz" cavities, three per ring, which produce harmonic 120 RF buckets. This is one third the harmonic number (360) produced by the standard "28 MHz" cavities.

Clean injection at the five energies requires that the bunches be narrow enough to fit on the flat top portion of the injection kicker pulse. It also requires that the rise and fall times of the pulse be short enough to ensure that bunches already in the machine are not affected. In this note the widths of bunches that are matched to harmonic 120 and 360 buckets at RHIC injection are calculated for various bunch areas at the five energies. The bunch areas range from 0.25 to 0.70 eV-s per nucleon. The calculated bunch widths give the time needed for the flat top portion of the kicker pulse and the time available for the kicker magnetic field to rise.

Stationary bucket areas and various other parameters at the five energies are given in **sections 1**, **2**, **3**, **4**, and **5**. These have been calculated in accordance with the formulae given in reference [5].

The relevant times associated with the injection kicker pulse are illustrated and discussed in **section 6**. Bucket and bunch parameters that satisfiy the conditions for clean injection are given in **sections 7**, **8**, and **9**. The main conclusions that follow from the data are summarized in **section 10**.

Sections 11 through **20** and **figures 8** through **20** contain additional information and should be regarded as appendices.

1 Stationary bucket area

The stationary bucket area is

$$A_S(\mathcal{H}, V) = 8\,\frac{R}{\mathcal{H}c}\left\{\frac{2eQVE}{\pi\mathcal{H}|\eta|}\right\}^{1/2} \tag{1}$$

where $\mathcal{H}$ is the harmonic number, V is the RF voltage per turn, and the other parameters are as defined in [5]. For

$$\mathcal{H} = h, \quad V = V_h \tag{2}$$

we have

$$A_S(h, V_h) = 8\,\frac{R}{hc}\left\{\frac{2eQV_hE}{\pi h|\eta|}\right\}^{1/2} \tag{3}$$

and for

$$\mathcal{H} = 3h, \quad V = V_{3h} \tag{4}$$

we have

$$A_S(3h, V_{3h}) = 8\,\frac{R}{3hc}\left\{\frac{2eQV_{3h}E}{\pi 3h|\eta|}\right\}^{1/2} \tag{5}$$

which gives bucket area ratio

$$\mathcal{R} = \frac{A_S(h, V_h)}{A_S(3h, V_{3h})} = 3\sqrt{3}\left(\frac{V_h}{V_{3h}}\right)^{1/2}. \tag{6}$$

In the standard setup of RHIC for gold-on-gold collisions, bunches of gold ions from AGS are injected into waiting stationary RF buckets produced by the "28 MHz" cavities in the machine. The cavities produce **harmonic 360** buckets and provide up to 400 KV per turn in each ring. At 400 KV the available bucket areas are 0.1987, 0.2570, 0.3704, 0.5431, and 0.8938 eV-s per nucleon for energies 3.85, 4.55, 5.75, 7.30, and 9.80 GeV per nucleon, respectively. Only the last area is sufficient to contain bunch areas that can be as high as 0.7 eV-s per nucleon.

The new "9 MHz" cavities produce **harmonic 120** buckets and provide up to 180 KV per turn in each ring. At 180 KV the available bucket areas are 0.693, 0.896, 1.291, 1.893, and 3.115 eV-s per nucleon for energies 3.85, 4.55, 5.75, 7.30, and 9.80 GeV per nucleon, respectively. Taking

$$V_h = 180\,\mathrm{KV}, \quad V_{3h} = 400\,\mathrm{KV} \tag{7}$$

in (6) gives

$$\mathcal{R} = 3.4857 \tag{8}$$

for the ratio of harmonic 120 to harmonic 360 bucket areas.

2 RHIC Injection Energies 1, 2, 3

	Energy 1	Energy 2	Energy 3	Unit
Q	79	79	79	
mc^2	183.433343902	183.433343902	183.433343902	GeV
W/A	2.91886627461	3.61886627461	4.81886627461	GeV
cp/A	3.73570475084	4.45370519741	5.67410697691	GeV
E/A	**3.850**	**4.550**	**5.750**	GeV
$B\rho$	31.0735574814	37.0458785389	47.1971692257	Tm
β	0.970312922296	0.978836307123	0.986801213376	
γ	4.13474444648	4.88651616403	6.17526767981	
η	-0.05658	-0.03997	-0.02431	
f	75.8748677296	76.5413647756	77.1641908707	KHz
h	120	120	120	
hf	9.10498412755	9.18496377307	9.25970290448	MHz
δC	0	0	0	mm

3 Corresponding AGS Extraction Parameters

	Energy 1	Energy 2	Energy 3	Unit
Q	77	77	77	
mc^2	183.434181300	183.434181300	183.434181300	GeV
W/A	2.91887959965	3.61888279525	4.81888827342	GeV
cp/A	3.73572180486	4.45372552920	5.67413288001	GeV
E/A	**3.85001757580**	**4.55002077139**	**5.75002624956**	GeV
$B\rho$	31.8808084103	38.0082826627	48.4232907832	Tm
β	0.970312922296	0.978836307123	0.986801213376	
γ	4.13474444652	4.88651616406	6.17526767985	
η	-0.04465	-0.02804	-0.01238	
h	12	12	12	
hf	4.32486746059	4.36285779221	4.39835887963	MHz
R	128.457981391	128.457981391	128.457981391	m

4 RHIC Injection Energies 4, 5, 6

	Energy 4	Energy 5	Energy 6	Unit
Q	79	79	79	
mc^2	183.433343902	183.433343902	183.433343902	GeV
W/A	6.36886627461	8.86482753983	8.86886627461	GeV
cp/A	7.24037222699	9.75160741084	9.75566450763	GeV
E/A	**7.300**	9.79596126523	**9.800**	GeV
$B\rho$	60.2253490540	**81.11378003**	81.1475269234	Tm
β	0.991831811917	0.995472230526	0.995475970166	
γ	7.83990505437	10.5204666076	10.5248040456	
η	-0.01436	-0.007126	-0.007119	
f	77.5575649979	77.8422322162	77.8425246421	KHz
h	120	120	120	
hf	9.30690779975	9.34106786594	9.34110295706	MHz
δC	0	0	0	mm

5 Corresponding AGS Extraction Parameters

	Energy 4	Energy 5	Energy 6	Unit
Q	77	77	77	
mc^2	183.434181300	183.434181300	183.434181300	GeV
W/A	6.36889534942	8.86486800852	8.86890676183	GeV
cp/A	7.24040528035	9.75165192780	9.75570904320	GeV
E/A	**7.300033325**	**9.79600598466**	**9.80004473797**	GeV
$B\rho$	61.7899259134	83.2210113689	83.2556349640	Tm
β	0.991831811917	0.995472230526	0.995475970166	
γ	7.83990505446	10.5204666071	10.5248040452	
η	-0.002429	0.004806	0.004813	
h	12	12	12	
hf	4.42078120488	4.43700723632	4.43702390460	MHz
R	128.457981391	128.457981391	128.457981391	m

6 Injection timing

Figure 1 illustrates the relevant times for the injection of gold bunches into RHIC RF buckets.

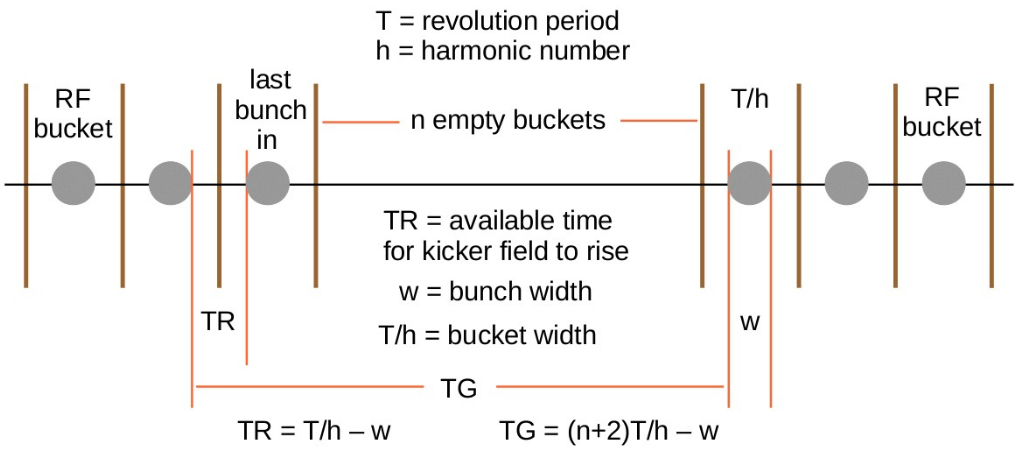

Figure 1: Injection of gold bunches into RHIC RF buckets. The revolution period is T and the RF harmonic number $h = 120$. One hundred and eleven of the 120 buckets are to be filled, leaving a gap of $n = 9$ empty buckets. For injection energies $E/A = 3.85$, 4.55, 5.75, 7.30, and 9.80 GeV per nucleon, the RF bucket widths are $T/h = 109.83$, 108.87, 107.99, 107.45, and 107.05 ns, respectively. The **time available** for the kicker magnetic field to rise is $T_R = T/h - w$, where w is the **width** of the bunch in the bucket. For injection of the last bunch, the kicker pulse must fit inside the **time gap** $T_G = (n + 2)T/h - w$.

The **actual rise time** and **flat top portion** of the pulsed kicker field have been measured for the setup in which a 40 ohm terminating resistor is used in the kicker modules instead of the standard 25 ohm resistor. This produces a "rise time of **55-60 ns** and a flat region (with small slope) of about **50 ns**" as reported in [6]. The incoming bunch from AGS therefore will be kicked cleanly into the RHIC acceptance if the time available for the rise of the kicker field satisfies

$$T_R \geq 60 \text{ ns} \tag{9}$$

and the bunch width satisfies

$$w \leq 50 \text{ ns}. \tag{10}$$

Since the bunch is contained in an RF bucket of width T/h we have

$$0 < T/h - w < T/h \tag{11}$$

and it follows that the available time gap T_G satisfies

$$(n+1)T/h < T_G < (n+2)T/h. \tag{12}$$

For $n = 9$ this gives

$$10T/h < T_G < 11T/h \tag{13}$$

where $T/h = 109.83$, 108.87, 107.99, 107.45, and 107.05 ns for energies $E/A = 3.85$, 4.55, 5.75, 7.30, and 9.80 GeV per nucleon, respectively. Figures 1 and 2 of reference [6] show that the kicker pulse easily fits inside the available time gap.

Figure 1 also illustrates the situation in which the harmonic number is $3h = 360$. In this case the time between bunch centers is the same, but each harmonic h bucket becomes a group of three harmonic $3h$ buckets with the bunch centered in the middle bucket of the group. The time available for the kicker field to rise is still $T_R = T/h - w$, where $h = 120$, but w is now the width of the bunch sitting in a harmonic $3h$ bucket. We then have

$$T_R \geq \frac{2}{3}\frac{T}{h} \tag{14}$$

and

$$w \leq \frac{1}{3}\frac{T}{h} \tag{15}$$

where, as before, $T/h = 109.83$, 108.87, 107.99, 107.45, and 107.05 ns for energies $E/A = 3.85$, 4.55, 5.75, 7.30, and 9.80 GeV per nucleon, respectively. Thus we see that conditions (9) and (10) are easily satisfied for all five energies in this case.

Values for w, T_R, and additional parameters are given in **sections 11** through **20** for bunches sitting in RHIC RF buckets at injection under various conditions. These data have been calculated in accordance with the formulae given in sections 3 and 26 of reference [5]. Results for w and T_R are summarized and plotted in **sections 7**, **8**, **9**.

7 Energies 1, 2, 3 in harmonic 120 buckets

In **sections 11** through **16**, parameters are given for gold bunches matched to harmonic 120 buckets at 180 KV for energies 1, 2, 3. The corresponding bucket areas are 0.693, 0.896, and 1.291 eV-s per nucleon, respectively. Here is a summary of bunch widths w and available times T_R.

For <u>0.25 eV-s</u> per nucleon bunches at energies $E/A = 3.85$, 4.55, 5.75 GeV per nucleon, we have respectively

$$w = 50.10,\ 43.06,\ 35.09 \text{ ns} \tag{16}$$

and

$$T_R = 59.73,\ 65.81,\ 72.90 \text{ ns.} \tag{17}$$

For <u>0.30 eV-s</u> per nucleon bunches

$$w = 55.63,\ 47.62,\ 38.67 \text{ ns} \tag{18}$$

and

$$T_R = 54.20,\ 61.25,\ 69.32 \text{ ns.} \tag{19}$$

For <u>0.35 eV-s</u> per nucleon bunches

$$w = 60.98,\ 51.95,\ 42.04 \text{ ns} \tag{20}$$

and

$$T_R = 48.85,\ 56.92,\ 65.95 \text{ ns.} \tag{21}$$

For <u>0.40 eV-s</u> per nucleon bunches

$$w = 66.26,\ 56.14,\ 45.23 \text{ ns} \tag{22}$$

and

$$T_R = 43.57,\ 52.73,\ 62.76 \text{ ns.} \tag{23}$$

For <u>0.45 eV-s</u> per nucleon bunches

$$w = 71.56,\ 60.23,\ 48.30 \text{ ns} \tag{24}$$

and

$$T_R = 38.27,\ 48.64,\ 59.69 \text{ ns.} \tag{25}$$

For <u>0.50 eV-s</u> per nucleon bunches

$$w = 77.01,\ 64.28,\ 51.27 \text{ ns} \tag{26}$$

and

$$T_R = 32.82,\ 44.59,\ 56.72 \text{ ns}. \tag{27}$$

For <u>0.55 eV-s</u> per nucleon bunches

$$w = 82.77,\ 68.32,\ 54.17 \text{ ns} \tag{28}$$

and

$$T_R = 27.06,\ 40.55,\ 53.82 \text{ ns}. \tag{29}$$

For <u>0.60 eV-s</u> per nucleon bunches

$$w = 89.17,\ 72.42,\ 57.02 \text{ ns} \tag{30}$$

and

$$T_R = 20.66,\ 36.45,\ 50.97 \text{ ns}. \tag{31}$$

These data are plotted in **Figures 2** and **3**, and are discussed in the text below each figure. Clean injection of a given bunch requires that the available time T_R be greater than the 60 ns rise time of the kicker field. The bunch width w also must be less than the 50 ns flat top portion of the kicker pulse. The plots show that in order to satisfy these two conditions, the bunch areas must be less than 0.25, 0.32, and 0.45 eV-s per nucleon for energies 3.85, 4.55, and 5.75 GeV per nucleon, respectively.

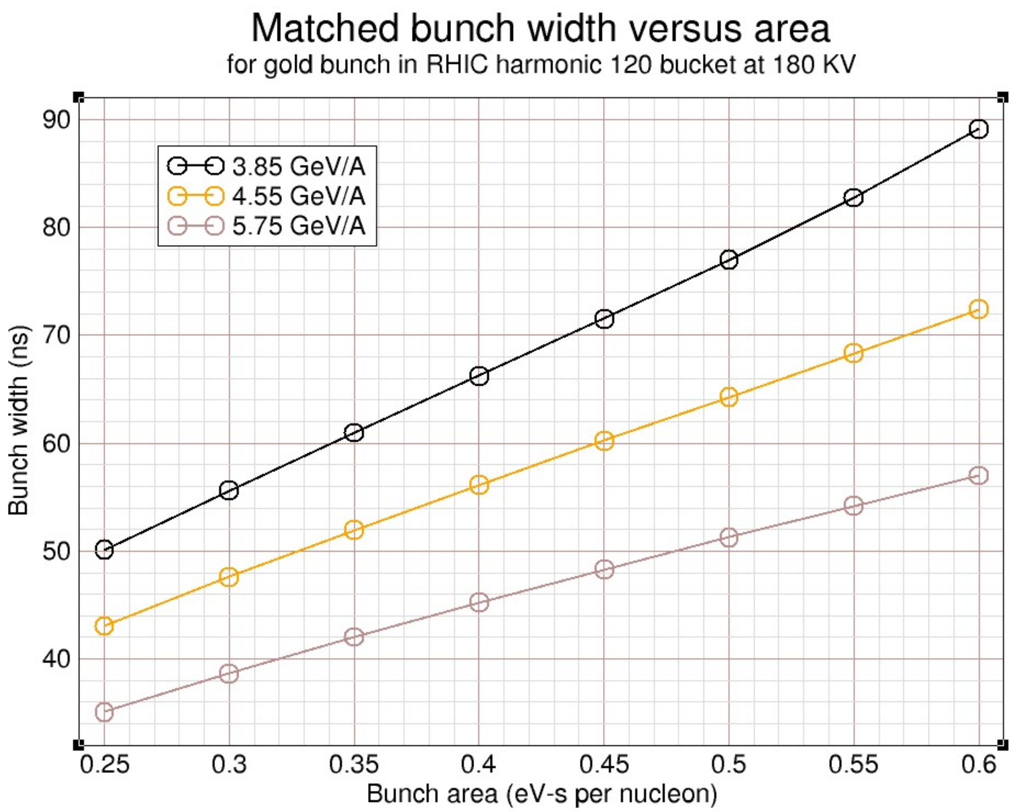

Figure 2: Matched bunch width w versus longitudinal emittance for gold bunches sitting in harmonic 120 buckets at 180 KV. The black, orange, and brown circles show the data for RHIC injection energies 3.85, 4.55, and 5.75 GeV per nucleon, respectively. The lines connecting the circles allow for interpolation. The flat top portion of the injection kicker pulse is 50 ns long as reported in [6]. Thus we see that at 3.85, 4.55, and 5.75 GeV per nucleon, bunches with areas of up to 0.25, 0.33, and 0.48 eV-s per nucleon, respectively, will fit on the flat top.

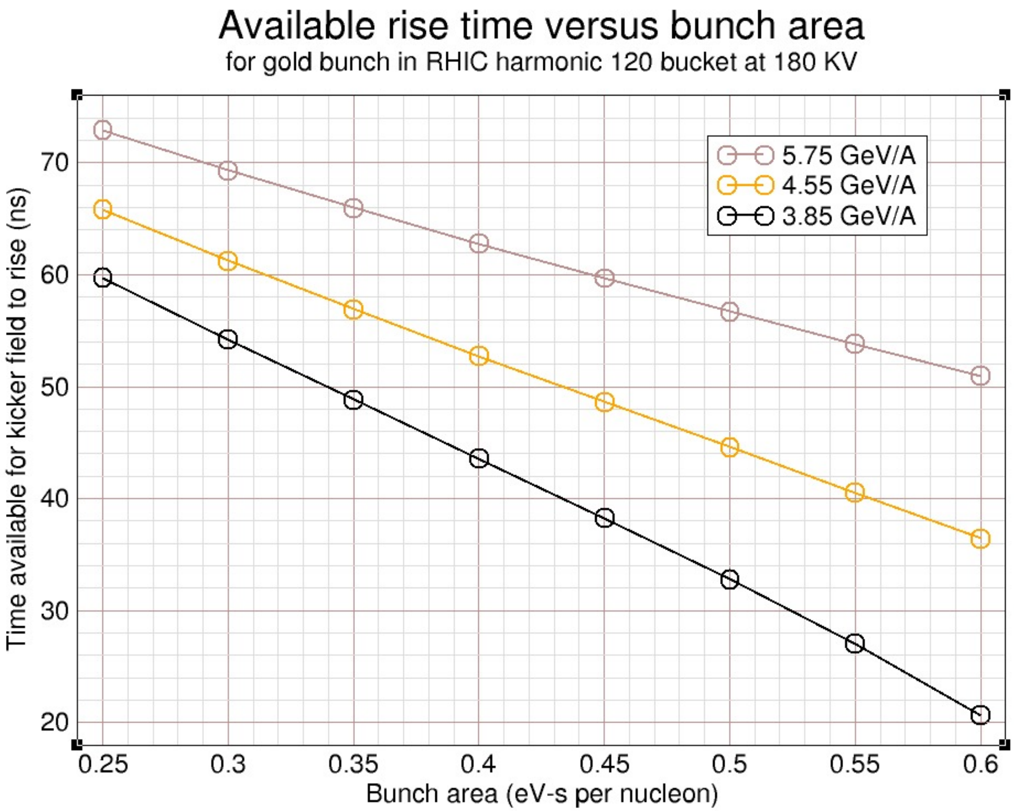

Figure 3: Available time T_R for kicker field to rise versus longitudinal emittance for gold bunches sitting in harmonic 120 buckets at 180 KV. The black, orange, and brown circles show the data for RHIC injection energies 3.85, 4.55, and 5.75 GeV per nucleon, respectively. The lines connecting the circles allow for interpolation. The actual rise time of the kicker field is 60 ns as reported in [6]. Thus we see that the available time T_R will be greater than rise time if the bunch areas are less than 0.25, 0.32, and 0.45 eV-s per nucleon for energies 3.85, 4.55, and 5.75 GeV per nucleon, respectively.

8 Energies 4 and 6 in harmonic 120 buckets

In **sections 17** and **19**, parameters are given for gold bunches matched to harmonic 120 buckets at 180 KV for energies 4 and 6. The corresponding bucket areas are 1.893 and 3.115 eV-s per nucleon, respectively. Here is a summary of bunch widths w and available times T_R.

For 0.30 eV-s per nucleon bunches at energies $E/A = 7.30$, 9.80 GeV per nucleon, we have respectively

$$w = 31.42,\ 24.18 \text{ ns} \tag{32}$$

and

$$T_R = 76.03,\ 82.87 \text{ ns.} \tag{33}$$

For 0.35 eV-s per nucleon bunches

$$w = 34.07,\ 26.18 \text{ ns} \tag{34}$$

and

$$T_R = 73.38,\ 80.87 \text{ ns.} \tag{35}$$

For 0.40 eV-s per nucleon bunches

$$w = 36.57,\ 28.05 \text{ ns} \tag{36}$$

and

$$T_R = 70.88,\ 79.00 \text{ ns.} \tag{37}$$

For 0.45 eV-s per nucleon bunches

$$w = 38.95,\ 29.82 \text{ ns} \tag{38}$$

and

$$T_R = 68.50,\ 77.23 \text{ ns.} \tag{39}$$

For 0.50 eV-s per nucleon bunches

$$w = 41.24,\ 31.51 \text{ ns} \tag{40}$$

and

$$T_R = 66.21,\ 75.54 \text{ ns.} \tag{41}$$

For 0.55 eV-s per nucleon bunches

$$w = 43.44,\ 33.13 \text{ ns} \tag{42}$$

and

$$T_R = 64.01,\ 73.92 \text{ ns}. \tag{43}$$

For 0.60 eV-s per nucleon bunches

$$w = 45.58,\ 34.69 \text{ ns} \tag{44}$$

and

$$T_R = 61.87,\ 72.36 \text{ ns}. \tag{45}$$

For 0.65 eV-s per nucleon bunches

$$w = 47.66,\ 36.19 \text{ ns} \tag{46}$$

and

$$T_R = 59.79,\ 70.86 \text{ ns}. \tag{47}$$

For 0.70 eV-s per nucleon bunches

$$w = 49.69,\ 37.66 \text{ ns} \tag{48}$$

and

$$T_R = 57.76,\ 69.39 \text{ ns}. \tag{49}$$

These data are plotted in **Figures 4** and **5**, and are discussed in the text below each figure. Clean injection of a given bunch requires that the available time T_R be greater than the 60 ns rise time of the kicker field. The bunch width w also must be less than the 50 ns flat top portion of the kicker pulse. The plots show that the bunch width condition is easily satisfied at both 7.30 and 9.80 GeV per nucleon. The available time condition is easily satisfied at 9.80 GeV per nucleon. At 7.30 GeV per nucleon, the condition is satisfied if the bunch area is less than 0.64 eV-s per nucleon.

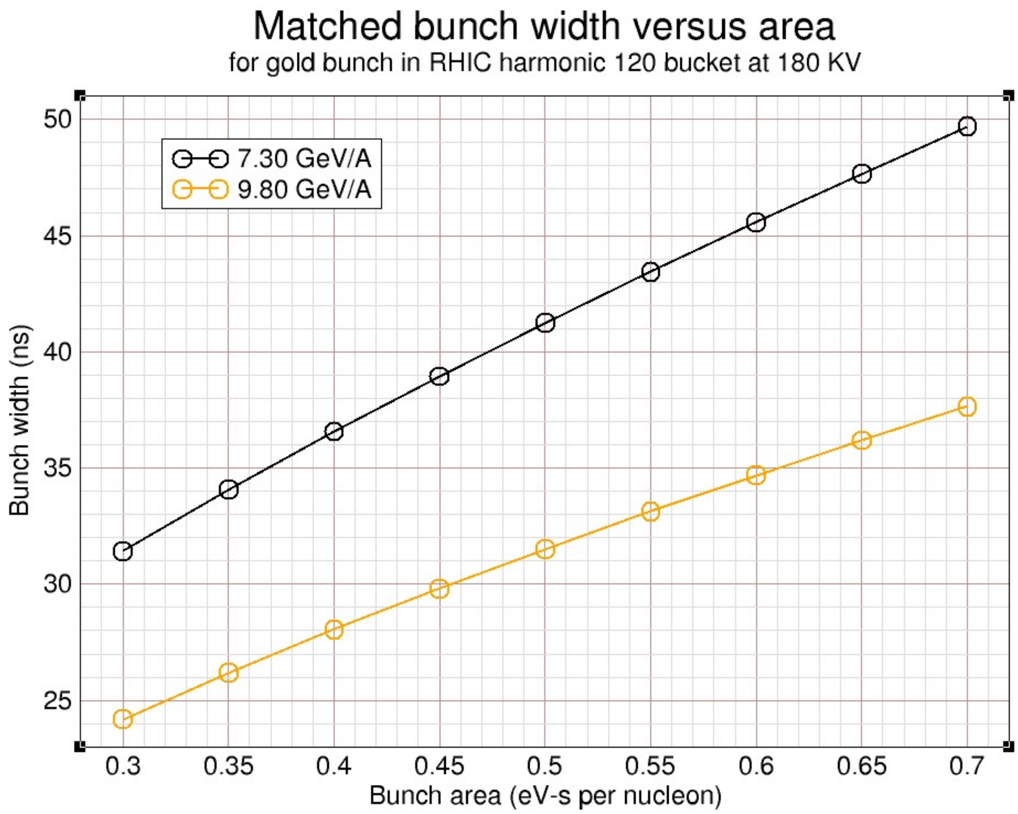

Figure 4: Matched bunch width w versus longitudinal emittance for gold bunches sitting in harmonic 120 buckets at 180 KV. The black and orange circles show the data for RHIC injection energies 7.30 and 9.80 GeV per nucleon, respectively. The lines connecting the circles allow for interpolation. The flat top portion of the injection kicker pulse is 50 ns long as reported in [6]. Thus we see that at 7.30 GeV per nucleon, bunches with areas up to 0.7 eV-s per nucleon will fit on the flat top. At 9.80 GeV per nucleon, bunches with areas up to 0.7 eV-s per nucleon will fit as long as the flat top length is greater than or equal to 38 ns.

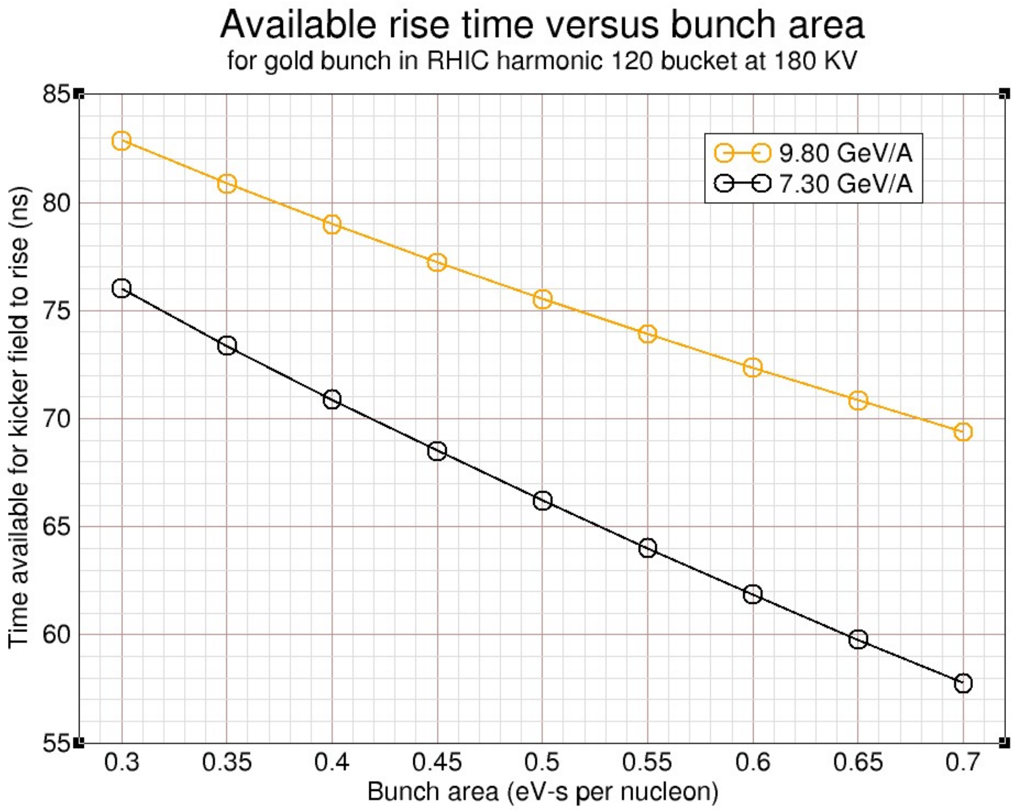

Figure 5: Available time T_R for kicker field to rise versus longitudinal emittance for bunches sitting in harmonic 120 buckets at 180 KV. The black and orange circles show the data for RHIC injection energies 7.30 and 9.80 GeV per nucleon, respectively. The lines connecting the circles allow for interpolation. The actual rise time of the kicker field is 60 ns as reported in [6]. Thus we see that at 7.30 eV-s per nucleon the available time T_R will be greater than the rise time if the bunch area is less than 0.64 eV-s per nucleon. At 9.80 GeV per nucleon, the available time is well above the rise time for bunches with areas up to 0.7 eV-s per nucleon.

9 Energies 4 and 6 in harmonic 360 buckets

In **sections 18** and **20**, parameters are given for gold bunches matched to harmonic 360 buckets at 400 KV for energies 4 and 6. Here is a summary of bunch widths w and available times T_R.

For 0.30 eV-s per nucleon bunches at energy $E/A =$ **7.30 GeV** per nucleon, we have

$$w = 21.01 \text{ ns}, \quad T_R = 86.44 \text{ ns}. \tag{50}$$

For 0.35 eV-s per nucleon bunches

$$w = 23.21 \text{ ns}, \quad T_R = 84.24 \text{ ns}. \tag{51}$$

For 0.40 eV-s per nucleon bunches

$$w = 25.48 \text{ ns}, \quad T_R = 81.97 \text{ ns}. \tag{52}$$

For 0.45 eV-s per nucleon bunches

$$w = 27.95 \text{ ns}, \quad T_R = 79.50 \text{ ns}. \tag{53}$$

For 0.50 eV-s per nucleon bunches

$$w = 30.88 \text{ ns}, \quad T_R = 76.57 \text{ ns}. \tag{54}$$

Here the bunch area is limited by the harmonic 360 bucket area which is 0.5431 eV-s per nucleon at 400 KV. The bucket width is 35.816 ns.

For 0.30 eV-s per nucleon bunches at energy $E/A =$ **9.80 GeV** per nucleon, we have

$$w = 15.63 \text{ ns}, \quad T_R = 91.42 \text{ ns}. \tag{55}$$

For 0.35 eV-s per nucleon bunches

$$w = 17.05 \text{ ns}, \quad T_R = 90.00 \text{ ns}. \tag{56}$$

For 0.40 eV-s per nucleon bunches

$$w = 18.43 \text{ ns}, \quad T_R = 88.62 \text{ ns}. \tag{57}$$

For 0.45 eV-s per nucleon bunches

$$w = 19.77 \text{ ns}, \quad T_R = 87.28 \text{ ns}. \tag{58}$$

For 0.50 eV-s per nucleon bunches

$$w = 21.10 \text{ ns}, \quad T_R = 85.95 \text{ ns}. \tag{59}$$

For 0.55 eV-s per nucleon bunches

$$w = 22.43 \text{ ns}, \quad T_R = 84.62 \text{ ns}. \tag{60}$$

For 0.60 eV-s per nucleon bunches

$$w = 23.77 \text{ ns}, \quad T_R = 83.28 \text{ ns}. \tag{61}$$

For 0.65 eV-s per nucleon bunches

$$w = 25.16 \text{ ns}, \quad T_R = 81.89 \text{ ns}. \tag{62}$$

For 0.70 eV-s per nucleon bunches

$$w = 26.60 \text{ ns}, \quad T_R = 80.45 \text{ ns}. \tag{63}$$

Here the bunches easily fit in the harmonic 360 bucket which has area 0.8938 eV-s per nucleon at 400 KV. The bucket width is 35.685 ns.

The data for energies 7.30 and 9.80 GeV per nucleon are plotted in **Figures 6** and **7** and are discussed in the text below the figures. Clean injection of a given bunch requires that the available time T_R be greater than the 60 ns rise time of the kicker field. The bunch width w also must be less than the 50 ns flat top portion of the kicker pulse. The plots show that the bunch width condition and the available time condition are easily satisfied at both 7.30 and 9.80 GeV per nucleon. However, at 7.30 GeV per nucleon the bunch area is limited to 0.5431 eV-s per nucleon by the available bucket area.

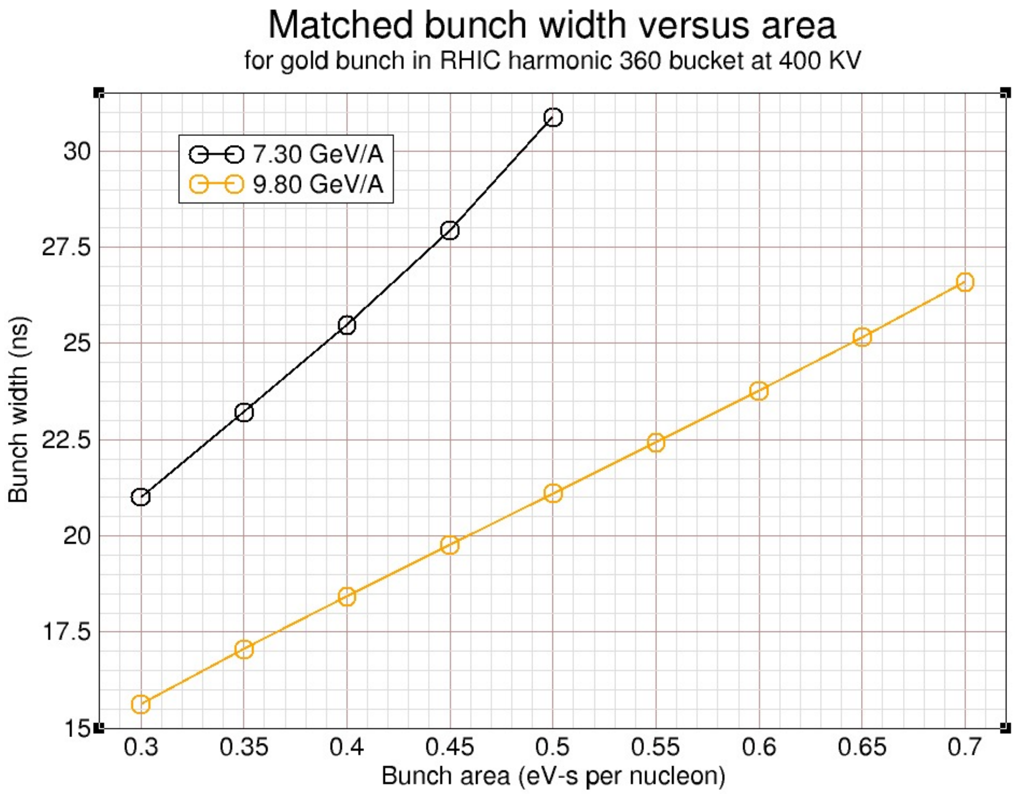

Figure 6: Matched bunch width w versus longitudinal emittance for gold bunches sitting in harmonic 360 buckets at 400 KV. The black and orange circles show the data for RHIC injection energies 7.30 and 9.80 GeV per nucleon, respectively. The lines connecting the circles allow for interpolation. The flat top portion of the injection kicker pulse is 50 ns long as reported in [6]. At 7.30 GeV per nucleon, the bunch area is limited by the available bucket area which is just 0.5431 eV-s per nucleon at 400 KV. The bucket width is 35.816 ns. Since this is less than 50 ns, any bunch sitting in the bucket will fit on the flat top. At 9.80 GeV per nucleon, bunches with areas up to 0.7 eV-s per nucleon fit as long as the flat top length is greater than 26 ns.

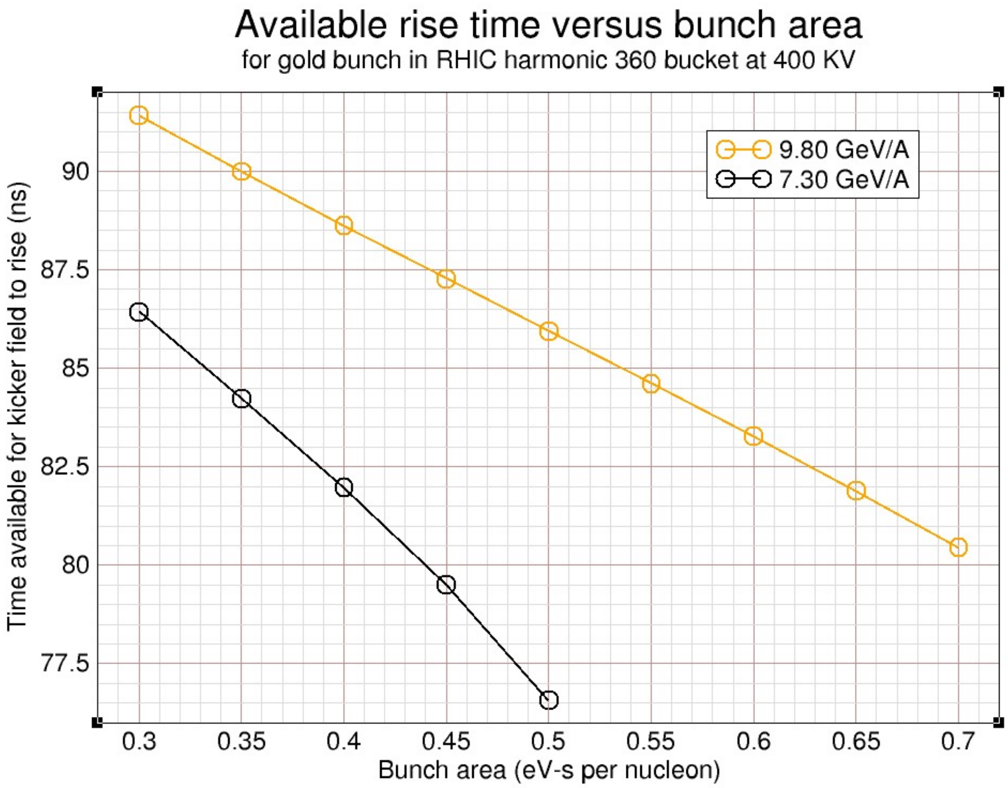

Figure 7: Available time T_R for kicker field to rise versus longitudinal emittance. The bunches are sitting in harmonic 360 buckets at 400 KV. The black and orange circles show the data for RHIC injection energies 7.30 and 9.80 GeV per nucleon, respectively. The lines connecting the circles allow for interpolation. The actual rise time of the kicker field is 60 ns as reported in [6]. At 7.30 GeV per nucleon, the bunch area is limited by the available bucket area which is just 0.5431 eV-s per nucleon at 400 KV. With this limitation the available time T_R is well above the rise time. At 9.80 GeV per nucleon, the available time is also well above the rise time for bunches with areas up to 0.7 eV-s per nucleon.

10 Summary and conclusions

Clean injection of a given bunch into RHIC requires that the **time available** for the kicker field to rise

$$T_R = T/h - w \tag{64}$$

be greater than the actual rise time $\mathcal{T}$ of the field. The **bunch width** w also must be less than the width $\mathcal{W}$ of the flat top portion of the kicker pulse. Here T is the revolution period of the bunch in RHIC and the harmonic number $h = 120$. Thus we must have

$$T_R \geq \mathcal{T} \tag{65}$$

and

$$w \leq \mathcal{W}. \tag{66}$$

These times are illustrated and discussed in **section 6**. In this note we have taken

$$\mathcal{T} = 60 \text{ ns} \tag{67}$$

and

$$\mathcal{W} = 50 \text{ ns}. \tag{68}$$

These are the numbers obtained from beam-based measurements of the kicker field [6]. Values of w and T_R for various matched bunches sitting in RHIC RF buckets at injection have been calculated and are listed and plotted in **sections 7**, **8**, and **9**.

Here are the main conclusions following from the requirement that conditions (65) and (66) be satisfied with $\mathcal{T}$ and $\mathcal{W}$ given by (67) and (68):

1. At 3.85, 4.55, and 5.75 GeV per nucleon, bunches sitting in **harmonic 120** buckets at 180 KV per turn must have areas less than 0.25, 0.32, and 0.45 eV-s per nucleon, respectively. The available bucket areas are 0.693, 0.896, and 1.291 eV-s per nucleon, respectively.

2. At 3.85, 4.55, and 5.75 GeV per nucleon, bunches sitting in **harmonic 360** buckets at 400 KV per turn easily satisfy the conditions, but the bunch areas are limited by the bucket areas which are 0.1987, 0.2570, and 0.3704, respectively.

3. At 7.30 GeV per nucleon, bunches sitting in **harmonic 120** buckets at 180 KV per turn must have areas less than 0.64 eV-s per nucleon. The available bucket area is 1.893 eV-s per nucleon.

4. At 7.30 GeV per nucleon, bunches sitting in **harmonic 360** buckets at 400 KV per turn easily satisfy the conditions, but the bunch area is limited by the bucket area which is 0.5432 eV-s per nucleon.

5. At 9.80 GeV per nucleon, bunches with areas from 0.3 to 0.7 eV-s per nucleon sitting in **harmonic 120** buckets at 180 KV per turn, or in **harmonic 360** buckets at 400 KV per turn, easily satisfy the conditions. The available bucket areas are 3.115 and 0.8938 eV-s per nucleon for the harmonic 120 and 360 buckets respectively.

The calculated values of w and T_R listed and plotted in **sections 7**, **8**, and **9** also can be used to find the kicker rise times $\mathcal{T}$ and flat top widths $\mathcal{W}$ necessary for the clean injection of bunches with given areas at any of the five energies. Here are some examples:

1. For a bunch with energy 3.85 GeV per nucleon and area 0.6 eV-s per nucleon sitting in a **harmonic 120** bucket at 180 KV, **figures 2** and **3** show that $\mathcal{W}$ must be at least <u>90 ns</u> and $\mathcal{T}$ at most <u>20 ns</u>. The available bucket area is 0.693 eV-s per nucleon.

2. For the same as above but with energy 4.55 GeV per nucleon, $\mathcal{W}$ must be at least <u>73 ns</u> and $\mathcal{T}$ at most <u>36 ns</u>. The available bucket area is 0.896 eV-s per nucleon.

3. For the same as above but with energy 5.75 GeV per nucleon, $\mathcal{W}$ must be at least <u>58 ns</u> and $\mathcal{T}$ at most <u>50 ns</u>. The available bucket area is 1.291 eV-s per nucleon.

4. For a bunch with energy 7.30 GeV per nucleon and area 0.7 eV-s per nucleon sitting in a **harmonic 120** bucket at 180 KV, **figures 4** and **5** show that $\mathcal{W}$ must be at least <u>50 ns</u> and $\mathcal{T}$ at most <u>57 ns</u>.

5. For the same as above but with energy 9.80 GeV per nucleon, $\mathcal{W}$ must be at least <u>38 ns</u> and $\mathcal{T}$ at most <u>69 ns</u>.

6. For a bunch with energy 9.80 GeV per nucleon and area 0.7 eV-s per nucleon sitting in a **harmonic 360** bucket at 400 KV, **figures 6** and **7** show that $\mathcal{W}$ must be at least <u>27 ns</u> and $\mathcal{T}$ at most <u>80 ns</u>.

11 3.85 GeV bunch in RHIC bucket at 60 KV

1. $h = 120$
2. $hf = 9.10498412755$ MHz
3. RF bucket width = **109.83 ns**
4. RHIC gap volts per turn = 60 KV
5. Bucket height = 563.3 MeV
6. Bucket area = 0.400 eV-s per nucleon

1. Matched bunch area <u>0.10 eV-s</u> per nucleon gives
   (a) Bunch width = **40.92 ns**
   (b) Time available for kicker rise: 109.83 − 40.92 = **68.91 ns**
   (c) Bunch height = 311.2 MeV
   (d) Bunch $\Delta p/p$ = 4.36e-04
   (e) AGS gap volts per turn for match = 19.22 KV
2. Matched bunch area <u>0.15 eV-s</u> per nucleon gives
   (a) Bunch width = **51.21 ns**
   (b) Time available for kicker rise: 109.83 − 51.21 = **58.62 ns**
   (c) Bunch height = 376.7 MeV
   (d) Bunch $\Delta p/p$ = 5.27e-04
   (e) AGS gap volts per turn for match = 17.96 KV
3. Matched bunch area <u>0.20 eV-s</u> per nucleon gives
   (a) Bunch width = **60.60 ns**
   (b) Time available for kicker rise: 109.83 − 60.60 = **49.23 ns**
   (c) Bunch height = 429.4 MeV
   (d) Bunch $\Delta p/p$ = 6.01e-04
   (e) AGS gap volts per turn for match = 16.62 KV

4. Matched bunch area <u>0.25 eV-s</u> per nucleon gives
   (a) Bunch width = **69.75 ns**
   (b) Time available for kicker rise: $109.83 - 69.75 =$ **40.08 ns**
   (c) Bunch height = 473.3 MeV
   (d) Bunch $\Delta p/p$ = 6.63e-04
   (e) AGS gap volts per turn for match = 15.15 KV
5. Matched bunch area <u>0.30 eV-s</u> per nucleon gives
   (a) Bunch width = **79.22 ns**
   (b) Time available for kicker rise: $109.83 - 79.22 =$ **30.61 ns**
   (c) Bunch height = 510.2 MeV
   (d) Bunch $\Delta p/p$ = 7.15e-04
   (e) AGS gap volts per turn for match = 13.48 KV
6. Matched bunch area <u>0.35 eV-s</u> per nucleon gives
   (a) Bunch width = **89.97 ns**
   (b) Time available for kicker rise: $109.83 - 89.97 =$ **19.86 ns**
   (c) Bunch height = 540.8 MeV
   (d) Bunch $\Delta p/p$ = 7.57e-04
   (e) AGS gap volts per turn for match = 11.44 KV

12 3.85 GeV bunch in RHIC bucket at 180 KV

1. $h = 120$
2. $hf = 9.10498412755$ MHz
3. RF bucket width = **109.83 ns**
4. RHIC gap volts per turn = 180 KV
5. Bucket height = 975.7 MeV
6. Bucket area = 0.693 eV-s per nucleon

1. Matched bunch area <u>0.10 eV-s</u> per nucleon gives
   (a) Bunch width = **30.59 ns**
   (b) Time available for kicker rise: 109.83 − 30.59 = **79.24 ns**
   (c) Bunch height = 413.4 MeV
   (d) Bunch $\Delta p/p$ = 5.79e-04
   (e) AGS gap volts per turn for match = 60.66 KV
2. Matched bunch area <u>0.15 eV-s</u> per nucleon gives
   (a) Bunch width = **37.88 ns**
   (b) Time available for kicker rise: 109.83 − 37.88 = **71.95 ns**
   (c) Bunch height = 503.1 MeV
   (d) Bunch $\Delta p/p$ = 7.05e-04
   (e) AGS gap volts per turn for match = 58.61 KV
3. Matched bunch area <u>0.20 eV-s</u> per nucleon gives
   (a) Bunch width = **44.25 ns**
   (b) Time available for kicker rise: 109.83 − 44.25 = **65.58 ns**
   (c) Bunch height = 577.1 MeV
   (d) Bunch $\Delta p/p$ = 8.08e-04
   (e) AGS gap volts per turn for match = 56.53 KV

4. Matched bunch area 0.25 eV-s per nucleon gives
   (a) Bunch width = **50.10 ns**
   (b) Time available for kicker rise: 109.83 − 50.10 = **59.73 ns**
   (c) Bunch height = 640.9 MeV
   (d) Bunch $\Delta p/p$ = 8.97e-04
   (e) AGS gap volts per turn for match = 54.33 KV
5. Matched bunch area 0.30 eV-s per nucleon gives
   (a) Bunch width = **55.63 ns**
   (b) Time available for kicker rise: 109.83 − 55.63 = **54.20 ns**
   (c) Bunch height = 697.0 MeV
   (d) Bunch $\Delta p/p$ = 9.76e-04
   (e) AGS gap volts per turn for match = 52.06 KV
6. Matched bunch area 0.35 eV-s per nucleon gives
   (a) Bunch width = **60.98 ns**
   (b) Time available for kicker rise: 109.83 − 60.98 = **48.85 ns**
   (c) Bunch height = 747.1 MeV
   (d) Bunch $\Delta p/p$ = 1.05e-03
   (e) AGS gap volts per turn for match = 49.68 KV
7. Matched bunch area 0.40 eV-s per nucleon gives
   (a) Bunch width = **66.26 ns**
   (b) Time available for kicker rise: 109.83 − 66.26 = **43.57 ns**
   (c) Bunch height = 792.3 MeV
   (d) Bunch $\Delta p/p$ = 1.11e-03
   (e) AGS gap volts per turn for match = 47.17 KV

8. Matched bunch area 0.45 eV-s per nucleon gives
    (a) Bunch width = **71.56 ns**
    (b) Time available for kicker rise: 109.83 − 71.56 = **38.27 ns**
    (c) Bunch height = 833.2 MeV
    (d) Bunch $\Delta p/p$ = 1.17e-03
    (e) AGS gap volts per turn for match = 44.53 KV
9. Matched bunch area 0.50 eV-s per nucleon gives
    (a) Bunch width = **77.01 ns**
    (b) Time available for kicker rise: 109.83 − 77.01 = **32.82 ns**
    (c) Bunch height = 870.2 MeV
    (d) Bunch $\Delta p/p$ = 1.22e-03
    (e) AGS gap volts per turn for match = 41.65 KV
10. Matched bunch area 0.55 eV-s per nucleon gives
    (a) Bunch width = **82.77 ns**
    (b) Time available for kicker rise: 109.83 − 82.77 = **27.06 ns**
    (c) Bunch height = 903.6 MeV
    (d) Bunch $\Delta p/p$ = 1.27e-03
    (e) AGS gap volts per turn for match = 38.47 KV
11. Matched bunch area 0.60 eV-s per nucleon gives
    (a) Bunch width = **89.17 ns**
    (b) Time available for kicker rise: 109.83 − 89.17 = **20.66 ns**
    (c) Bunch height = 933.2 MeV
    (d) Bunch $\Delta p/p$ = 1.31e-03
    (e) AGS gap volts per turn for match = 34.75 KV

The boundaries of matched bunches in RHIC RF buckets are illustrated in **Figures 8**, **9**, **10**.

13 Mismatched 3.85 GeV bunch in 180 KV bucket

1. $h = 120$
2. $hf = 9.10498412755$ MHz
3. RF bucket width = **109.83 ns**
4. RHIC gap volts per turn = 180 KV
5. Bucket height = 975.7 MeV
6. Bucket area = 0.693 eV-s per nucleon

Assume AGS bunch is matched to AGS RF bucket at extraction:

1. AGS bunch area = <u>0.25 eV-s</u> per nucleon
   (a) AGS gap volts per turn = 80.0 KV
   (b) AGS bunch width = **45.38 ns**
   (c) AGS bunch height = 693.7 MeV
   (d) Area of filamented bunch in RHIC = <u>0.297 eV-s</u> per nucleon
   (e) Width of filamented bunch in RHIC = **55.31 ns**
   (f) Filamented bunch $\Delta p/p$ = 9.72e-04
2. AGS bunch area = <u>0.35 eV-s</u> per nucleon
   (a) AGS gap volts per turn = 120.0 KV
   (b) AGS bunch width = **48.59 ns**
   (c) AGS bunch height = 907.6 MeV
   (d) Area of filamented bunch in RHIC = <u>0.556 eV-s</u> per nucleon
   (e) Width of filamented bunch in RHIC = **83.49 ns**
   (f) Filamented bunch $\Delta p/p$ = 1.27e-03

The boundaries of matched, mismatched, and filamented bunches in RHIC RF buckets are illustrated in **Figures 8** through **20**.

14 4.55 GeV bunch in RHIC bucket at 60 KV

1. $h = 120$
2. $hf = 9.18496377307$ MHz
3. RF bucket width = **108.87 ns**
4. RHIC gap volts per turn = 60 KV
5. Bucket height = 735.0 MeV
6. Bucket area = 0.517 eV-s per nucleon

1. Matched bunch area <u>0.10 eV-s</u> per nucleon gives
   (a) Bunch width = **35.35 ns**
   (b) Time available for kicker rise: 108.87 − 35.35 = **73.52 ns**
   (c) Bunch height = 358.9 MeV
   (d) Bunch $\Delta p/p$ = 4.18e-04
   (e) AGS gap volts per turn for match = 17.57 KV
2. Matched bunch area <u>0.15 eV-s</u> per nucleon gives
   (a) Bunch width = **43.97 ns**
   (b) Time available for kicker rise: 108.87 − 43.97 = **64.90 ns**
   (c) Bunch height = 435.7 MeV
   (d) Bunch $\Delta p/p$ = 5.07e-04
   (e) AGS gap volts per turn for match = 16.74 KV
3. Matched bunch area <u>0.20 eV-s</u> per nucleon gives
   (a) Bunch width = **51.65 ns**
   (b) Time available for kicker rise: 108.87 − 51.65 = **57.22 ns**
   (c) Bunch height = 498.4 MeV
   (d) Bunch $\Delta p/p$ = 5.80e-04
   (e) AGS gap volts per turn for match = 15.86 KV

4. Matched bunch area <u>0.25 eV-s</u> per nucleon gives
   (a) Bunch width = **58.85 ns**
   (b) Time available for kicker rise: 108.87 − 58.85 = **50.02 ns**
   (c) Bunch height = 551.8 MeV
   (d) Bunch $\Delta p/p$ = 6.42e-04
   (e) AGS gap volts per turn for match = 14.94 KV
5. Matched bunch area <u>0.30 eV-s</u> per nucleon gives
   (a) Bunch width = **65.86 ns**
   (b) Time available for kicker rise: 108.87 − 65.86 = **43.01 ns**
   (c) Bunch height = 598.0 MeV
   (d) Bunch $\Delta p/p$ = 6.96e-04
   (e) AGS gap volts per turn for match = 13.95 KV
6. Matched bunch area <u>0.35 eV-s</u> per nucleon gives
   (a) Bunch width = **72.94 ns**
   (b) Time available for kicker rise: 108.87 − 72.94 = **35.93 ns**
   (c) Bunch height = 638.4 MeV
   (d) Bunch $\Delta p/p$ = 7.43e-04
   (e) AGS gap volts per turn for match = 12.88 KV
7. Matched bunch area <u>0.40 eV-s</u> per nucleon gives
   (a) Bunch width = **80.37 ns**
   (b) Time available for kicker rise: 108.87 − 80.37 = **28.50 ns**
   (c) Bunch height = 673.7 MeV
   (d) Bunch $\Delta p/p$ = 7.85e-04
   (e) AGS gap volts per turn for match = 11.68 KV

15 4.55 GeV bunch in RHIC bucket at 180 KV

1. $h = 120$
2. $hf = 9.18496377307$ MHz
3. RF bucket width = **108.87 ns**
4. RHIC gap volts per turn = 180 KV
5. Bucket height = 1273.1 MeV
6. Bucket area = 0.896 eV-s per nucleon

1. Matched bunch area <u>0.10 eV-s</u> per nucleon gives
   (a) Bunch width = **26.54 ns**
   (b) Time available for kicker rise: 108.87 − 26.54 = **82.33 ns**
   (c) Bunch height = 475.6 MeV
   (d) Bunch $\Delta p/p$ = 5.54e-04
   (e) AGS gap volts per turn for match = 54.70 KV
2. Matched bunch area <u>0.15 eV-s</u> per nucleon gives
   (a) Bunch width = **32.77 ns**
   (b) Time available for kicker rise: 108.87 − 32.77 = **76.10 ns**
   (c) Bunch height = 579.7 MeV
   (d) Bunch $\Delta p/p$ = 6.75e-04
   (e) AGS gap volts per turn for match = 53.34 KV
3. Matched bunch area <u>0.20 eV-s</u> per nucleon gives
   (a) Bunch width = **38.16 ns**
   (b) Time available for kicker rise: 108.87 − 38.16 = **70.71 ns**
   (c) Bunch height = 666.1 MeV
   (d) Bunch $\Delta p/p$ = 7.76e-04
   (e) AGS gap volts per turn for match = 51.96 KV

4. Matched bunch area <u>0.25 eV-s</u> per nucleon gives
   (a) Bunch width = **43.06 ns**
   (b) Time available for kicker rise: 108.87 − 43.06 = **65.81 ns**
   (c) Bunch height = 741.0 MeV
   (d) Bunch $\Delta p/p$ = 8.63e-04
   (e) AGS gap volts per turn for match = 50.47 KV
5. Matched bunch area <u>0.30 eV-s</u> per nucleon gives
   (a) Bunch width = **47.62 ns**
   (b) Time available for kicker rise: 108.87 − 47.62 = **61.25 ns**
   (c) Bunch height = 807.5 MeV
   (d) Bunch $\Delta p/p$ = 9.40e-04
   (e) AGS gap volts per turn for match = 48.99 KV
6. Matched bunch area <u>0.35 eV-s</u> per nucleon gives
   (a) Bunch width = **51.95 ns**
   (b) Time available for kicker rise: 108.87 − 51.95 = **56.92 ns**
   (c) Bunch height = 867.4 MeV
   (d) Bunch $\Delta p/p$ = 1.01e-03
   (e) AGS gap volts per turn for match = 47.48 KV
7. Matched bunch area <u>0.40 eV-s</u> per nucleon gives
   (a) Bunch width = **56.14 ns**
   (b) Time available for kicker rise: 108.87 − 56.14 = **52.73 ns**
   (c) Bunch height = 922.1 MeV
   (d) Bunch $\Delta p/p$ = 1.07e-03
   (e) AGS gap volts per turn for match = 45.88 KV

8. Matched bunch area 0.45 eV-s per nucleon gives
   (a) Bunch width = **60.23 ns**
   (b) Time available for kicker rise: 108.87 − 60.23 = **48.64 ns**
   (c) Bunch height = 972.3 MeV
   (d) Bunch $\Delta p/p$ = 1.13e-03
   (e) AGS gap volts per turn for match = 44.25 KV
9. Matched bunch area 0.50 eV-s per nucleon gives
   (a) Bunch width = **64.28 ns**
   (b) Time available for kicker rise: 108.87 − 64.28 = **44.59 ns**
   (c) Bunch height = 1019 MeV
   (d) Bunch $\Delta p/p$ = 1.19e-03
   (e) AGS gap volts per turn for match = 42.54 KV
10. Matched bunch area 0.55 eV-s per nucleon gives
   (a) Bunch width = **68.32 ns**
   (b) Time available for kicker rise: 108.87 − 68.32 = **40.55 ns**
   (c) Bunch height = 1061 MeV
   (d) Bunch $\Delta p/p$ = 1.24e-03
   (e) AGS gap volts per turn for match = 40.77 KV
11. Matched bunch area 0.60 eV-s per nucleon gives
   (a) Bunch width = **72.42 ns**
   (b) Time available for kicker rise: 108.87 − 72.42 = **36.45 ns**
   (c) Bunch height = 1101 MeV
   (d) Bunch $\Delta p/p$ = 1.27e-03
   (e) AGS gap volts per turn for match = 38.88 KV

16 5.75 GeV bunch in RHIC bucket at 180 KV

1. $h = 120$
2. $hf = 9.25970290448$ MHz
3. RF bucket width = **107.99 ns**
4. RHIC gap volts per turn = 180 KV
5. Bucket height = 1850.0 MeV
6. Bucket area = 1.291 eV-s per nucleon

1. Matched bunch area <u>0.10 eV-s</u> per nucleon gives
   (a) Bunch width = **21.82 ns**
   (b) Time available for kicker rise: 107.99 − 21.82 = **86.17 ns**
   (c) Bunch height = 577.3 MeV
   (d) Bunch $\Delta p/p$ = 5.23e-04
   (e) AGS gap volts per turn for match = 40.33 KV
2. Matched bunch area <u>0.15 eV-s</u> per nucleon gives
   (a) Bunch width = **26.87 ns**
   (b) Time available for kicker rise: 107.99 − 26.87 = **81.12 ns**
   (c) Bunch height = 704.7 MeV
   (d) Bunch $\Delta p/p$ = 6.39e-04
   (e) AGS gap volts per turn for match = 39.65 KV
3. Matched bunch area <u>0.20 eV-s</u> per nucleon gives
   (a) Bunch width = **31.20 ns**
   (b) Time available for kicker rise: 107.99 − 31.20 = **76.79 ns**
   (c) Bunch height = 811.1 MeV
   (d) Bunch $\Delta p/p$ = 7.35e-04
   (e) AGS gap volts per turn for match = 38.97 KV

4. Matched bunch area <u>0.25 eV-s</u> per nucleon gives
   (a) Bunch width = **35.09 ns**
   (b) Time available for kicker rise: $107.99 - 35.09 =$ **72.90 ns**
   (c) Bunch height = 903.8 MeV
   (d) Bunch $\Delta p/p$ = 8.19e-04
   (e) AGS gap volts per turn for match = 38.26 KV

5. Matched bunch area <u>0.30 eV-s</u> per nucleon gives
   (a) Bunch width = **38.67 ns**
   (b) Time available for kicker rise: $107.99 - 38.67 =$ **69.32 ns**
   (c) Bunch height = 986.6 MeV
   (d) Bunch $\Delta p/p$ = 8.94e-04
   (e) AGS gap volts per turn for match = 37.55 KV

6. Matched bunch area <u>0.35 eV-s</u> per nucleon gives
   (a) Bunch width = **42.04 ns**
   (b) Time available for kicker rise: $107.99 - 42.04 =$ **65.95 ns**
   (c) Bunch height = 1062 MeV
   (d) Bunch $\Delta p/p$ = 9.63e-04
   (e) AGS gap volts per turn for match = 36.79 KV

7. Matched bunch area <u>0.40 eV-s</u> per nucleon gives
   (a) Bunch width = **45.23 ns**
   (b) Time available for kicker rise: $107.99 - 45.23 =$ **62.76 ns**
   (c) Bunch height = 1131 MeV
   (d) Bunch $\Delta p/p$ = 1.03e-03
   (e) AGS gap volts per turn for match = 36.07 KV

8. Matched bunch area <u>0.45 eV-s</u> per nucleon gives
   (a) Bunch width = **48.30 ns**
   (b) Time available for kicker rise: 107.99 − 48.30 = **59.69 ns**
   (c) Bunch height = 1195 MeV
   (d) Bunch $\Delta p/p$ = 1.08e-03
   (e) AGS gap volts per turn for match = 35.30 KV
9. Matched bunch area <u>0.50 eV-s</u> per nucleon gives
   (a) Bunch width = **51.27 ns**
   (b) Time available for kicker rise: 107.99 − 51.27 = **56.72 ns**
   (c) Bunch height = 1255 MeV
   (d) Bunch $\Delta p/p$ = 1.14e-03
   (e) AGS gap volts per turn for match = 34.54 KV
10. Matched bunch area <u>0.55 eV-s</u> per nucleon gives
   (a) Bunch width = **54.17 ns**
   (b) Time available for kicker rise: 107.99 − 54.17 = **53.82 ns**
   (c) Bunch height = 1311 MeV
   (d) Bunch $\Delta p/p$ = 1.19e-03
   (e) AGS gap volts per turn for match = 33.74 KV
11. Matched bunch area <u>0.60 eV-s</u> per nucleon gives
   (a) Bunch width = **57.02 ns**
   (b) Time available for kicker rise: 107.99 − 57.02 = **50.97 ns**
   (c) Bunch height = 1364 MeV
   (d) Bunch $\Delta p/p$ = 1.24e-03
   (e) AGS gap volts per turn for match = 32.92 KV

17 7.30 GeV bunch in RHIC bucket at 180 KV

1. $h = 120$
2. $hf = 9.30690779975$ MHz
3. RF bucket width = **107.45 ns**
4. RHIC gap volts per turn = 180 KV
5. Bucket height = 2776 MeV
6. Bucket area = 1.893 eV-s per nucleon

1. Matched bunch area <u>0.30 eV-s</u> per nucleon gives
   (a) Bunch width = **31.42 ns**
   (b) Time available for kicker rise: 107.45 − 31.42 = **76.03 ns**
   (c) Bunch height = 1209 MeV
   (d) Bunch $\Delta p/p$ = 8.54e-04
   (e) AGS gap volts per turn for match = 12.91 KV
2. Matched bunch area <u>0.35 eV-s</u> per nucleon gives
   (a) Bunch width = **34.07 ns**
   (b) Time available for kicker rise: 107.45 − 34.07 = **73.38 ns**
   (c) Bunch height = 1302 MeV
   (d) Bunch $\Delta p/p$ = 9.21e-04
   (e) AGS gap volts per turn for match = 12.76 KV
3. Matched bunch area <u>0.40 eV-s</u> per nucleon gives
   (a) Bunch width = **36.57 ns**
   (b) Time available for kicker rise: 107.45 − 36.57 = **70.88 ns**
   (c) Bunch height = 1389 MeV
   (d) Bunch $\Delta p/p$ = 9.82e-04
   (e) AGS gap volts per turn for match = 12.60 KV

4. Matched bunch area <u>0.45 eV-s</u> per nucleon gives
   (a) Bunch width = **38.95 ns**
   (b) Time available for kicker rise: 107.45 − 38.95 = **68.50 ns**
   (c) Bunch height = 1470 MeV
   (d) Bunch $\Delta p/p$ = 1.04e-03
   (e) AGS gap volts per turn for match = 12.44 KV

5. Matched bunch area <u>0.50 eV-s</u> per nucleon gives
   (a) Bunch width = **41.24 ns**
   (b) Time available for kicker rise: 107.45 − 41.24 = **66.21 ns**
   (c) Bunch height = 1546 MeV
   (d) Bunch $\Delta p/p$ = 1.09e-03
   (e) AGS gap volts per turn for match = 12.26 KV

6. Matched bunch area <u>0.55 eV-s</u> per nucleon gives
   (a) Bunch width = **43.44 ns**
   (b) Time available for kicker rise: 107.45 − 43.44 = **64.01 ns**
   (c) Bunch height = 1617 MeV
   (d) Bunch $\Delta p/p$ = 1.14e-03
   (e) AGS gap volts per turn for match = 12.10 KV

7. Matched bunch area <u>0.60 eV-s</u> per nucleon gives
   (a) Bunch width = **45.58 ns**
   (b) Time available for kicker rise: 107.45 − 45.58 = **61.87 ns**
   (c) Bunch height = 1685 MeV
   (d) Bunch $\Delta p/p$ = 1.19e-03
   (e) AGS gap volts per turn for match = 11.92 KV

8. Matched bunch area <u>0.65 eV-s</u> per nucleon gives

   (a) Bunch width = **47.66 ns**
   (b) Time available for kicker rise: 107.45 − 47.66 = **59.79 ns**
   (c) Bunch height = 1749 MeV
   (d) Bunch $\Delta p/p$ = 1.24e-03
   (e) AGS gap volts per turn for match = 11.75 KV

9. Matched bunch area <u>0.70 eV-s</u> per nucleon gives

   (a) Bunch width = **49.69 ns**
   (b) Time available for kicker rise: 107.45 − 49.69 = **57.76 ns**
   (c) Bunch height = 1811 MeV
   (d) Bunch $\Delta p/p$ = 1.28e-03
   (e) AGS gap volts per turn for match = 11.58 KV

18 7.30 GeV bunch in RHIC bucket at 400 KV

1. $h = 360$
2. $hf = 27.92072339925$ MHz
3. RF bucket width = **35.816 ns**
4. RHIC gap volts per turn = 400 KV
5. Bucket height = 2346 MeV
6. Bucket area = 0.5431 eV-s per nucleon

1. Matched bunch area 0.30 eV-s per nucleon gives
   (a) Bunch width = **21.01 ns**
   (b) Time available for kicker rise: $107.45 - 21.01 =$ **86.44 ns**
   (c) Bunch height = 1869 MeV
   (d) Bunch $\Delta p/p = 1.32$e-03
   (e) AGS gap volts per turn for match = 63.88 KV
2. Matched bunch area 0.35 eV-s per nucleon gives
   (a) Bunch width = **23.21 ns**
   (b) Time available for kicker rise: $107.45 - 23.21 =$ **84.24 ns**
   (c) Bunch height = 1997 MeV
   (d) Bunch $\Delta p/p = 1.41$e-03
   (e) AGS gap volts per turn for match = 58.50 KV
3. Matched bunch area 0.40 eV-s per nucleon gives
   (a) Bunch width = **25.48 ns**
   (b) Time available for kicker rise: $107.45 - 25.48 =$ **81.97 ns**
   (c) Bunch height = 2109 MeV
   (d) Bunch $\Delta p/p = 1.49$e-03
   (e) AGS gap volts per turn for match = 52.72 KV

4. Matched bunch area 0.45 eV-s per nucleon gives
   (a) Bunch width = **27.95 ns**
   (b) Time available for kicker rise: 107.45 − 27.95 = **79.50 ns**
   (c) Bunch height = 2208 MeV
   (d) Bunch $\Delta p/p$ = 1.56e-04
   (e) AGS gap volts per turn for match = 46.21 KV
5. Matched bunch area 0.50 eV-s per nucleon gives
   (a) Bunch width = **30.88 ns**
   (b) Time available for kicker rise: 107.45 − 30.88 = **76.57 ns**
   (c) Bunch height = 2291 MeV
   (d) Bunch $\Delta p/p$ = 1.62e-03
   (e) AGS gap volts per turn for match = 38.42 KV

19 9.80 GeV bunch in RHIC bucket at 180 KV

1. $h = 120$
2. $hf = 9.34110295706$ MHz
3. RF bucket width = **107.05 ns**
4. RHIC gap volts per turn = 180 KV
5. Bucket height = 4503 MeV
6. Bucket area = 3.115 eV-s per nucleon

1. Matched bunch area 0.30 eV-s per nucleon gives
   (a) Bunch width = **24.18 ns**
   (b) Time available for kicker rise: 107.05 − 24.18 = **82.87 ns**
   (c) Bunch height = 1564 MeV
   (d) Bunch $\Delta p/p$ = 8.18e-04
   (e) AGS gap volts per turn for match = 53.13 KV
2. Matched bunch area 0.35 eV-s per nucleon gives
   (a) Bunch width = **26.18 ns**
   (b) Time available for kicker rise: 107.05 − 26.18 = **80.87 ns**
   (c) Bunch height = 1687 MeV
   (d) Bunch $\Delta p/p$ = 8.82e-04
   (e) AGS gap volts per turn for match = 52.73 KV
3. Matched bunch area 0.40 eV-s per nucleon gives
   (a) Bunch width = **28.05 ns**
   (b) Time available for kicker rise: 107.05 − 28.05 = **79.00 ns**
   (c) Bunch height = 1801 MeV
   (d) Bunch $\Delta p/p$ = 9.42e-04
   (e) AGS gap volts per turn for match = 52.37 KV

4. Matched bunch area <u>0.45 eV-s</u> per nucleon gives
   (a) Bunch width = **29.82 ns**
   (b) Time available for kicker rise: 107.05 − 29.82 = **77.23 ns**
   (c) Bunch height = 1908 MeV
   (d) Bunch $\Delta p/p$ = 9.97e-04
   (e) AGS gap volts per turn for match = 52.00 KV
5. Matched bunch area <u>0.50 eV-s</u> per nucleon gives
   (a) Bunch width = **31.51 ns**
   (b) Time available for kicker rise: 107.05 − 31.51 = **75.54 ns**
   (c) Bunch height = 2009 MeV
   (d) Bunch $\Delta p/p$ = 1.05e-03
   (e) AGS gap volts per turn for match = 51.60 KV
6. Matched bunch area <u>0.55 eV-s</u> per nucleon gives
   (a) Bunch width = **33.13 ns**
   (b) Time available for kicker rise: 107.05 − 33.13 = **73.92 ns**
   (c) Bunch height = 2104 MeV
   (d) Bunch $\Delta p/p$ = 1.10e-03
   (e) AGS gap volts per turn for match = 51.20 KV
7. Matched bunch area <u>0.60 eV-s</u> per nucleon gives
   (a) Bunch width = **34.69 ns**
   (b) Time available for kicker rise: 107.05 − 34.69 = **72.36 ns**
   (c) Bunch height = 2194 MeV
   (d) Bunch $\Delta p/p$ = 1.15e-03
   (e) AGS gap volts per turn for match = 50.80 KV

8. Matched bunch area 0.65 eV-s per nucleon gives
   (a) Bunch width = **36.19 ns**
   (b) Time available for kicker rise: $107.05 - 36.19 =$ **70.86 ns**
   (c) Bunch height = 2280 MeV
   (d) Bunch $\Delta p/p = 1.19\text{e-}03$
   (e) AGS gap volts per turn for match = 50.44 KV
9. Matched bunch area 0.70 eV-s per nucleon gives
   (a) Bunch width = **37.66 ns**
   (b) Time available for kicker rise: $107.05 - 37.66 =$ **69.39 ns**
   (c) Bunch height = 2363 MeV
   (d) Bunch $\Delta p/p = 1.24\text{e-}03$
   (e) AGS gap volts per turn for match = 50.00 KV

20 9.80 GeV bunch in RHIC bucket at 400 KV

1. $h = 360$
2. $hf = 28.02330887118$ MHz
3. RF bucket width = **35.685 ns**
4. RHIC gap volts per turn = 400 KV
5. Bucket height = 3875 MeV
6. Bucket area = 0.8938 eV-s per nucleon

1. Matched bunch area 0.30 eV-s per nucleon gives
   (a) Bunch width = **15.63 ns**
   (b) Time available for kicker rise: 107.05 − 15.63 = **91.42 ns**
   (c) Bunch height = 2460 MeV
   (d) Bunch $\Delta p/p = 1.29$e-03
   (e) AGS gap volts per turn for match = 302.2 KV
2. Matched bunch area 0.35 eV-s per nucleon gives
   (a) Bunch width = **17.05 ns**
   (b) Time available for kicker rise: 107.05 − 17.05 = **90.00 ns**
   (c) Bunch height = 2643 MeV
   (d) Bunch $\Delta p/p = 1.38$e-03
   (e) AGS gap volts per turn for match = 290.8 KV
3. Matched bunch area 0.40 eV-s per nucleon gives
   (a) Bunch width = **18.43 ns**
   (b) Time available for kicker rise: 107.05 − 18.43 = **88.62 ns**
   (c) Bunch height = 2810 MeV
   (d) Bunch $\Delta p/p = 1.47$e-03
   (e) AGS gap volts per turn for match = 278.5 KV

4. Matched bunch area <u>0.45 eV-s</u> per nucleon gives
   (a) Bunch width = **19.77 ns**
   (b) Time available for kicker rise: 107.05 − 19.77 = **87.28 ns**
   (c) Bunch height = 2963 MeV
   (d) Bunch $\Delta p/p$ = 1.55e-03
   (e) AGS gap volts per turn for match = 266.4 KV
5. Matched bunch area <u>0.50 eV-s</u> per nucleon gives
   (a) Bunch width = **21.10 ns**
   (b) Time available for kicker rise: 107.05 − 21.10 = **85.95 ns**
   (c) Bunch height = 3104 MeV
   (d) Bunch $\Delta p/p$ = 1.62e-03
   (e) AGS gap volts per turn for match = 253.8 KV
6. Matched bunch area <u>0.55 eV-s</u> per nucleon gives
   (a) Bunch width = **22.43 ns**
   (b) Time available for kicker rise: 107.05 − 22.43 = **84.62 ns**
   (c) Bunch height = 3234 MeV
   (d) Bunch $\Delta p/p$ = 1.69e-03
   (e) AGS gap volts per turn for match = 240.8 KV
7. Matched bunch area <u>0.60 eV-s</u> per nucleon gives
   (a) Bunch width = **23.77 ns**
   (b) Time available for kicker rise: 107.05 − 23.77 = **83.28 ns**
   (c) Bunch height = 3355 MeV
   (d) Bunch $\Delta p/p$ = 1.75e-03
   (e) AGS gap volts per turn for match = 227.5 KV

8. Matched bunch area <u>0.65 eV-s</u> per nucleon gives
   (a) Bunch width = **25.16 ns**
   (b) Time available for kicker rise: 107.05 − 25.16 = **81.89 ns**
   (c) Bunch height = 3467 MeV
   (d) Bunch $\Delta p/p$ = 1.81e-03
   (e) AGS gap volts per turn for match = 213.0 KV
9. Matched bunch area <u>0.70 eV-s</u> per nucleon gives
   (a) Bunch width = **26.60 ns**
   (b) Time available for kicker rise: 107.05 − 26.60 = **80.45 ns**
   (c) Bunch height = 3570 MeV
   (d) Bunch $\Delta p/p$ = 1.87e-03
   (e) AGS gap volts per turn for match = 198.0 KV

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[5] C.J. Gardner, "FY2016 Parameters for Gold Ions in Booster, AGS, and RHIC", C-A/AP/Note 574, October 2016.

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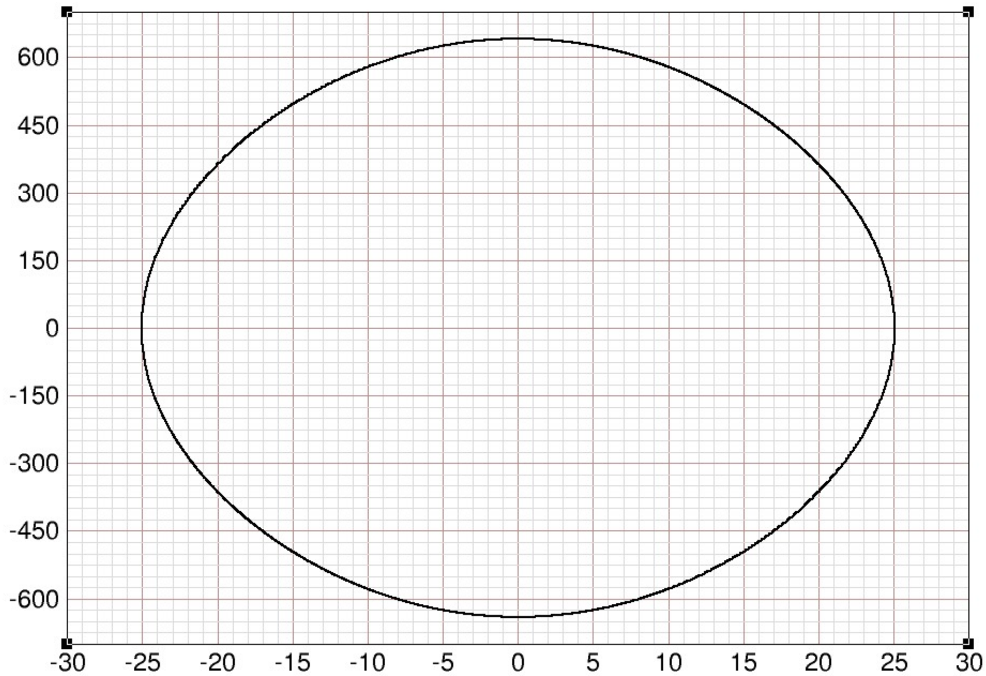

Figure 8: Boundary of gold bunch matched to RHIC RF bucket. The area enclosed is 0.25 eV-s per nucleon. RF voltage is 180 KV per turn. Harmonic number is 120. Energy is 3.85 GeV per nucleon. The horizontal axis gives the time deviation in ns. The vertical axis gives the energy deviation in MeV. (Divide by 197 to get the energy deviation in MeV per nucleon.)

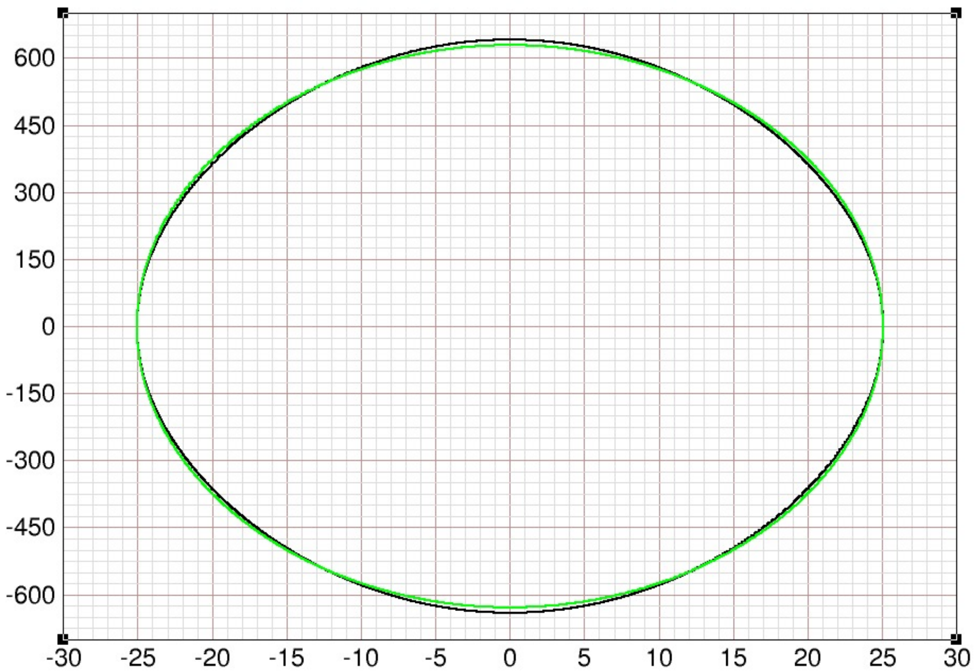

Figure 9: Same as previous figure, but with green curve showing the boundary of an incoming gold bunch from AGS. The longitudinal emittance of the incoming bunch is 0.25 eV-s per nucleon, the same as that of the matched bunch. The AGS RF voltage at extraction has been adjusted so that the incoming bunch has the same width as the matched bunch.

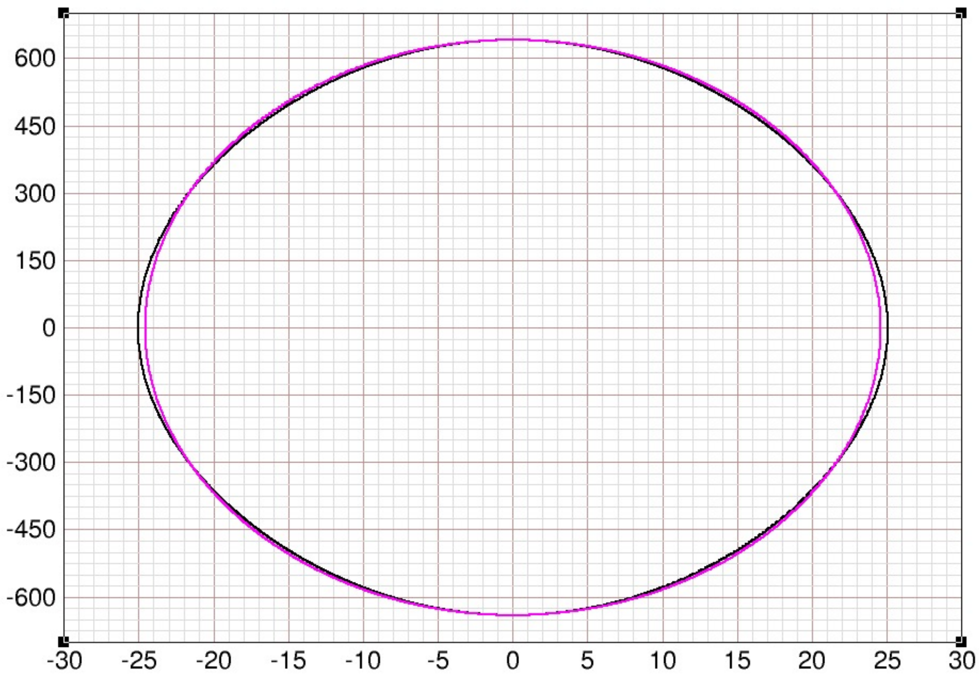

Figure 10: Here, on the other hand, the AGS RF voltage has been adjusted so that the incoming bunch has the same height as the matched bunch. The longitudinal emittance of the incoming bunch is the same as before. This gives the magenta curve shown above. This curve and the green curve shown in the previous figure closely follow the black boundary of the matched bunch, and in both cases we can say that the incoming AGS bunch is matched to the waiting RF bucket in RHIC.

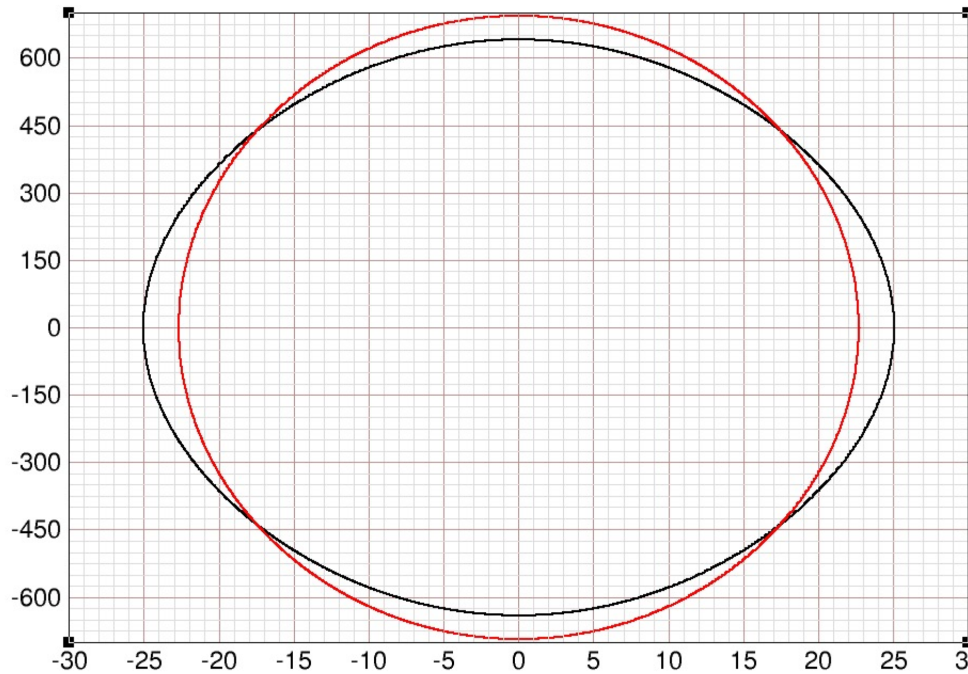

Figure 11: Here, in order to get a narrower incoming bunch, the AGS RF voltage has been raised to give the red curve shown above. The areas enclosed by the two curves are both 0.25 eV-s per nucleon. However, the red curve is not matched to the RHIC RF bucket and this will cause the incoming bunch to filament. The boundary of the filamented bunch is shown in the next figure.

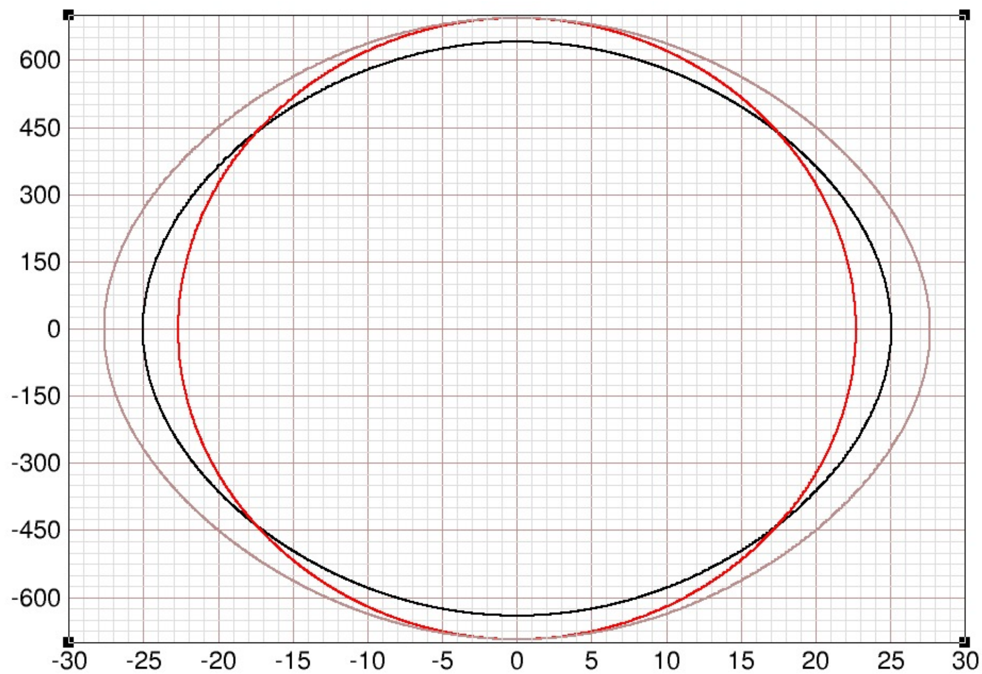

Figure 12: Same as the previous figure, but with brown curve showing the boundary of the filamented bunch. The areas enclosed by the red and black curves are both 0.25 eV-s per nucleon. The area enclosed by the brown curve is 0.297 eV-s per nucleon. One might have hoped that reducing the width of the incoming bunch (red curve) would give more time for the RHIC injection kicker field to rise, but here we see that there is really no net gain. The width of the filamented bunch is larger than the width of the matched bunch (black curve) by about the same amount that the width of the incoming bunch is smaller.

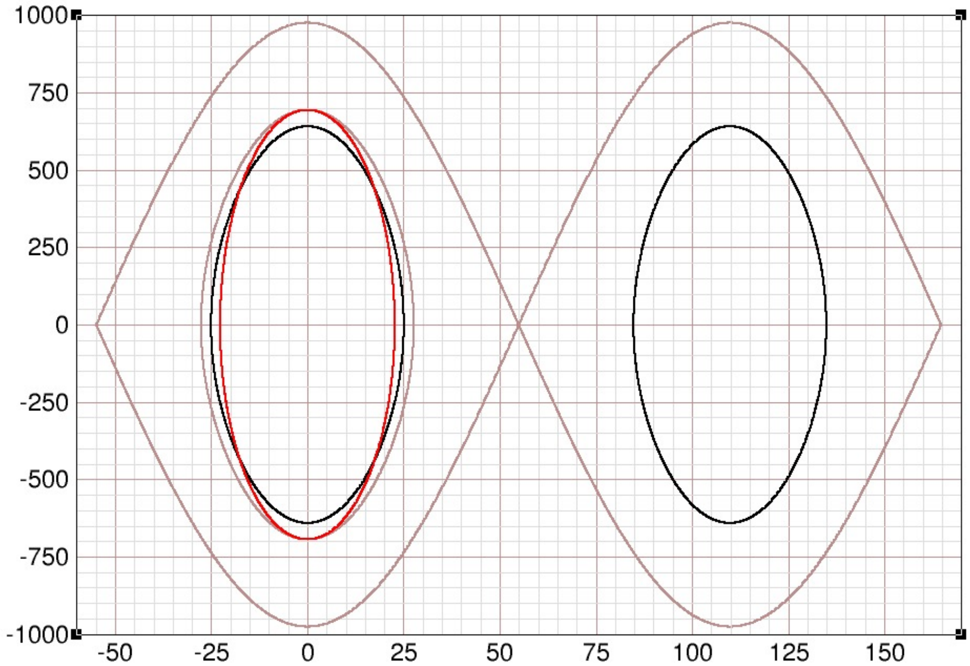

Figure 13: The bunches of the previous figures are shown here as they sit in the RHIC RF buckets. The black curves are matched to the buckets and enclose an area of 0.25 eV-s per nucleon. The red curve in the left bucket is boundary of the incoming mismatched bunch from AGS. The brown curve is the boundary of the filamented bunch. As already noted, the RHIC RF voltage is 180 KV per turn. Harmonic number is 120. Energy is 3.85 GeV per nucleon. The horizontal axis gives the time deviation in ns. The vertical axis gives the energy deviation in MeV. (Divide by 197 to get the energy deviation in MeV per nucleon.)

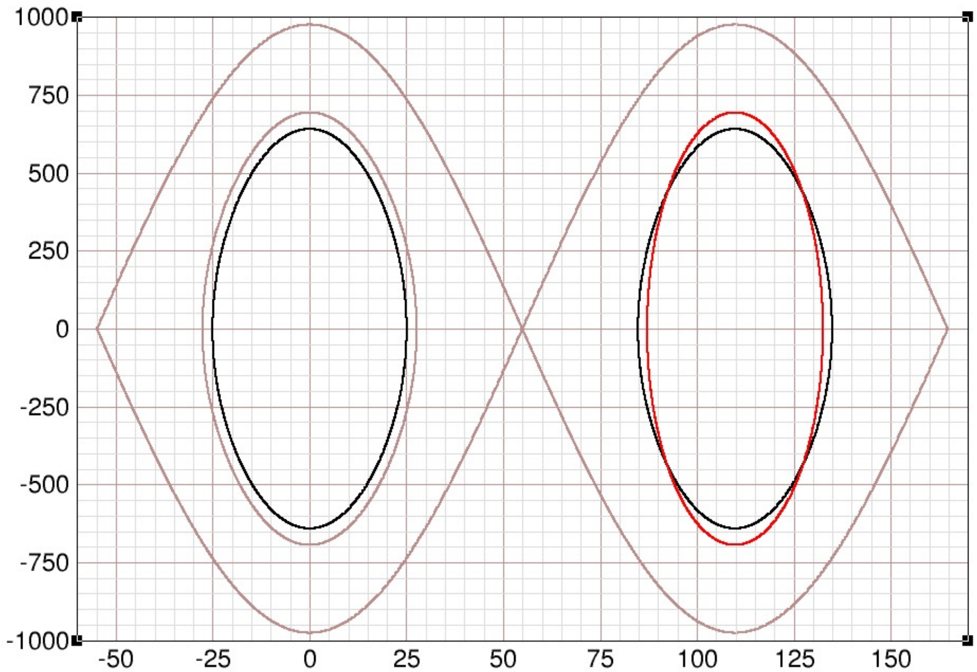

Figure 14: As in the previous figure, the brown curve in the left bucket is the boundary of the filamented bunch. The red curve in the right bucket is the boundary of the incoming mismatched bunch from AGS. The area enclosed by the brown curve is 0.297 eV-s per nucleon. The area enclosed by the red curve is 0.25 eV-s per nucleon.

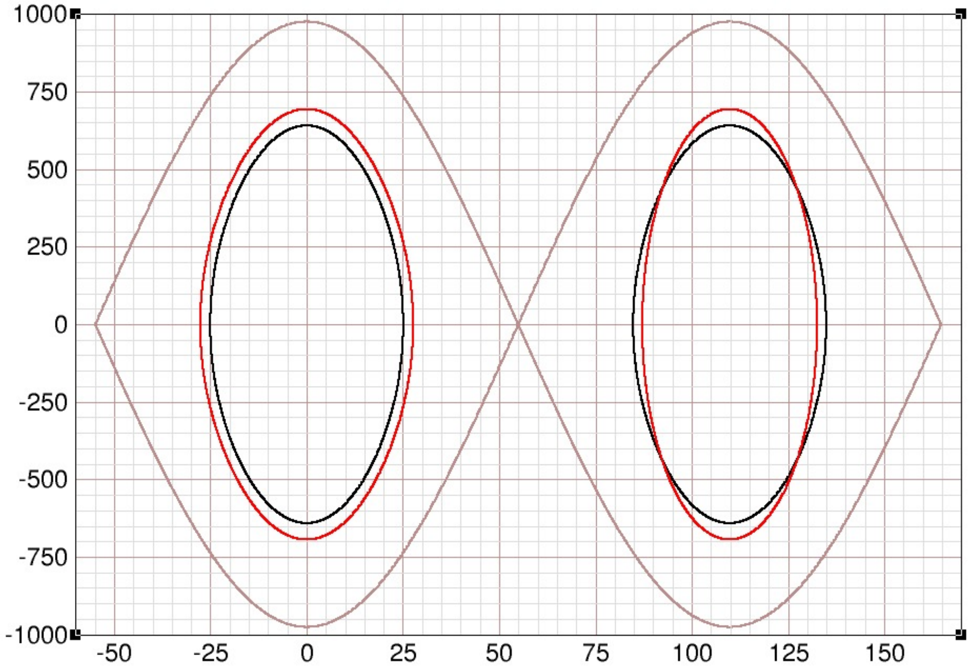

Figure 15: Same as previous figure, but with filamented bunch now outlined in red. Here we see that the distance between the two red bunches is just about the same as the distance between the two black bunches. So, as already observed in **Figure 12**, there is no significant difference in the time available for the rise of the injection kicker field.

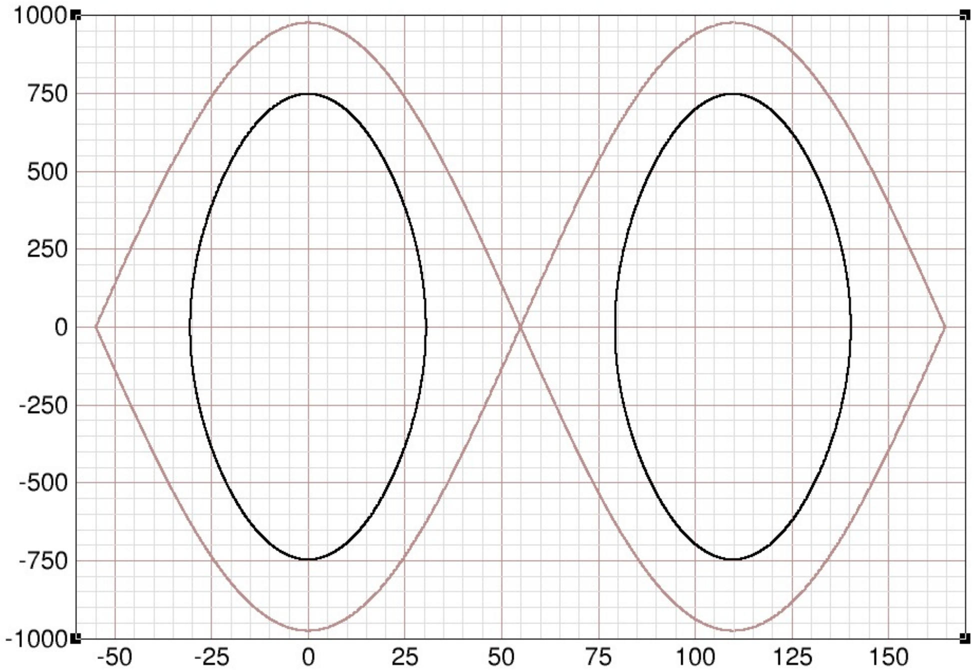

Figure 16: We consider now incoming bunches with a longitudinal emittance of 0.35 eV-s per nucleon. Here the black curves are matched to the buckets and each enclose an area of 0.35 eV-s per nucleon. The width of the enclosed matched bunch is 60.98 ns. This leaves $109.83 - 60.98 =$ 48.85 ns for the injection kicker field to rise. As before, the RHIC RF voltage is 180 KV per turn. Harmonic number is 120. Energy is 3.85 GeV per nucleon. The horizontal axis gives the time deviation in ns. The vertical axis gives the energy deviation in MeV. (Divide by 197 to get the energy deviation in MeV per nucleon.)

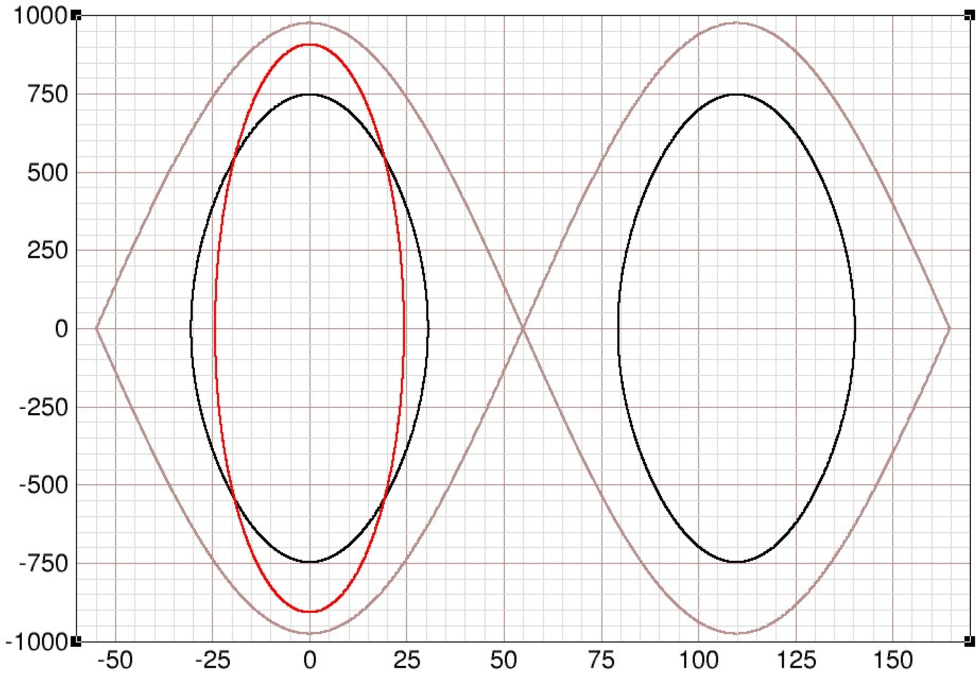

Figure 17: Here the red curve in the left bucket is the boundary of an incoming mismatched bunch from AGS. It encloses an area of 0.35 eV-s per nucleon, just like the black curves. The width of the mismatched bunch is 48.59 ns.

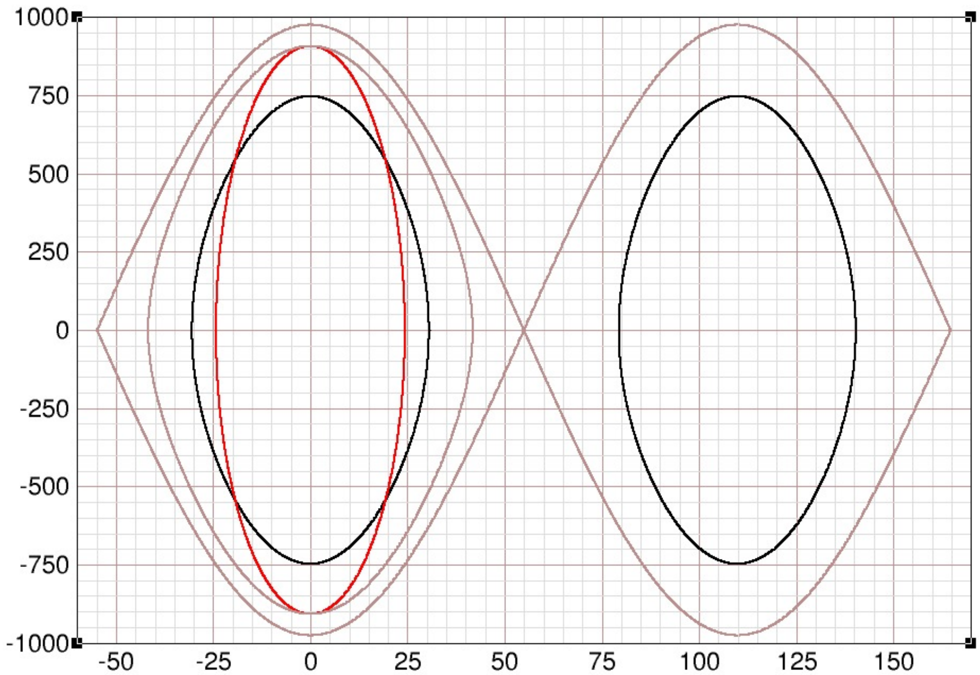

Figure 18: Here the brown curve in the left bucket is the boundary of the filamented bunch. It encloses an area of 0.556 eV-s per nucleon. The width of the filamented bunch is 83.49 ns.

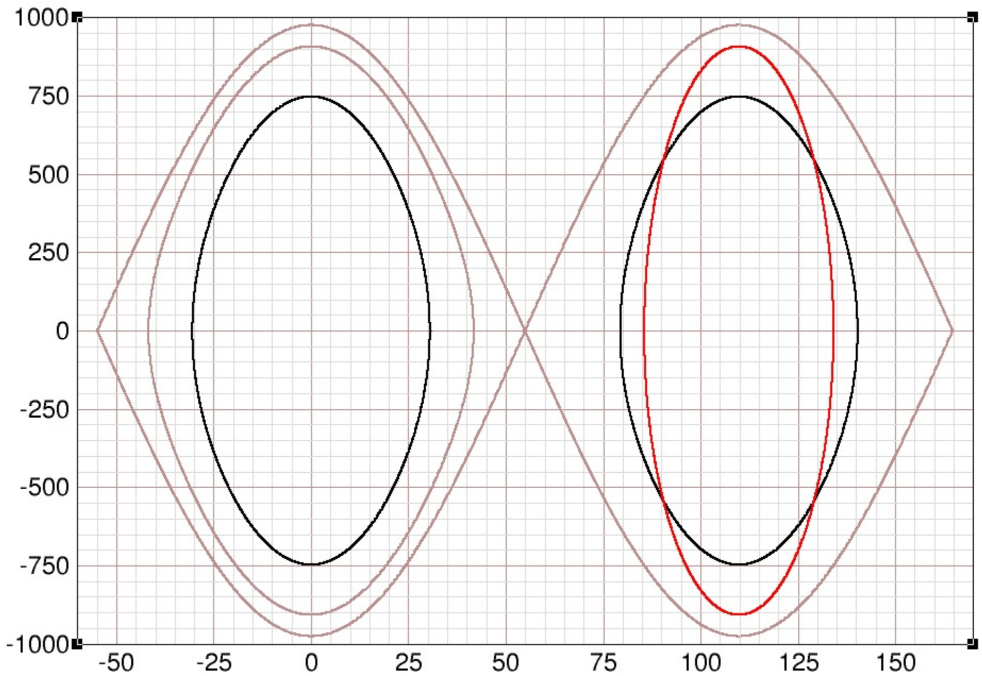

Figure 19: As in the previous figure, the brown curve in the left bucket is the boundary of the filamented bunch. The red curve in the right bucket is the boundary of the incoming mismatched bunch from AGS. The area enclosed by the brown curve is 0.556 eV-s per nucleon. The area enclosed by the red curve is 0.35 eV-s per nucleon.

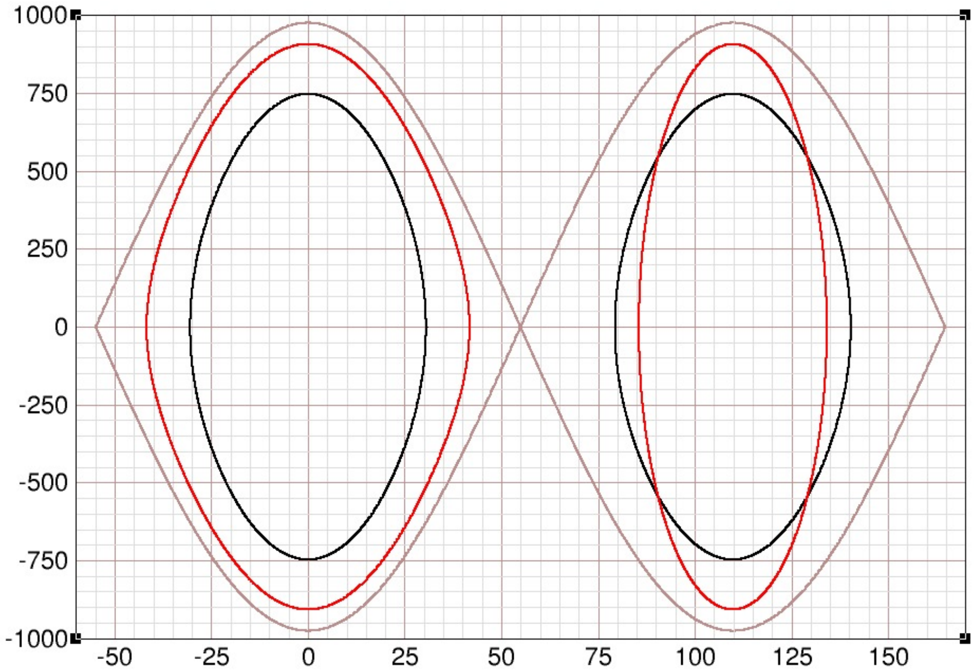

Figure 20: Same as previous figure, but with filamented bunch now outlined in red. Here we see that the distance between the filamented bunch and the incoming mismatched bunch is significantly less than the distance between the two matched bunches, thereby reducing the time available for the injection kicker field to rise.